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An Introduction to Number Theory:

Prime Numbers and Their Applications

A thesis

presented to

the faculty of the Department of Mathematics

East Tennessee State University

In partial fulfillment

of the requirements for the degree

Master of Science in Mathematical Science

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by

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August 2006

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Dr. Teresa Haynes

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Keywords: Number Theory, Prime Numbers, Greatest Common Factor,

Least Common Multiple, Common Denominators

ABSTRACT

An Introduction to Number Theory:

Prime Numbers and Their Applications

by

Crystal Anderson

The author has found, during her experience teaching students on the fourth grade level, that some concepts of number theory haven't even been introduced to the students. Some of these concepts include prime and composite numbers and their applications. Through personal research, the author has found that prime numbers are vital to the understanding of the grade level curriculum. Prime numbers are used to aide in determining divisibility, finding greatest common factors, least common multiples, and common denominators. Through experimentation, classroom examples, and homework, the author has introduced students to prime numbers and their applications.

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DEDICATION

This thesis is dedicated to the many people who have supported me throughout my educational journey. First and foremost, I want to think my heavenly Father for being my guide and my strength through it all. To my daughter, Alyssa Anderson, I don't know what I would've done without your smiling face, your prayers, and your sacrifice of time. To my mother, thank you for encouragement, support, sacrifice of time, and most importantly your prayers. I couldn't have done it without you. To all the other people who have prayed and supported me throughout the way, I want to say thank you.

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1 INTRODUCTION

1.1 An Introduction to Number Theory

Number Theory is a captivating and measureless field of mathematics. It is sometimes referred to as the "higher arithmetic," related to the properties of whole numbers [2]. The famous German mathematician Karl Friedrich Gauss once said that the complex study of numbers "is just this which gives the higher arithmetic that magical charm which has made it the favorite science of the greatest mathematicians, not to mention its inexhaustible wealth, wherein it so greatly surpasses other parts of mathematics [2]." He also stated that mathematics is the "queen of the sciences [2]." Number Theory can be subdivided into several categories. Some of these categories include: prime and composite numbers, greatest common factors, and least common multiples.

1.2 The Choice

The author has found that students experience difficulty with the concept of prime numbers, divisibility, and fractions. The author has discovered, while talking with other colleagues, that the concept of prime numbers has been overlooked. The majority of students at grade level four haven't even heard of the word "prime." Therefore, neither have they been introduced to prime applications. So with this in mind, the author chose to create a unit on primes and their applications in order to help students with division and fractions. We will begin the study of a few subcategories of number theory by looking at divisibility. Another important category, prime numbers and composite numbers, will be looked at in the next section followed by a look at applications of prime numbers.

2 DIVISIBILITY

2.1 Introduction

A key concept of number theory is divisibility. Being able to determine divisibility will help in advanced division, determining the greatest common factor and the least common multiple, as well as adding and subtracting fractions with unlike denominators and finding the simplest form of a fraction. In simple words, we say that a natural number, n, is divisible by another number, d, if the first number, n, divided by the second number, d, leaves a remainder of 0 or equivalently, n is divisible by d if there is a natural number k such that n = d x k [1]. For example, 4 divides 24 because 24 = 4 x 6. The natural number here is 6. On the other hand, 23 divided by 6 gives the quotient 3 with a remainder of 5. Therefore, divisibility doesn't hold true. That is, 23 = 6 x k has no solution in the natural numbers.

When one is trying to determine divisibility, it remains "simple" with small numbers, such as the ones mentioned above. On the other hand, it becomes more difficult when trying to decide if a larger number, such as 653, is divisible by a smaller natural number. In this case, we need a method to determine divisibility. Over the years, mathematicians have provided us with divisibility tests to help us work with larger numbers [4].

Divisibility Test	Example
A number is divisible by 2 if the last digit is an even number. (0, 2, 4, 6, or 8)	174 is divisible by 2 since the last digit, 4, is an even number.
A number is divisible by 3 if the sum of the number's digits is divisible by 3.	369 is divisible by 3, because 3+6+9=18, and 18 is divisible by 3.
A number is divisible by 4 if the last two digits of a number are divisible by 4.	224 is divisible by 4, because the number formed by the last two digits, 24 is divisible by 4.
A number is divisible by 5 if the last digit is a 0 or a 5.	255 is divisible by 5, because the last digit is 5.
A number is divisible by 6 if it is divisible by both 2 and 3.	246 is divisible by 6 because the last digit is even making it divisible by 2 and the sum of the digits is 12, which is divisible by 3.
A number is divisible by 7 if you double the one's digit and subtract it from the remaining number, and the resulting number is divisible by 7.	245 is divisible by 7 because if we double the last number, 5, it becomes 10. Then we subtract 10 from the remaining two numbers, 24, and get 14. 14 is divisible by 7.
A number is divisible by 8 if the number formed by the last 3 digits is divisible by 8.	5024 is divisible by 8 because 024 is divisible by 8.
A number is divisible by 9 if the sum of the digits is divisible by 9.	9315 is divisible by 9 because 9+3+1+5=18, which is divisible by 9.
A number is divisible by 10 if the last digit is 0.	12050 is divisible by 10 because the last digit is 0.
A number is divisible by 11, if we start with the ones place and add every other number and subtract that number from the sum of the remaining numbers and the resulting number is divisible by 11. This difference may be positive, negative, or zero.	824472 is divisible by 11. If we start with the number in the ones digit and add every other number (2+4+2=8) and add every other remaining number (7+4+8=19) and find the difference of the two (19-8=11), we find that 11 is divisible by 11.
A number is divisible by 12 if the last two numbers form a number divisible by 4 and the sum of the numbers form a number divisible by 3.	7824 is divisible by 12, because 24 is divisible by 4 and 7+8+2+4=21, which is divisible by 3.

2.2 Divisibility Arguments

The divisibility tests are all based on the fact that our system of numerals is written in base 10. In the following arguments, assume that X is an integer and X is written in ***expanded form***:

$$X = C_k \ldots C_3C_2C_1C_0 = C_k\cdot 10^k + \ldots + C_3\cdot 10^3 + C_2\cdot 10^2 + C_1\cdot 10 + C_0$$

These two expressions stand for the digit representation and the place value expanded form representation that describe the digit representation. For example, in the number 456,789, 4 is C_5, 5 is C_4, 6 is C_3, 7 is C_2, 8 is C_1, and 9 is C_0. $456{,}786 = 4 \times 10^5 + 5 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 8 \times 10^1 + 6$. Note that the coefficients C_i are integers between zero and 9 ($0 \le C_i \le 9$).

Divisibility by 2: Since 2 divides 10, 2 divides each term in the expanded form to the left of the units' digit, C_0. Therefore, 2 will divide T if and only if 2 divides the one's digit, C_0.

Divisibility by 3: First, note that each power of 10^t , where $t > 0$, $z_t = 10^t - 1$ is divisible by 9. For example, $z_1 = 10 - 1 = 9$, $z_2 = 10^2 - 1 = 99$, $z_3 = 10^3 - 1 = 999$, and so on. Since $z_k = 10^k - 1$, $10^k = z_k + 1$. Therefore, for any $k > 0$, $10^k = z_k + 1$. Second, note that

$$\begin{aligned} X &= C_k\cdot 10^k + \ldots + C_3\cdot 10^3 + C_2\cdot 10^2 + C_1\cdot 10 + C_0 \\ &= C_k\cdot(z_k + 1) + \ldots + C_3\cdot(z_3 + 1) + C_2\cdot(z_2 + 1) + C_1\cdot(z_1 + 1) + C_0 \\ &= (C_k\cdot z_k + C_k) + \ldots + (C_3\cdot z_3 + C_3) + (C_2\cdot z_2 + C_2) + (C_1\cdot z_1 + C_1) + C_0 \\ &= [C_k\cdot z_k + \ldots + C_3\cdot z_3 + C_2\cdot z_2 + C_1\cdot z_1] + [C_k + \ldots + C_3 + C_2 + C_1 + C_0] \end{aligned}$$

The first bracketed expression is divisible by 3 since each z_k is divisible by 3. So X can be divisible by 3 only when the second bracketed expression is divisible by 3, namely the sum of the digits of X.

Divisibility by 4: Since 10^k is divisible by 4 for all $k > 1$, the expression for T, namely

$$T = C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0 \ ,$$

is divisible by 4 whenever the last two terms are divisible by 4. That is, 4 must divide the number $C_1C_0 = C_1 \cdot 10 + C_0$.

<u>Divisibility by 5</u>: Since 10^k is divisible by 5 for all $k > 0$, the expression for T, namely

$$T = C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0 \ ,$$

is divisible by 5 whenever the last term is divisible by 5. That is, 5 must divide the digit C_0 .

<u>Divisibility by 6</u>: Since 6 = 2 x 3, T is divisible by 6 only when T is divisible by both 2 and 3.

<u>Divisibility by 7</u>: Consider the number Y obtained from T by deleting the units' digit, C_0, and then subtracting the double of C_0 from it. That is,

$$Y = C_k \ldots C_3C_2C_1 - 2C_0 = C_k \cdot 10^{k-1} + \ldots + C_3 \cdot 10^2 + C_2 \cdot 10 + C_1 - 2C_0$$

If Y is divisible by 7, then

$$Y = C_k \cdot 10^{k-1} + \ldots + C_3 \cdot 10^2 + C_2 \cdot 10 + C_1 - 2C_0 = 7w \ .$$

Therefore,

$$C_k \cdot 10^{k-1} + \ldots + C_3 \cdot 10^2 + C_2 \cdot 10 + C_1 = 7w + 2C_0$$

or, multiplying both sides by 10,

$$C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 = 70w + 20C_0 \ .$$

Now add C_0 to both sides to obtain

$$T = C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0 = 70w + 21C_0 \ .$$

Since the right side is clearly divisible by 7, so is the left side.

<u>Divisibility by 8</u>: Since 10^t is divisible by 8 for all $t > 2$, the expression for T, namely

$$T = C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0 \ ,$$

is divisible by 8 whenever the last three terms are divisible by 8. That is, 8 must divide the number $C_2C_1C_0 = C_2 \cdot 10^2 + C_1 \cdot 10 + C_0$.

Divisibility by 9: Refer to divisibility by 3 argument. Replace 3 by 9, and one can clearly see that T is divisible by 9 only when the sum of the digits is divisible by 9.

Divisibility by 10: Since 10^k is divisible by 10 for all k > 0, the expression for T, namely

$$T = C_k \cdot 10^k + \ldots + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0 \ ,$$

is divisible by 10 whenever the last term is zero. That is, 10 divides T only when the units' digit, C_0, is zero.

Divisibility by 11: This test is based on the observation that $z_{2t} = 10^{2t} - 1$ is divisible by 11 for each t > 0 and $z_{(2t+1)} = 10^{(2t+1)} + 1$ is divisible by 11 for each t > 0. For example, if t = 2, then $z_4 = 10^4 - 1 = 10{,}000 - 1 = 9999$ which is divisible by 11. And $z_5 = 10^5 + 1 = 100{,}000 + 1 = 100001$ which is divisible by 11. As an illustration, consider the five-digit number given by

$$T = C_4C_3C_2C_1C_0 = C_4 \cdot 10^4 + C_3 \cdot 10^3 + C_2 \cdot 10^2 + C_1 \cdot 10 + C_0$$

$$= C_4 \cdot [(10^4 - 1) + 1] + C_3 \cdot [(10^3 + 1) - 1] + C_2 \cdot [(10^2 - 1) + 1] + C_1 \cdot [(10 + 1) - 1] + C_0$$

$$= C_4 \cdot (10^4 - 1) + C_4 + C_3 \cdot (10^3 + 1) - C_3 + C_2 \cdot (10^2 - 1) + C_2 + C_1 \cdot (10 + 1) - C_1 + C_0$$

$$= [C_4 \cdot (10^4 - 1) + C_3 \cdot (10^3 + 1) + C_2 \cdot (10^2 - 1) + C_1 \cdot (10 + 1)]$$

$$+ [(C_4 + C_2 + C_0) - (C_3 + C_1)]$$

The expression inside the pair of brackets on the left is divisible by 11 since each term is divisible by 11. Consequently, T can be divisible by 11 only if the expression inside the pair of brackets on the right (bold letters) is divisible by 11. And this bold-lettered expression is just the difference between the sum of the odd digits and the sum of the even digits. This difference may be positive, negative, or zero.

