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Chromatic Number of the Alphabet Overlap Graph, $