Divisibility by 12: A number T is divisible by 12 if it is divisible by both 3 and 4. Refer to the divisibility tests for 3 and 4.

Teaching students that these are the rules for divisibility and expecting them to understand them and apply them would be a "perfect scenario" for a teacher. Unfortunately, throughout the author's years of experience, this scenario only exists in a perfect classroom. On the other hand, teaching students the rules and showing them how the rules work seems to be a more effective way of helping students understand divisibility rules. This could be shown by a hands-on activity. Provide students with manipulatives that they could count and put in groups. Allow them to begin to work out division problems. If patterns are not being found, the teacher could provide problems that would be easy to determine patterns. If students are allowed to find a pattern on their own, it will be a lot easier for them to retain this skill for further use.

Obviously, it would be difficult to allow a student to check divisibility with a large number, but if they understand the concept of divisibility with smaller numbers they will be more likely to transfer this information to work with larger numbers. Provide each student with connecting cubes, or any other manipulative that can be connected and counted.

Start with a number such as eighteen. Explain to students that they will be illustrating how the divisibility rules work. Have students connect eighteen cubes. Explain to them that they will be determining if each divisibility rule is true. Show students how to equally divide the cubes into different size groups. Remind them that groups must be equal.

Start with eighteen cubes. Let students determine if eighteen is divisible by two. Instruct students to divide the cubes into equal groups of two.

□ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □ □

Students will be able to recognize that eighteen is divisible by two. Let them check their divisibility rules, which state that any even number is divisible by two. Let them determine if eighteen is divisible by two.

Now, let's check to see if eighteen is divisible by 3. Let students predict the outcome by looking at the divisibility rules.

□ □ □   □ □ □   □ □ □   □ □ □   □ □ □   □ □ □

Students will be able to recognize that eighteen is divisible by three. Let students check to see if their predictions are correct.

Continue student prompted questions and answers to determine divisibility. Let's proceed to check divisibility of eighteen by four.

□ □ □ □   □ □ □ □   □ □ □ □   □ □ □ □   □ □

Allow student responses to this outcome. Is eighteen divisible by four? Why or why not? Have students continue checking divisibility rules with this procedure. As they work, allow them to fill in a chart that they can use to compare their results with their divisibility chart.

Have students check divisibility with other numbers such as 25, 16, 42, and 39. After you have proved that the divisibility rules work, allow students to put their knowledge to work by completing some problems without manipulatives.

2.3 Divisibility Charts for In Class Examples

Divisibility of 18

2	3	4	5	6	7	8	9	10	11	12

Divisibility of 25

2	3	4	5	6	7	8	9	10	11	12

Divisibility of 16

2	3	4	5	6	7	8	9	10	11	12

Divisibility of 42

2	3	4	5	6	7	8	9	10	11	12

Divisibility of 39

2	3	4	5	6	7	8	9	10	11	12

2.4 Answer Key for Divisibility Charts

Divisibility of 18

2	3	4	5	6	7	8	9	10	11	12
Yes	Yes	No	No	Yes	No	No	Yes	No	No	No

Divisibility of 25

2	3	4	5	6	7	8	9	10	11	12
No	No	No	Yes	No	No	No	No	No	No	No

Divisibility of 16

2	3	4	5	6	7	8	9	10	11	12
Yes	No	Yes	No	No	No	Yes	No	No	No	No

Divisibility of 42

2	3	4	5	6	7	8	9	10	11	12
Yes	Yes	No	No	Yes	Yes	No	No	No	No	No

Divisibility of 39

2	3	4	5	6	7	8	9	10	11	12
No	Yes	No	No	No	No	No	No	No	No	No

2.5 In Class Examples

1. Use divisibility test to determine whether 150 is divisible by 2, 3, 4, 5, 6, 9, and 10.

2. Use divisibility tests to determine whether 163 is divisible by 2, 3, 4, 5, 6, 9, and 10.

3. Use divisibility tests to determine whether 224 is divisible by 2, 3, 4, 5, 6, 9, and 10.

4. Use divisibility tests to determine whether 7,168 is divisible by 2, 3, 4, 5, 6, 9, and 10.

5. Use divisibility tests to determine whether 6, 679 is divisible by 2, 3, 4, 5, 6, 9, and 10.

2.6 Divisibility Homework

Use the following information to determine what color each part of the clown needs to be.

1. Use the divisibility tests to determine whether 9,042 is divisible by 2, 3, 4, 5, and 6. If this number is divisible by all of these numbers, color the clown's hair orange. If this number isn't divisible by all of these numbers, color the clown's hair red.

2. Use the divisibility tests to determine whether 35,120 is divisible by 2, 3, 5, 6, and 9. If this number is divisible by all of these numbers, color the clown's hat blue. If this number isn't divisible by all of these numbers, color the clown's hat orange.

3. Use the divisibility tests to determine whether 477 is divisible by 3, 6, and 9. If this number is divisible by all of these numbers, color the clown's flower brown. If this number isn't divisible by all of these numbers, color the clown's flower yellow.

4. If a number is divisible by 9, it is also divisible by 3. If this statement is true, color the clown's bowtie red. If this statement is false, color the clown's bowtie black.

5. If the ones place of a number is an even number, it is always divisible by 2. If this statement is true color the clown's lips pink. If this is not true, color the clown's lips red.

6. If a number ends with a 0 it is not divisible by 5. If this statement is true color the clown's cheeks red. If this statement is false color the clown's cheeks purple.

7. Finish coloring the clown's face.

Figure 1:
Clown Divisibility
Homework

2.7 Divisibility Homework

Answer Key

In Class Examples

1. Use divisibility test to determine whether 150 is divisible by 2, 3, 4, 5, 6, 9, and 10.

- The ones digit is 0, an even number, therefore 150 is divisible by 2.
- The sum of the digits 1, 5, and 0 is 6, a number divisible by 3, therefore 150 is divisible by 3.
- The last two digits, 50, is not divisible by 4, therefore 150 isn't divisible by 4.
- The ones digits is 0, therefore 150 is divisible by 5.
- 150 is divisible by 2 and 3, therefore it is also divisible by 6.
- The sum of the digits 1, 5, and 0 is 6, a number not divisible by 9, therefore 150 isn't divisible by 9.
- The ones digit is 0, therefore 150 is divisible by 10.

2. Use divisibility test to determine whether 163 is divisible by 2, 3, 4, 5, 6, 9, and 10.

- The ones digit is 3, an odd number, therefore 163 isn't divisible by 2.
- The sum of the digits 1, 6, and 3 is 10, a number not divisible by 3, therefore 163 isn't divisible by 3.
- The last two digits, 63, are not divisible by 4, therefore, 163 isn't divisible by 4.
- The number in the ones place isn't 0 or 5, therefore 163 isn't divisible by 5.
- The number 163, isn't divisible by both 2 and 3, therefore 163 isn't divisible by 6.
- The sum of the digits 1, 6, and 3, is 10, a number not divisible by 9, therefore, 163 isn't divisible by 9.
- The number in the ones place isn't a zero, therefore 163 isn't divisible by 10.

3. Use divisibility tests to determine whether 224 is divisible by 2, 3, 4, 5, 6, 9, and 10.

- The ones digit is 4, an even number, therefore 224 is divisible by 2.
- The sum of the digits 2, 2, and 4, is 8, a number not divisible by 3, therefore 224 isn't divisible by 3.
- The last two digits, 24, is a number divisible by 4, therefore 224 is divisible by 4.

- The ones digit isn't a 0 or a 5, therefore 224 isn't divisible by 5.
- The number 224, isn't divisible by both 2 and 3, therefore 224 isn't divisible by 6.
- The sum of the digits, 2, 2, and 4, is 8, a number not divisible by 9, therefore 224 isn't divisible by 9.
- The ones digit isn't 0, therefore 224 isn't divisible by 10.

4. Use divisibility tests to determine whether 7, 168 is divisible by 2, 3, 4, 5, 6, 9, and 10.
    - The number in the ones digit, 8, is even, therefore 7,168 is divisible by 2.
    - The sum of the digits, 7, 1, 6, and 8 is 22, a number not divisible by 3, therefore 7,168 isn't divisible by 3.
    - The last two digits, 68, is a number not divisible by 4, therefore 7,168 isn't divisible by 4.
    - The number in the ones place isn't a 0 or a 5, therefore 7,168 isn't divisible by 5.
    - The number isn't divisible by both 2 and 3, therefore 7,168 isn't divisible by 6.
    - The sum of the digits, 7, 1, 6, and 8 is 22, a number not divisible by 9, therefore 7,168 isn't divisible by 9.
    - The number in the ones place isn't 0, therefore 7,168 isn't divisible by 10.

5. Use divisibility tests to determine whether 5,253 is divisible by 2, 3, 4, 5, 6, 9, and 10.
    - The number in the ones place, 3, is not an even number, therefore 5,253 isn't divisible by 2.
    - The sum of the digits, 5, 2, 5, and 3 is 15, a number divisible by 3, therefore 5, 253 is divisible by 3.
    - The last two digits, 53, is a number not divisible by 4, therefore 5, 253 isn't divisible by 4.
    - The ones digit isn't 0 or 5, therefore 5, 253 isn't divisible by 5.
    - The number isn't divisible by both 2 and 3, therefore 5, 253 isn't divisible by 6.
    - The sum of the digits, 5, 2, 5, and 3 is 15, a number not divisible by 9, therefore 5,253 isn't divisible by 9.
    - The ones digit doesn't contain a 0, therefore 5, 253 isn't divisible by 10.

Answer Key (continued)

- Divisibility Homework

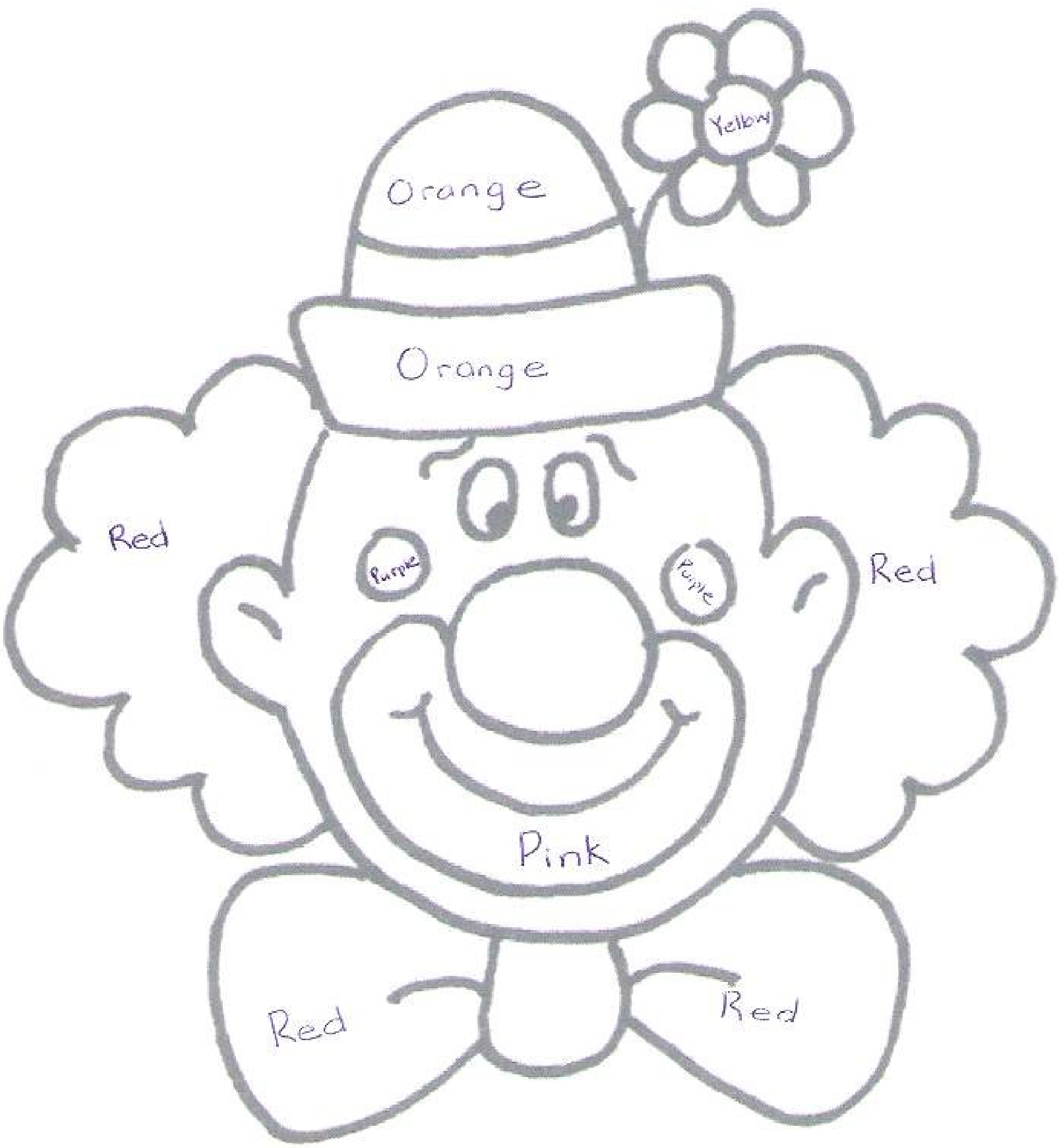


Figure 2:
Clown Divisibility Homework
Answer Key

3 PRIMES

3.1 Introduction

In order to understand the subject of prime numbers, we first have to understand a few vocabulary words. A natural number includes any counting number 1, 2, 3, ... A prime is any natural number greater than one that has no divisors other than itself and one. For example, 5 is a prime because it can't be divided by any other number than itself and one. On the other hand, a composite number includes any natural number greater than one that is not prime [1]. For example, 42 is a composite number because 2, 6, and 7 are all divisors of this number. Therefore it does have divisors other than itself and one.

To insure that the concepts of primes and composites have been mastered, we need to provide a few problems to test the students.

First, let's find all of the primes between one and one hundred by experimentation. Review the concept of prime and composite numbers. Then give each student a bag of one hundred countable items and a chart on which they can record their results. Let each student begin his or her experimentation of finding all of the prime numbers between one and one hundred. Remind each student that he/she will be working with numbers starting with one and continuing in order to one hundred to determine if they can be divided into equal groups. If they can divide the counters into an equal group other than one in each group, then they will know that that number isn't a prime number. Let's begin with one counter. Allow students to determine if 1 is a prime or composite number. Continue on by working with two counters. The students will be able to make two even groups of one. They should realize that two is the first prime number. After students record their answers in the chart, they will begin working with three counters. Students should realize that they can not divide the counters into even groups other than one in each group, making three a prime number. Allow students to continue in this manner until they have found all of the prime numbers between one and one hundred. Then, have students compare their results with the class.

3.2 In Class Example

Prime Number Experiment

Record your findings in the following chart. If the number is prime, write a "P" in the provided space, and if the number is composite, write a "C" in the provided space.

Remember: Prime numbers are natural numbers greater than one that can't be divided by any number other than itself and one.

Composite numbers are natural numbers greater than one that are not prime.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Explain why or why not the following numbers are prime or composite numbers.

1. 13

2. 23

3. 49

4. 71

5. 970

6. 9996

7. 67895

8. 100259874

9. Why is the number 1 not a prime and not a composite?

10. Why are the numbers 2 and 3 the only pair of primes that are next to each other?

3.3 Prime Homework

Prime Number Experiment

Record your findings in the following chart. If the number is prime, write a "P" in the provided space, and if the number is composite, write a "C" in the provided space.

Remember: Prime numbers are natural numbers greater than one that can't be divided by any number other than itself and one.

Composite numbers are natural numbers greater than one that are not prime.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100

Label each of the following as prime or composite. Explain your answer.

1. 89

2. 120

3. 456

4. 1003456

5. 830213

6. 1000230

7. 20056575

8. 8888082

9. 1520356

10. 10005628978

3.4 Answer Key

In Class Examples

Prime Number Experiment

Record your findings in the following chart. If the number is prime, write a "P" in the provided space, and if the number is composite, write a "C" in the provided space.

Remember: Prime numbers are natural numbers greater than one that can't be divided by any number other than itself and one.

Composite numbers are natural numbers greater than one that are not prime.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
x	P	P	C	P	C	P	C	C	C	P	C	P	C	C	C	P	C	P	C

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
C	C	P	C	C	C	C	C	P	C	P	C	C	C	C	C	P	C	C	C

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
P	C	P	C	C	C	P	C	C	C	C	C	P	C	C	C	C	C	P	C

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
P	C	C	C	C	C	P	C	C	C	P	C	P	C	C	C	C	C	P	C

81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
C	C	P	C	C	C	C	C	P	C	C	C	C	C	C	C	P	C	C	C

Explain why or why not the following numbers are prime or composite numbers.

1. 13- Prime. This number is only divisible by 1 and itself.
2. 23- Prime. This number is only divisible by 1 and itself.
3. 49- Composite. The number is a multiple of 7. 49= 7 x 7
4. 71- Prime. This number is only divisible by 1 and itself.
5. 970- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
6. 67895- Composite. The number in the ones place is a 5. So, this number is divisible by other numbers than 1 and itself.
7. 9996- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
8. 100259874- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
9. One is neither a prime nor a composite, by definition.
10. The set of counting numbers are of the form 1, 2, 3, 4, 5, 2k, 2k + 1 … (for all k > 2). Hence, any pair of adjacent numbers are either 2k, 2k + 1 or 2k + 1, 2k + 2. In either case, one of the numbers is divisible by 2. Therefore, if the numbers are not 2 and 3, then k > 1. If the pair of numbers is 2k and 2k + 1, then 1, 2, and k divide 2k (not prime). If the pair of numbers are 2k + 1 and 2k + 2 = 2(k + 1), then 1, 2, and k+1 divide 2k + 2 (not prime). So 2 and 3 are the only primes adjacent to each other.

3.5 Prime Homework

Answer Key

Label each of the following as prime or composite. Explain your answer.

1. 89- Prime. This number is only divisible by 1 and itself.
2. 120- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
3. 456- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
4. 1003456- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
5. 830223- Composite. The sum of the digits in this number is 18. The sum is divisible by three, another number other than 1 or itself.
6. 1000230- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
7. 20056575- Composite. The number in the ones place is a 5. So, this number is divisible by 5, another number other than 1 and itself.
8. 8888082- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
9. 1520356- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.
10. 10005628978- Composite. The number in the ones place is an even number. So, this number is divisible by 2, a number other than 1 and itself.

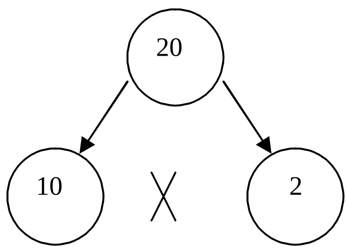
4 PRIME FACTORIZATION

4.1 Introduction

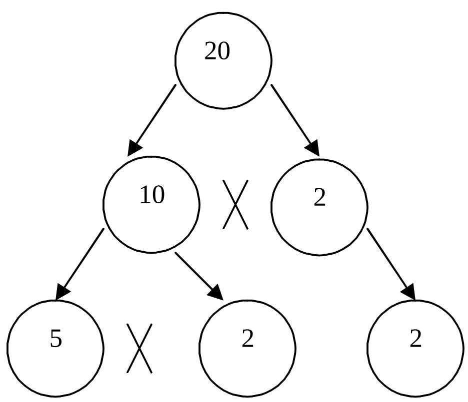
Prime numbers are often thought of as the building blocks for multiplication [1]. Every composite number is the product of primes. For example: 50=5x5x2, in which 2 and 5 are both prime numbers. The author has found that drawing a picture can often help with this concept. Therefore, we will begin by drawing factor trees to write the prime factorization of numbers.

Example 1: Find the prime factors of 20 as follows:

Step 1: Find 2 factors of 20.



Step 2: Continue factoring each number until only prime factors remain.

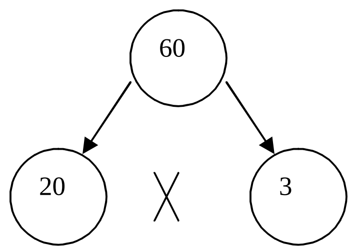


So 20 = 5 x 2 x 2 = 2 x 2 x 5

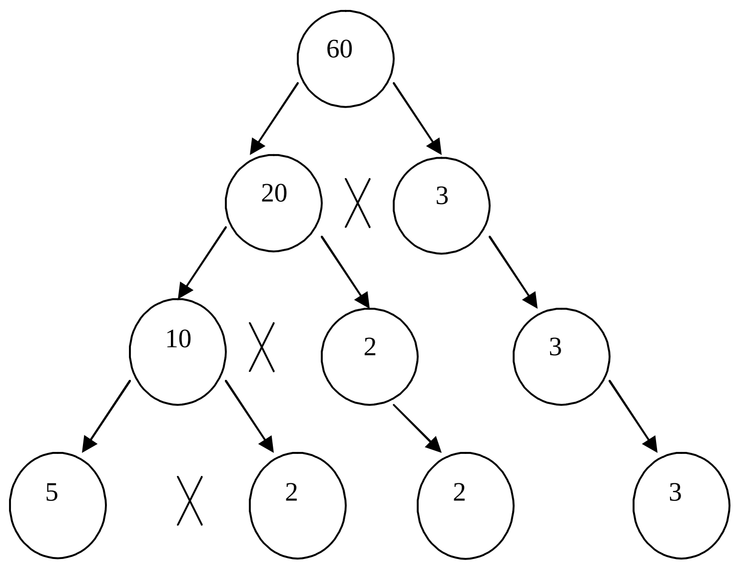
By convention, once the prime factors are found, the product of the primes is written in ascending order.

Example 2: Find the prime factors of 60.

Step 1: Find 2 factors of 60.



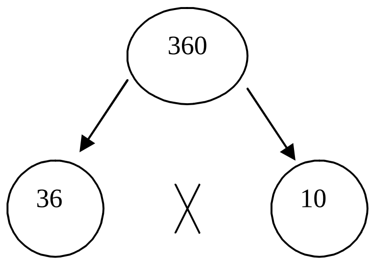
Step 2: Continue factoring each number until only prime factors remain.



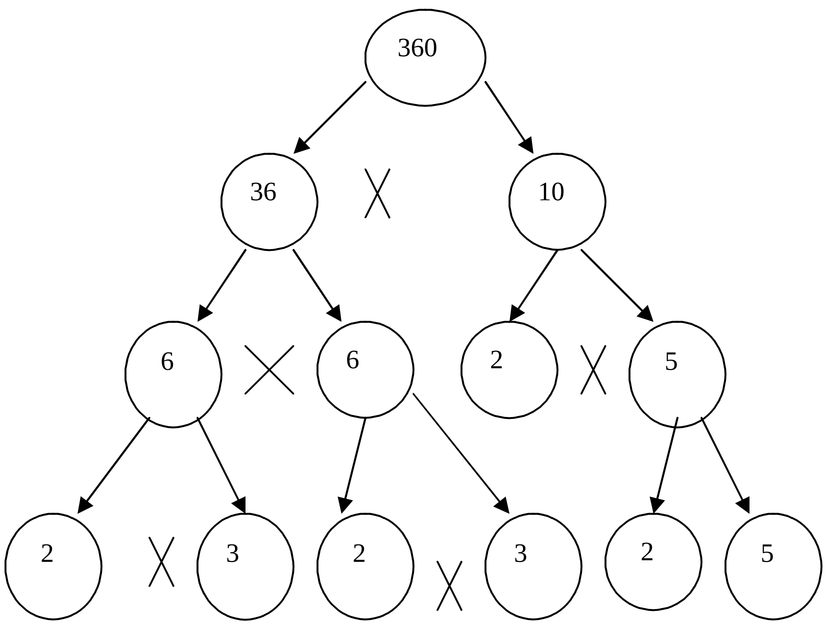
So 60 = 2 x 2 x 3 x 5

Example 3: Find the prime factors of 360:

Step 1: Find 2 factors of 360.



Step 2: Continue factoring each number until only prime factors remain.



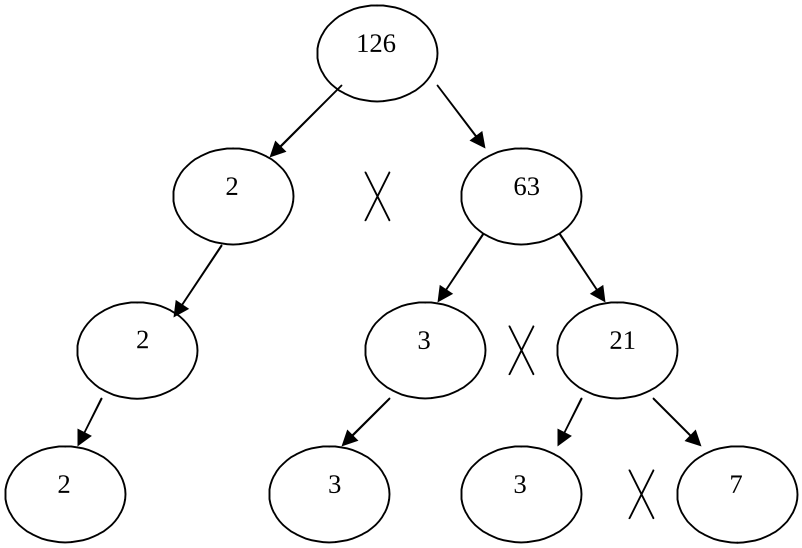
So 360= 2 x 2 x 2 x 3 x 3 x 5

During the years of 276 to 196 B.C., a well known Greek mathematician, Eratosthenes, discovered a method to identify prime numbers in a string of numbers. In any given sequence of counting numbers, starting at the beginning, mark out those divisible by two, the first prime, and every other number after that. This method allows one to quickly see all of the numbers divisible by two, therefore identifying these numbers as composites. Go back to the beginning, mark out the first number divisible by three, the second prime, and every third number after that. Marking these numbers will allow you to find all numbers divisible by three, which would make them composites. Then go back to the beginning, and mark out every number divisible by five, the third prime, and every fifth number after that. Continue to check each number in the sequence to see if they are divisible by the next primes. This will eliminate the composite numbers so that one can determine the primes in the number sequence. This process became known as the Sieve of Eratosthenes [2].

The Sieve of Erotosthenes can also be used to determine if any single number is prime or composite. For example, is 583 prime or composite? First, determine the largest prime to check divisibility by. What is the square root of 583? Since $23 \times 23 = 529 < 583$, we need only check the primes up through 23. The reason is, if 583 is the product of two primes, one bigger than 583 and one smaller than 583, then we would have found the smaller prime first (before the larger prime). So there is no need to check divisibility by primes larger than the square root of the number, in this case, the square root of 583 is approximately 24.1. This procedure and thought process is completed to show when one can stop checking primes. If we didn't complete this strategy, we could check primes for a long time. Determining the square root shows one how far he or she needs to go to finish checking primes. There is no need in checking any prime number larger than the square root because it will result in numbers larger than the original beginning number. Let's proceed with checking 583. Well, 583 is not an even number so it isn't divisible by 2. The sum of the digits 5, 8, and 3 = 16, which is not divisible by 3. It does not end in a 0 or a 5, so it's not divisible by 5. Continue this thought process using the primes 7, 11, 13, 17, 19, and 23. If this number is divisible by any of these primes the number is composite, but if it is not, it is a prime? We will discover that 583 is a prime number.

4.2 In Class Example

Draw a tree diagram to show the factors of 126.



So, 126 = 2 x 3 x 3 x 7

Which numbers between 70 and 82 are primes?

First list the numbers in the sequence.

70 71 72 73 74 75 76 77 78 79 80 81 82

Begin with the first number that is divisible by 2, marking it and every other number after it out.

Note that these numbers are marked in red.

70 71 72 73 74 75 76 77 78 79 80 81 82

Then, go back to the beginning and find the first number divisible by three, and mark it out and every third one after that. If the number is already marked, you still need to count it. These are marked in blue.

70 71 72 73 74 75 76 77 78 79 80 81 82

Now determine the first number divisible by 5 and mark it out along with every fifth number after that. These are marked in yellow.

70	71	72	73	74	75	76	77	78	79	80	81	82

Now determine the first number to be divisible by 7, and mark every 7th number out after that. These are marked in purple.

70	71	72	73	74	75	76	77	78	79	80	81	82

Now determine the first number to be divisible by 11, and mark every 11th number out after that. These are marked in green.

70	71	72	73	74	75	76	77	78	79	80	81	82

Continue on with this method until you have found all of the primes in the number sequence. How do you know when to stop? If a number *n* is not prime, then it has to be divisible by a number that is less than or equal to the square root of *n*. Since the largest number in our sequence is 82, we must determine the square root to find out the last prime that we need to check divisibility with. 7 x 7 = 42 and 11 x 11 = 121. Therefore, 7 is the largest prime that needs to be checked. In this example, after you have checked divisibility of 2, 3, 5, and 7 you will find the primes in this sequence of numbers. We have found that 71, 73, and 79 are the only primes between 70 and 80.

4.3 Prime Factorization Homework

Using the Sieve of Eratosthenes method, determine the primes in each of the number list.

1. Find the primes between 100 and 155. What is the largest prime you need to check?

2. Find the primes between 12 and 25. What is the largest prime you need to check?

3. Find the primes between 45 and 85. What is the largest prime you need to check?

4. Determine if 477 is a prime number. What is the largest prime you need to check?

5. Is 587 a prime or composite? What is the largest prime you need to check?

Draw a tree diagram to show the factorization of each of the following numbers.

6. 58

7. 120

8. 450

4.4 Prime Factorization Homework

Answers Key

1. List all of the numbers in this string. First determine with what prime number we need to stop checking divisibility. One hundred fifty-five is our largest number, so let's find the largest prime, p, such that $p^2 \leq 155$. 11 x 11= 121 and 13 x13= 169, so we can stop checking divisibility with 11.

Cross out the first to be divisible by 2 and every other number after that, using red.

100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138
139	140	141	142	143	144	145	146	147	148	149	150	151
152	153	154	155									

Cross out the first to be divisible by 3 and every 3rd number after that, using blue. Remember, some numbers can be divisible by more than one prime and that numbers color may change throughout this process.

100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138
139	140	141	142	143	144	145	146	147	148	149	150	151
152	153	154	155									

Cross out the first to be divisible by 5 and every 5th number after that, using yellow.

100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138
139	140	141	142	143	144	145	146	147	148	149	150	151
152	153	154	155									

Cross out the first number to be divisible by 7, and every 7th number after that, using green.

100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138
139	140	141	142	143	144	145	146	147	148	149	150	151
152	153	154	155									

Cross out the first number to be divisible by 11 and every 11th number after that, using purple.

100	101	102	103	104	105	106	107	108	109	110	111	112
113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138
139	140	141	142	143	144	145	146	147	148	149	150	151
152	153	154	155									

We have discovered that 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, and 151 are the primes for this string of numbers.

2. List all of the numbers is this string. First determine with what prime number we need to stop checking divisibility. Twenty-five is our largest number, so let's find the largest prime, p, such that $p^2 \leq 25$. 5 x 5= 25, so we can stop checking divisibility with 5.

Cross out the first number to be divisible by 2, and every other number after that, using red.

12 13 14 15 16 17 18 19 20 21 22 23 24 25

Cross out the first number to be divisible by 3, and every 3rd number after that, using blue.

12 13 14 15 16 17 18 19 20 21 22 23 24

25

Cross out every number divisible by 5, and every 5th number after that using yellow.

12 13 14 15 16 17 18 19 20 21 22 23 24

25

We have discovered that between the numbers 12 and 25 there are four prime numbers. (13, 17, 19, 23)

3. List all of the numbers is this string. As before, first determine with what prime number we need to stop checking divisibility. So let's find the largest prime, p, such that $p^2 \leq 85$.

7 x 7 = 49 and 11 x 11 = 121, so we can stop checking divisibility with the prime 7.

Cross out the first number to be divisible by 2 and every other number after that, using red.

45 46 47 48 49 50 51 52 53 54 55 56 57

58 59 60 61 62 63 64 65 66 67 68 69 70

71 72 73 74 75 76 77 78 79 80 81 82 83

84 85

Cross out the first number to be divisible by 3, and every 3rd number after that, using blue.

45	46	47	48	49	50	51	52	53	54	55	56	57
58	59	60	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	81	82	83
84	85											

Cross out every number divisible by 5, and every 5th number after that using yellow.

45	46	47	48	49	50	51	52	53	54	55	56	57
58	59	60	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	81	82	83
84	85											

Cross out the first number to be divisible by 7, and every 7th number after that, using green.

45	46	47	48	49	50	51	52	53	54	55	56	57
58	59	60	61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80	81	82	83
84	85											

We discover that 47, 53, 59, 61, 67, 71, 73, 79, and 83 are prime.

4. Is 477 prime or composite?

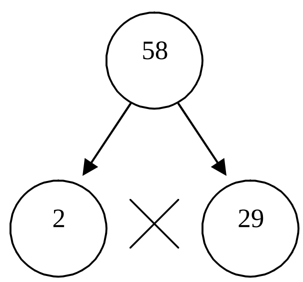
First determine with what primes we need to check divisibility. Remember, in order to do this, we must first find the square root of 477. 17 x 17 = 289 and 23 x 23 = 529, so we can stop checking divisibility with the prime 17. Let's start with the first prime, 2. The number in the ones place isn't even so it isn't divisible by 2. The sum of the digits in 477 is 18, so it is divisible by 3. Therefore, the number 477 isn't prime, but composite.

5. Is 587 prime or composite?

First determine with what primes we need to check divisibility. Remember, in order to do this, we must first find the square root of 587. Because 23 x 23 = 529 and 29 x 29 = 841, we can stop checking divisibility with the prime 23. Let's start with the first prime, 2. The number in the ones place isn't even so it isn't divisible by 2. The sum of the digits in 587 is 20, so it is not divisible by 3. The last digit isn't a 0 or a 5, so it isn't divisible by 5. If we take the last digit, 7, and double it, we get 14. Then subtract it from the rest of the number, 58, we get 44, which isn't divisible by 7. Also, it isn't divisible by 11, 13, 17, 19, or 23, so it is a prime number.

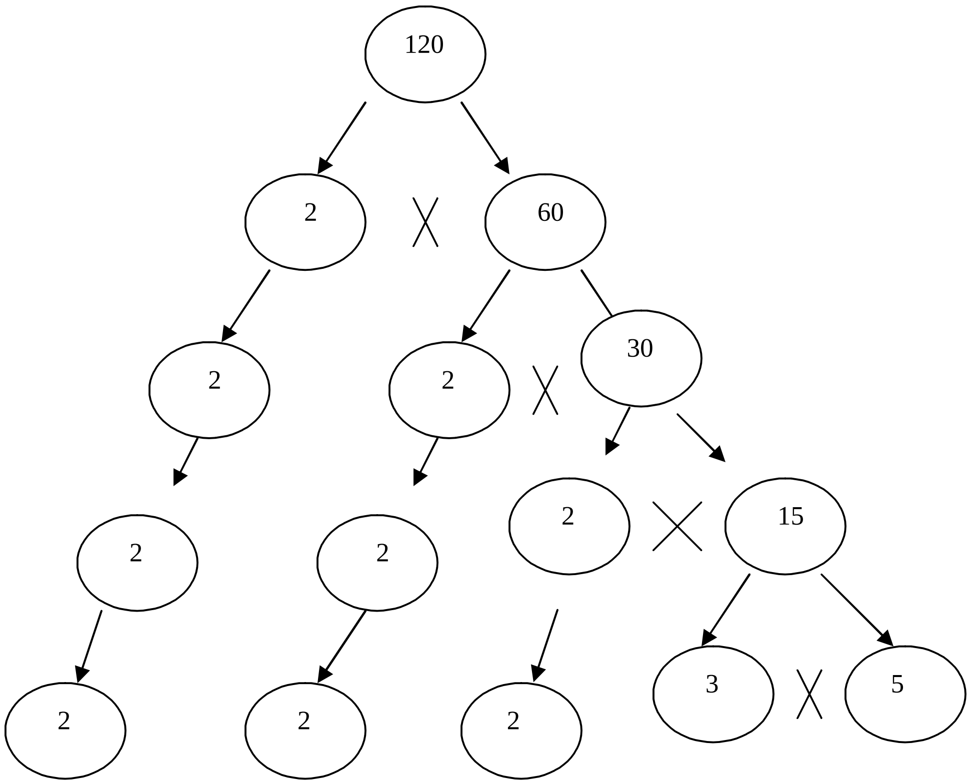
Draw a tree diagram to show the factorization of each of the following numbers.

6. 58



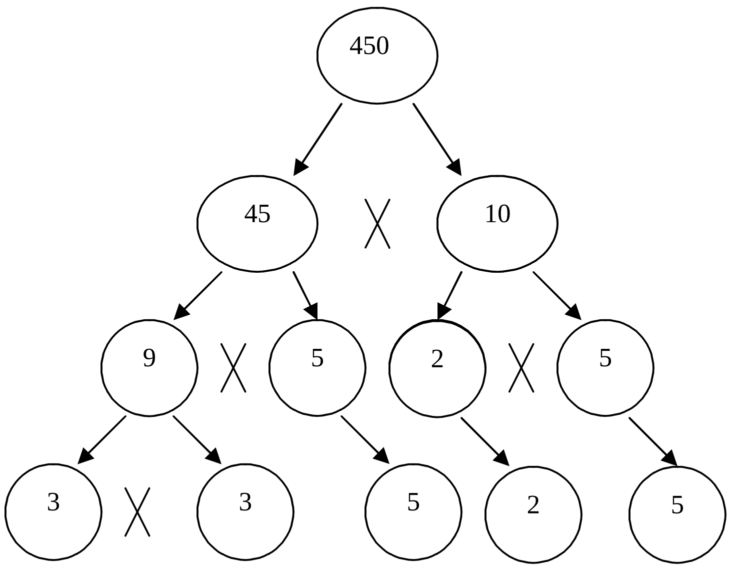
So 58= 2 x 29

7. 120



So 120= 2 x 2 x 2 x 3 x 5

8. 450



So 450= 2 x 3 x 3 x 5 x 5

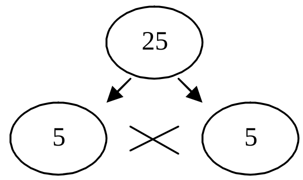
5 APPLICATIONS OF PRIMES

5.1 Greatest Common Factor

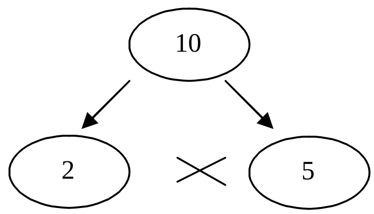
Understanding the concept of primes and divisibility can help students with many aspects of mathematics. Primes are both very useful in determining the greatest common factor and the lowest common multiple. Determining these two numbers can, in turn, help students to solve problems dealing with adding and subtracting fractions, as well as putting a fraction in lowest terms, also referred to as reducing fractions.

Multiplying two or more numbers together will result in a number, called the product. The numbers that are multiplied together are factors of the product obtained after multiplying the numbers. For example 3 x 4 = 12, therefore 3 and 4 are factors of 12. Factoring a number involves tearing it apart into smaller numbers to find all of its factors. Let's look at 24. The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24. How is this determined? Begin with 1. Every number has two factors, 1 and itself (1 x 24 = 24). Then one determines if the number is divisible by the next number in line, 2. Twenty four is an even number; therefore it is divisible by 2, and 2 x 12 = 24. Continue with the remaining counting numbers until all factors are found. In this case, 3 x 8 = 24, 4 x 6 = 24 will give us the remaining factors. Remembering the divisibility rules discussed in the previous lessons will help to determine factors of numbers.

The Greatest Common Factor (GCF) of a set of numbers, also known as the Greatest Common Divisor (GCD), is the greatest factor common to all the numbers in the set. Usually the set of interest has only two numbers in it [3]. Determining the greatest common factor can be accomplished in a couple of different ways. The prime factorization method referred to in previous lessons can be used to determine the GCF. For example, find the GCF of 10 and 25.

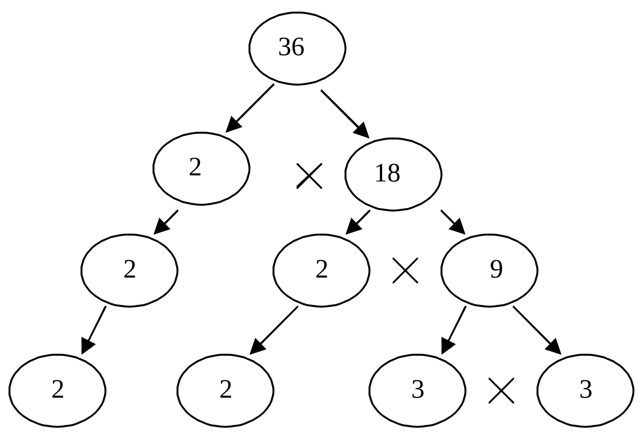


So 25 = 5 x 5

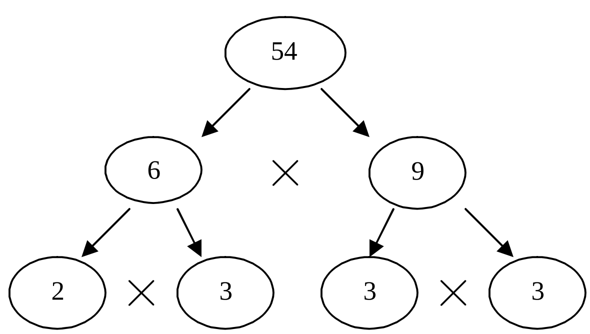


So 10 = 2 x 5

The numbers 10 and 25 have only one 5 in common, so the GCF is 5. Now let's try one that may be a little tricky. Find the GCF of 36 and 54 using prime factorization.



So 36 = 2 x 2 x 3 x 3



So 54 = 2 x 3 x 3 x 3

The numbers 54 and 36 have one 2 and two 3's in common. So when we multiply 2 x 3 x 3 we get 18, the GCF of 54 and 36.

We can also find the GCF of two or more numbers by simply listing the factors or divisors of the numbers and finding the greatest factor that each number shares. Using the same numbers that we used in the above examples find the largest number that divides each.

Example 1: Find the GCF of 5 and 10.

Factors or divisors of 5: 1, 5

Factors or divisors of 10: 1, 2, 5, 10

Each number has the common divisor of 1 and 5. Since 5 is larger than 1, 5 is the GCF of these two numbers.

Example 2: Find the GCF of 36 and 54.

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Each number has the common factor of 1, 2, 3, 6, 9, and 18. Since 18 is the largest of these numbers, it is the GCF of 36 and 54.

Although both of these methods can be used, the author has found that students often leave out factors when using the listing method and make mistakes using the prime factorization method. Therefore, we want to introduce a quicker and easier way to help students determine the GCF of a pair or group of numbers, referred to in the author's classroom as the "L method." Let's look at the following:

2	36	54
3	18	27
3	6	9
	2	3

Write down the numbers for which you are trying to determine the GCF, in this case 36 and 54. Choose <u>any</u> divisor of both of the numbers. In this case, we chose 2. Beneath the numbers that you started with, 36 and 54, write the quotient obtained after dividing the divisor into the beginning numbers, in this case 18 and 27. In this array, 36/2 = 18 and 54/2 = 27. Next, find a divisor that will divide 18 and 27. In this array 3 was chosen and is written beneath the first divisor. Now divide 18 and 27 by 3. We will get 6 and 9. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the GCF, find the product of the numbers in the left column; in this array 2 x 3 x 3 = 18. So 18 is the GCF. This method has been found, by the author, to be the most effective strategy in finding the GCF of two numbers.

The reason this technique works is that we are sequentially finding common factors of both numbers until no common factors are left. Hence, the product of these common factors is the largest among all common factors.

5.2 Greatest Common Factor Homework

Using the listing method, find the greatest common factor of the following.

1. 4, 36

2. 12, 36

Using the prime factorization method, find the greatest common factor of the following.

3. 10, 25

4. 36, 90

Using the “L method”, determine the GCF of the following numbers.

5. 60, 125

6. 180, 78

7. 96, 154

8. 126, 162

5.3 Greatest Common Factor Homework

Answer Key

Using the listing method, find the greatest common factor of the following.

1. 4, 36

Factors of 4: 1, 2, 4

Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36

Common Factors: 1, 2, 4

GCF: 4

2. 12, 36

Factors of 12: 1, 2, 3, 4, 6, 12

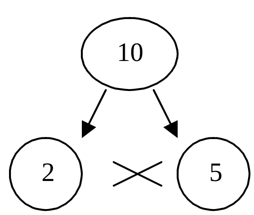
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 36

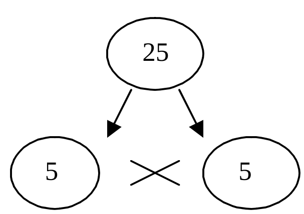
Common Factors: 1, 2, 3, 4, 6, 12

GCF: 12

Using the prime factorization method, find the greatest common factor of the following.

3. 10, 25



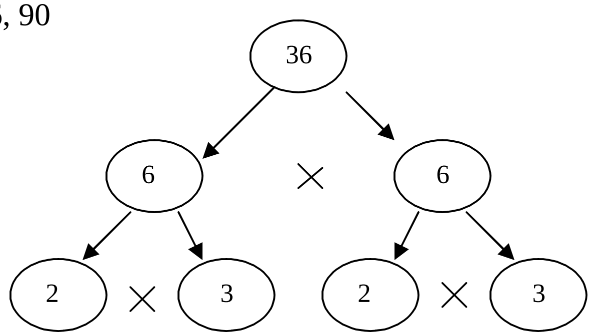


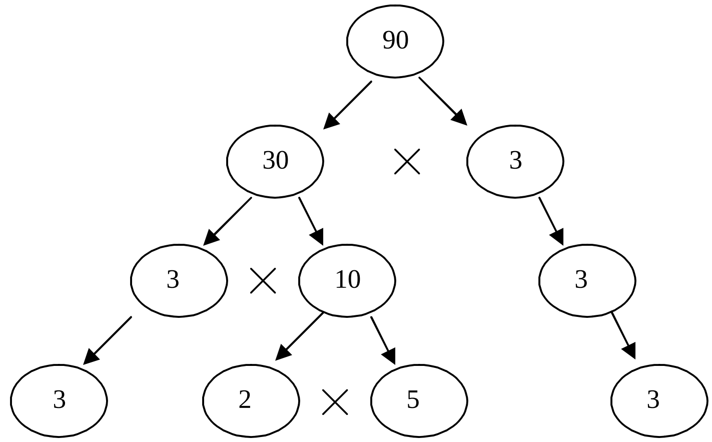
So 10= 2 x 5

So 25= 5 x 5

The numbers 10 and 25 share the factor 5, so the GCF is 5.

4. 36, 90





So 36 = 2 x 2 x 3 x 3

So 90 = 2 x 3 x 3 x 5

The numbers 36 and 90 share one 2 and two 3's. If we multiply these prime factors, 2 x 3 x 3, we get 18, the GCF of 36 and 90.

Using the "L method", determine the GCF of the following numbers.

5. 60, 125

5	60	125
	12	25

There are no common divisors of 12 and 25 other than 1. Since 5 is the only number in the left column, it is the GCF of 60 and 125.

6. 180, 78

3	180	78
2	60	26
	30	13

When we multiply the numbers in the left column, 3 x 2, and we find that the GCF of 180 and 78 is 6.

7. 96, 154

2	96	154
	48	77

2 is the only number in the left column, therefore the GCF of 96 and 154 is 2.

8. 126, 162

2	126	162
3	63	81
3	21	27
	7	9

When we multiply the numbers in the left column, 2 x 3 x 3, we find that the GCF of 126 and 162 is 18.

5.4 Least Common Multiple

Next let's take a glance at the least common multiple. A multiple is a number that one obtains when multiplying a number by another. The least common multiple (LCM) is simply the smallest multiple, other than zero, of two or more numbers. The LCM can also be referred to as the smallest number that two or more numbers can divide into evenly other than 0 [3]. For example, let's find the multiples of 3. Start by multiplying 3 by 0 and continue to multiply 3 by the remaining counting numbers. So the multiples of 3 are 0, 3, 6, 9, 12, 15, 18 …

Just as we used several methods to find the GCF of numbers, we can also use these methods to find the LCM of two or more numbers. Let's start with the simplest, the listing method.

Example 1: Find the LCM of 6 and 12.

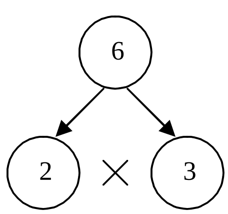
Multiples of 6: 0, 6, 12, 18, 24, 30, 36, 42 …

Multiples of 12: 0, 12, 24, 36, 48 …

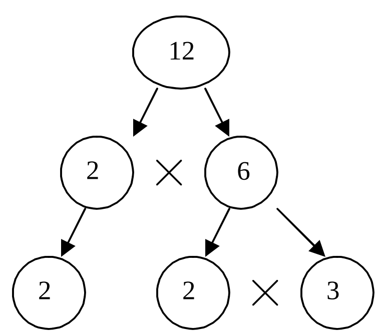
We find that the LCM of 6 and 12 is 12.

Using prime factorization can also help one determine the LCM of a number. Use the previous lessons on prime factorization during this method.

Example 1: Find the LCM of 6 and 12.



So 6 = 2 x 3



So 12 = 2 x 2 x 3

Now count the maximum number of times that each prime factor occurs in each number. The prime 2 and 3 both appear once in the factorization of 6 and the prime 2 appears twice in the factorization of 12 along with one 3. The maximum number of 2's is two and the maximum number of 3's is one. Multiply these numbers to get the LCM.

2 x 2 x 3 = 12 and this is the LCM of 6 and 12.

We can also use the "L method" to determine the LCM. Let's look at the following:

2	6	12
3	3	6
	1	2

Write down the numbers for which you are trying to determine the LCM. Choose any divisor of both of the numbers. Beneath the numbers that you started with, 6 and12, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 6/2 = 3 and 12/2 = 6. Next, find a divisor that will divide 3 and 6. In this array 3 was chosen and is written beneath the first divisor. Now divide 3 and 6 by 3. We will get 1 and 2. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the LCM, find the product of the numbers in the left column and the numbers in the lowest row, which forms an "L". In this array 2 x 3 x 1 x 2 = 12, so 12 is the LCM. Although using this method to find the GCF has been found to be the most effective in the author's classroom, it is sometimes confusing to find the LCM using this strategy.

5.5 Least Common Multiple

Homework

Use the “listing method” to find the LCM for each of the following numbers.

1. 3, 4, 6

2. 2, 3, 18

3. 5, 8, 10

4. 60, 126

Use the prime factorization method to find the LCM of the following numbers.

5. 4, 6, 8

6. 12, 18

Use the “L method” to find the LCM of the following numbers.

7. 60, 126

7. 12, 36

8. 24, 48

9. 4, 8

10. 18, 42

5.6 Least Common Multiple

Answers to Homework

Use the "listing method" to find the LCM for each of the following numbers.

1. 3, 4, 6

   Multiples of 3: 0, 3, 6, 9, 12, 15

   Multiples of 4: 0, 4, 8, 12, 16, 24

   Multiples of 6: 0, 6, 12, 18, 24, 30, 36

Other than 0, the lowest common multiple of 3, 4, and 6 is 12.

2. 2, 3, 18

   Multiples of 2: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18

   Multiples of 3: 0, 3, 6, 9, 12, 15, 18, 21, 24, 27

   Multiples of 18: 0, 18, 36

Other than 0, the lowest common multiple of 2, 3, and 18 is 18.

3. 5, 8, 10

   Multiples of 5: 0, 5, 10, 15, 20, 25, 30, 35, 40, 45

   Multiples of 8: 0, 8, 16, 24, 32, 40, 48, 56, 64, 72

   Multiples of 10: 0, 10, 20, 30, 40, 50, 60, 70

Other than 0, the lowest common multiple of 5, 8, and 10 is 40.

4. 60, 126

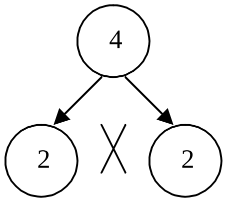
   Multiples of 60: 0, 60, 120, 180, 240, 300, 360, 420, 480, 540, 600, 660, 720, 780, 840, 900, 960, 1020, 1080, 1140, 1200, 1260

   Multiples of 126: 0, 126, 252, 378, 504, 630, 756, 882, 1008, 1134, 1260

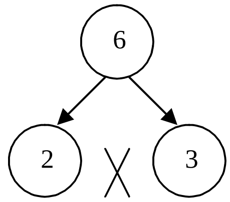
Other than 0, the lowest common multiple of 60 and 126 is 1260.

Use the prime factorization method to find the LCM of the following numbers.

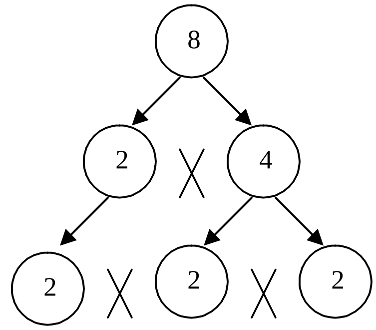
5. 4, 6, 8



So. 4 = 2 x 2



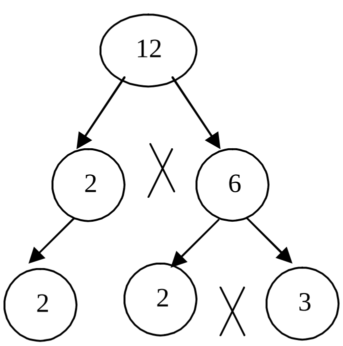
So 6= 2 x 3



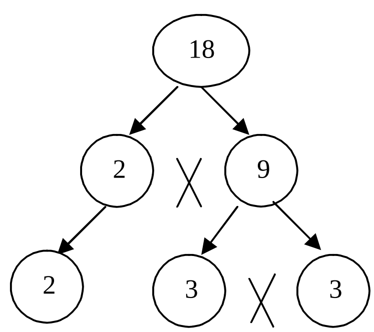
So 8= 2 x 2 x 2

Now count the maximum number of times that each prime factor occurs in each number. The prime 2 appears twice in the factorization of 4, once in the factorization of 6, and three times in the factorization of 8. The prime 3 occurs once in the factorization of 6. The maximum number of 2's is three and the maximum number of 3's is one. Multiply these numbers to get the LCM. 2 x 2 x 2 x 3 = 24 this is the LCM of 4, 6 and 8.

6. 12, 18



So 12= 2x2x3



So 18= 2x3x3

Now count the maximum number of times that each prime factor occurs in each number. The prime 2 appears twice in the factorization of 12 and once in the factorization of 18. The prime 3 occurs once in the factorization of 12 and twice in the factorization of 18. The maximum number of 2's is two and the maximum number of 3's is two. Multiply these numbers to get the LCM: 2 x 2 x 3 x 3 = 36 . Therefore, 36 is the LCM of 12 and 18.

Use the "L method" to find the LCM of the following numbers.

7. 60, 126

2	60	126
3	30	63
	10	21

Write down the numbers of which you are trying to determine the LCM. Choose any divisor of both of the numbers. Beneath the numbers that you started with, 60 and 126, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 60/2 = 30 and 126/2 = 63. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the LCM, find the product of the numbers in the left column and the lowest row, which forms an "L". In this array 2 x 3 x 10 x 21= 1260, so 1260 is the LCM.

8. 12, 36

2	12	36
2	6	18
3	3	9
	1	3

Write down the numbers of which you are trying to determine the LCM. Choose any divisor of both of the numbers. Beneath the numbers that you started with, 12 and 36, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 12/2 = 6 and 36/5 = 18. Continue on with this process until 1 is the only divisor between the two numbers remaining.

To determine the LCM, find the product of the numbers in the left column and the lowest row, which forms an "L". In this array 2 x 2 x 3 x 1 x 3= 36, so 36 is the LCM.

9. 24, 48

2	24	48
2	12	24
2	6	12
3	3	6
	1	2

Write down the numbers of which you are trying to determine the LCM. Choose <u>any</u> divisor of both of the numbers. Beneath the numbers that you started with, 24 and 48, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 24/2 = 12 and 48/2 = 24. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the LCM, find the product of the numbers in the left column and the lowest row, which forms an "L". In this array, 2 x 2 x 2 x 3 x 1 x 2 = 48. So 48 is the LCM.

10. 4, 8

2	4	8
2	2	4
	1	2

Write down the numbers of which you are trying to determine the LCM. Choose <u>any</u> divisor of both of the numbers. Beneath the numbers that you started with, 4 and 8, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 4/2 =2 and 8/2 = 4. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the LCM, find the product of the numbers in the left column and the lowest row, which forms an “L”. In this array 2 x 2 x 1 x 2 = 8, so 8 is the LCM

11. 18, 42

2	18	42
3	9	21
	3	7

Write down the numbers of which you are trying to determine the LCM. Choose any divisor of both of the numbers. Beneath the numbers that you started with, 18 and 42, write the quotient obtained after dividing the divisor into the beginning numbers. In this array, 18/2 =9 and 42/2 = 21. Continue on with this process until 1 is the only divisor between the two numbers remaining. To determine the LCM, find the product of the numbers in the left column and the lowest row, which forms an “L”. In this array 2 x 3 x 3 x 7 = 126. Therefore, 126 is the LCM of 18 and 42.

6 Fractions

6.1 Introduction to Fractions

A fraction is a number that may be used to describe part of a whole. In the classroom the concept of fractions isn't as hard to establish as the concept of reducing and finding common denominators. Therefore, the author found it necessary to include a unit of using the greatest common factor and lowest common multiple to help in solving these kinds of problems in the classroom. The author will provide information on how to use the greatest common multiple to reduce fractions and how to use the lowest common multiple to find equivalent fractions to add and subtract unlike fractions.

6.2 Reducing Fractions

Reducing or simplifying fractions is a hard concept for students to grasp in the classroom. The author has found that introducing the idea of using the greatest common factor to simplify fractions has been very effective in the understanding of simplifying fractions. Let's begin by providing a fraction for the student to reduce. Students need to be aware that reducing a fraction simply means to make the numerator and denominator as small as possible so that 1 is the only number that can divide into both numbers evenly.

Example 1: 12/36

Find the greatest common factor of 12 and 36. Remember that you can use the prime factorization method, the "listing method," or the "L method." (Refer to previous lesson on finding the greatest common factor). In this example, we will use the "L method."

2	12	36
2	6	18
3	3	9
	1	3

Multiply the number on the left side to get the GCF.

2 x 2 x 3=12, therefore 12 is the GCF

Now divide the numerator and denominator by the GCF to find the simplest form of the fraction 12/36.

12/12=1 and 36/12=3, therefore the simplest form of 12/36 is 1/3.

We can also find the simplest form of a fraction with prime factorization.

12= 2 x 2 x 3

36= 2 x 2 x 3 x 3

First, we list the prime factors of the numerator and denominator. Next, we divide or cancel the factor of two that both the numerator and denominator have in common. In other words since 2/2 = 1, we say these are canceled out. Now multiply the remaining factors in the numerator and denominator to find the simplest form of the fraction. Therefore we have 1 for the new numerator and in the denominator we are left with a 3. The simplest form of 12/36 is the equivalent fraction 1/3.

Example 2: 30/45

Find the GCF of 30 and 45 using the one of the methods provided in earlier lessons.

30= 2 x 3 x 5 45= 3 x 3 x 5

First, we list the prime factors of the numerator and denominator. Next, we divide or cancel the factor of three that both the numerator and denominator have in common. In other words since 3/3 = 1, we say these are canceled out. Then cancel out the fives. Now multiply the remaining factors in the numerator and denominator to find the simplest form of the fraction. Therefore we have 2 for the numerator and 3 for the denominator. The simplest form of 30/45 is 2/3.

6.3 Fraction Homework

Find the simplest form or reduce each fraction.

1. 3/12

2. 7/49

3. 6/8

4. 18/42

5. 21/36

6. 180/225

6.4 Fraction Homework

Answer Key

Find the simplest form or reduce each fraction. Make sure to accept any method that the student uses to find the answer. The author will provide different methods for each answer. Remember that any of the methods could be used to find the answer.

1. 3/12

3	3	12	GCF= 3
	1	4	

Divide the numerator and the denominator by 3. 3/3=1 and 12/3 = 4

3/12 = 1/4

2. 7/49

7= 7   49= 7 x 7

First, we list the prime factors of the numerator and denominator. Next, we divide or cancel the factor of seven that both the numerator and denominator have in common. In other words since 7/7 = 1, we say these are canceled out. Now multiply the remaining factors in the numerator and denominator to find the simplest form of the fraction. Therefore we have 1 for the numerator and 7 for the denominator. The simplest form of 7/49=1/7.

3. 6/8

2	6	8
	3	4

GCF= 2

Divide the numerator and the denominator by 2. 6/2=3 and 8/2=4

6/8 = 3/4

4. 18/42

2	18	42
3	9	21
	3	7

GCF= 2 x 3=6

Divide the numerator and denominator by 6. 18/6=3 and 42/6=7

18/42 = 3/7

5. 21/36

21=3 x 7 36=2 x 2 x 3 x 3

First, we list the prime factors of the numerator and denominator. Next, we divide or cancel the factor of three that both the numerator and denominator have in common. Now multiply the remaining factors in the numerator and denominator to find the simplest form of the fraction. Therefore we have 7 for the numerator and 2 x 2 x3=12 for the denominator. The simplest form of 21/36 is 7/12.

7. 180/225

3	180	225
3	60	75
5	20	25
	4	5

GCF= 3 x 3 x 5=45

Divide the numerator and denominator by 45. 180/45=4 and 225/45=5. So, 180/225= 4/5

7 Common Denominators

7.1 Fractions with Unlike Denominators

Adding or subtracting fractions with like denominators is a concept that most students grasp pretty quickly. On the other hand, trying to teach students to add or subtract fractions with unlike denominators is a task that several teachers struggle with year after year. Today, the author hopes to make that task a little easier by providing a means of finding common or equivalent fractions using the lowest common multiple. If you have forgotten this procedure, refer to the previous lessons on finding the lowest common multiple of a number.

Example 1: Add 2/12 + 2/6

First one needs to remind students that fractions have to be alike to add or subtract them. Remind students that "like fractions" are ones with like denominators. In the above example, the denominators 6 and 12 are not alike, therefore they are unlike fractions. We have to find an equivalent fraction in order to add these two fractions.

"But why?" is a common question heard throughout many classrooms today. We are going to introduce a couple of different methods to answer this question. You can use this little story to make this concept much clearer for your students. If you have 3 apples and Joe has 2 apples, how many apples do you have altogether? Then ask your students, if I had 3 apples, 2 bananas, and 4 grapefruits, would I have 9 applebananagrapefruits? Your students will most likely answer no to this question. Explain to them that you now have 9 pieces of fruit. It was easy to add the apples together because they are the same, but when we added the different pieces of fruit together, we had to classify them into another group of something that they had in common, namely fruit [5].

When adding fractions together, we also have to find something that they have in common. Imagine that you have a large pizza cut into 12 equal pieces and another large pizza cut into 6 pieces. You and a friend both eat two pieces of pizza, but you eat from the large pizza that is sliced into 12 and your friend eats from the pizza that is sliced into 6. You try to decide how much you ate all together. How can you add the amount of pizza eaten if the slices are different in size? You have to find a denominator that both fractions have in common and changing the original fractions into equivalent fractions that have common denominators.

Now let's try to figure this out. Your friend ate 2/6 of his pizza and you ate 2/12 of your pizza.

Method 1: Multiply the numerator and denominator by a number of your choice to find an equivalent fraction. Think: is there anyway that you can make the denominator, 6, look like the denominator, 12? 2 x 6=12, so multiply the numerator and denominator by 2.

$$\frac{2 \times 2}{6 \times 2} = \frac{4}{12}$$

Now 4/12 and 2/12 are like denominators, therefore we can add them together.

4/12 + 2/12 = 6/12

Method 2: Find the LCM of the denominators.

$$\begin{array}{r|rr} 2 & 6 & 12 \\ \hline 3 & 3 & 6 \\ \hline & 1 & 2 \end{array}$$

The LCM= 2 x 3 x 1 x 2= 12

Now that we have the LCM, we can work with 2/6 + 2/12. Since the LCM is 12, we must work with both denominators to make them a 12. 2/12 already has the denominator of 12, so we don't have to do anything to this fraction. Now, let's take a look at 2/6. How can we make the denominator of 6 look like 12? We know that 6 x 2= 12, so let's multiply the numerator and the denominator by 2 so we can have a denominator of 12.

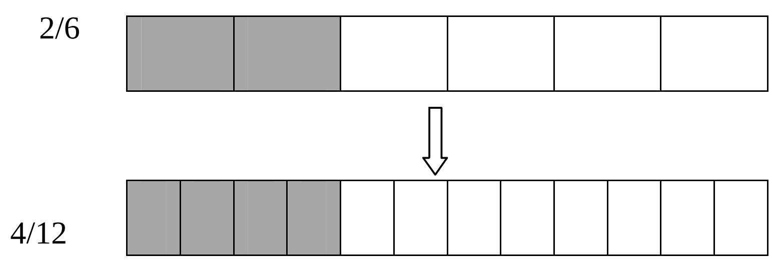
2 x 2 = 4 6 x 2 = 12

Now that we have found an equivalent fraction for 2/6, which is 4/12, we can add our original problem 2/6 + 2/12 to obtain 4/12 + 2/12 = 6/12

One may find that a student may still be struggling with the concept of finding equivalent fractions. So you may want to provide manipulatives to introduce the concept of equivalent or equal fractions. You can give students a fraction and allow them to work with the fraction strips to find equal fractions. Then you can introduce the above concepts to show them what is happening. Let's look at the following examples:

Example 1: 2/6 + 2/12=

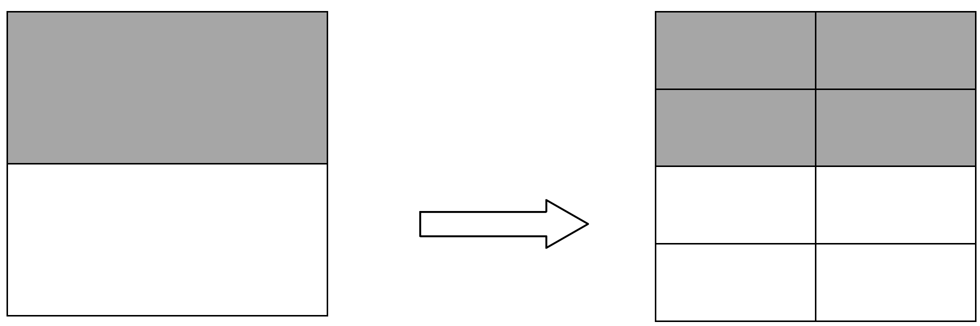
Let's find an equivalent fraction for 2/6. Start with a picture divided into six equal pieces. Shade two sections showing 2/6. Then work with the picture trying to divide it into twelve equal pieces so that it will have the same denominator as 2/12. Allow students to offer any suggestions. Explain why or why not the suggestions may work.



Students should be able to discover that 2/6 is equivalent to 4/12. This will enable them to add 4/12 + 2/12, which will give them the answer to the original problem, 2/6 + 2/12.

Example 2: 1/2 + 3/8

Let's find an equivalent fraction for 1/2. Start with a picture divided into two equal pieces. Shade one section showing 1/2. Then work with the picture trying to divide it into eight equal pieces so that it will have the same denominator as 3/8. Allow students to offer any suggestions. Explain why or why not the suggestions may work.

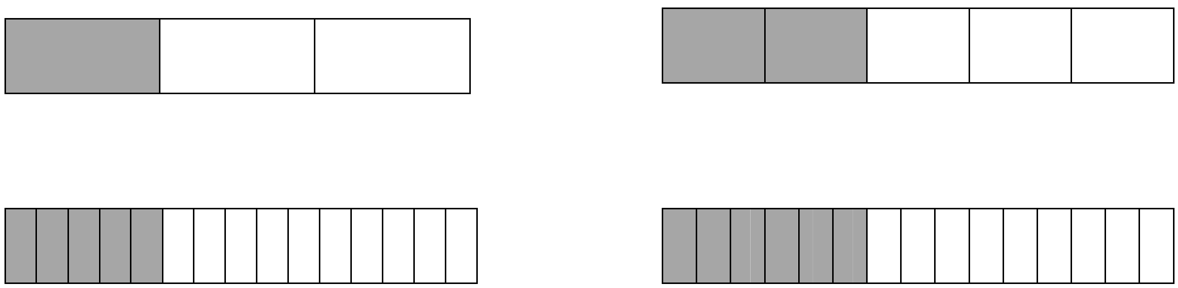


1/2 4/8

Students should be able to discover that 1/2 is equivalent to 4/8. This will enable them to add 3/8 + 4/8, which will give them the answer to the original problem, 1/2 + 3/8.

Example 3: 1/3 + 2/5

Let's find an equivalent fraction for 1/3 and 2/5. Start with a picture divided into three equal pieces. Shade one section showing 1/3. Then work with the picture trying to divide it into five equal pieces so that it will have the same denominator as 1/3. Allow students to offer any suggestions. Explain why or why not the suggestions may work. After awhile, students should discover that there is no way to divide thirds into fifths. Start to explain that sometimes you may have to change both pictures. Now have students draw an example of both fractions, 1/3 and 2/5. Let them work with ways to divide each picture into the same number of pieces.



Students should discover that 1/3 = 5/15 and 2/5 = 6/15. Now, they can add 5/15 + 6/15, to get the answer to the original problem 1/3 + 2/5. So 1/3 + 2/5 = 5/15 + 6/15 = 11/15.

After demonstrating how to find equivalent fractions, allow students to work with each other and discover finding other equivalent fractions.

7.2 Adding and Subtracting with Unlike Denominators

Homework

Draw a picture to help you find the answers to the following fraction addition and subtraction problems.

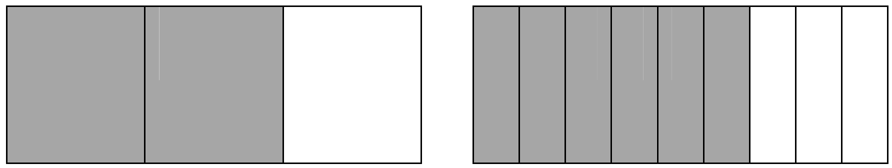
1. 2/3 + 3/9=
2. 2/4 + 1/8=
3. 1/3 + 1/4=
4. 6/12 + 2/4=
5. 2/5 + 3/10=
6. 3/6 - 1/18=
7. 2/3 – 5/12=
8. 3/8 - 7/24=
9. 12/27 – 1/3=
10. 6/8 – 9/16=
11. 3/5 – 1/3=

7.3 Adding and Subtracting with Unlike Denominators Homework

Answer Key

Draw a picture to help you find the answers to the following fraction addition and subtraction problems.

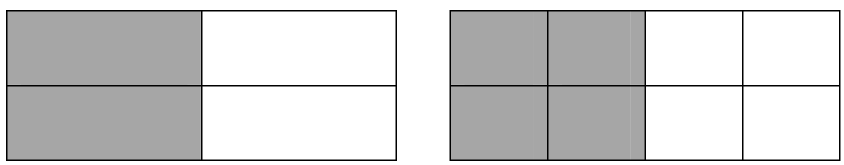
1. 2/3 + 3/9=



2/3 6/9

We find that 2/3 is equivalent to 6/9. Now we can add 6/9 + 3/9 to get the answer for the original problem, 2/3 + 3/9. Therefore we find that 2/ 3 + 3/9= 6/9 +3/9 = 9/9 or 1.

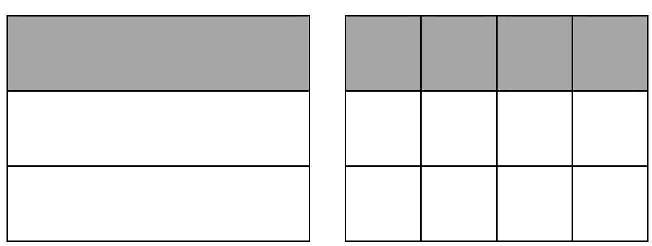
2. 2/4 + 1/8=



2/4 4/8

We find that 2/4 is equivalent to 4/8. Now we can add 4/8 + 1/8 to get the answer for the original problem, 2/4 + 1/8. Therefore we find that 2/4 + 1/8 = 4/8 + 1/8 = 5/8.

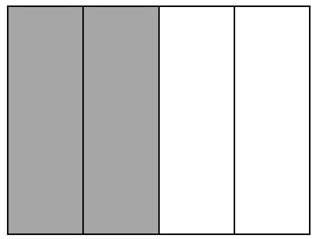
3. 1/3 + 1/12=

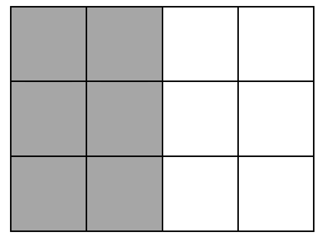


1/3 4/12

We find that 1/3 is equivalent to 4/12. Now we can add 4/12 + 1/12 to get the answer for the original problem, 1/3 + 1/12. Therefore we find that 1/3 + 1/12= 4/12 + 1/12 = 5/12.

4. 5/12 + 2/4=

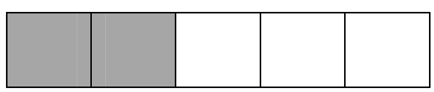


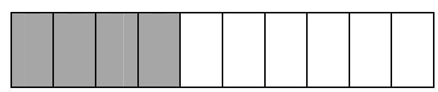


2/4 6/12

We find that 2/4 is equivalent to 6/12. Now we can add 6/12 + 5/12 to get the answer for the original problem, 5/12 + 6/12. Therefore we find that 5/12 + 2/4 = 5/12 + 6/12 = 11/12.

5. 2/5 + 3/10=

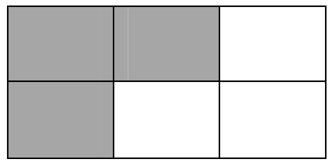




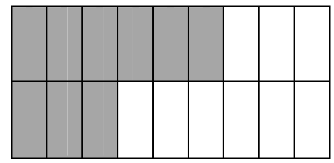
2/5 4/10

We find that 2/5 is equivalent to 4/10. Now we can add 4/10 + 3/10 to get the answer for the original problem, 2/5 + 3/10. Therefore we find that 2/5 + 3/10 = 4/10 + 3/10= 7/10.

6.   3/6 - 1/18=



3/6

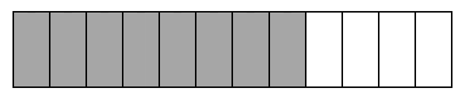


9/18

We find that 3/6 is equivalent to 9/18. Now we can subtract 9/18 - 1/18 to get the answer for the original problem, 3/6 – 1/18. Therefore we find that 3/6 – 1/18 = 9/18 – 1/18 = 8/18 = 4/9.

7.   2/3 - 5/12=

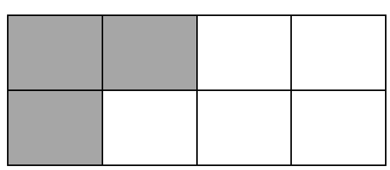
2/3



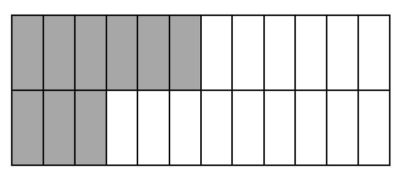
8/12

We find that 2/3 is equivalent to 8/12. Now we can subtract 8/12 - 5/12 to get the answer for the original problem, 2/3 – 5/12. Therefore we find that 2/3 – 5/12 = 8/12 – 5/12 = 3/12 = 1/4.

8.   3/8 – 7/24=



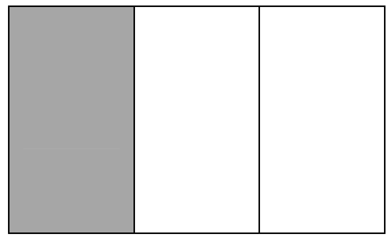
3/8



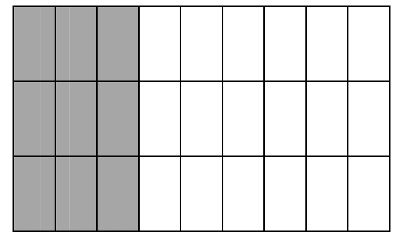
9/24

We find that 3/8 is equivalent to 9/24. Now we can find 9/24 – 7/24 to get the answer for the original problem, 3/8 – 7/24. Therefore we find that 3/8 – 7/24 = 9/24 – 7/24 = 2/24 = 1/12.

9. 12/27 – 1/3=



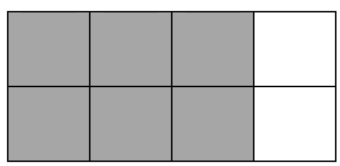
1/3



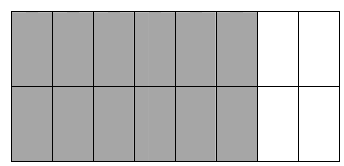
9/27

We find that 1/3 is equivalent to 9/27. Now we can subtract 12/27 – 9/27 to get the answer for the original problem, 12/27 – 1/3. Therefore we find that 12/27-1/3 = 12/27 – 9/27 = 3/27 = 1/9.

10. 6/8 – 9/16=



6/8



12/16

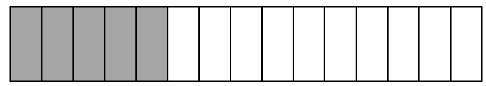
We find that 6/8 is equivalent to 12/16. Now we can subtract 12/16 – 9/16 to get the answer for the original problem, 6/8 – 9/16. Therefore we find that 6/8 – 9/16= 12/16 - 9/16 = 3/16.

11. 3/5 – 1/3 =

3/5

9/15

1/3



5/15

We find that 3/5 is equivalent to 9/15 and 1/3 is equivalent to 5/15. Now we can subtract 9/15 – 5-15 to get the answer for the original problem, 3/5 – 1/3. Therefore we find that 3/5 – 1/3 = 9/15 – 5 /15 = 4/15.

8 SUMMARY

The author wanted to test the information within this thesis. Due to the fact that the author works within the school system daily, she was able to test parts of applications during her fraction unit. The author had never approached the teaching of fractions in the manner suggested in the thesis. Students seemed to grasp the idea of common denominators, least common multiples, and greatest common factors in a way they had not in the past years of the author's teaching experience. Students expressed their love of hands on and exploration activities that allowed them to find answers on their own. This approach seemed to be a success.

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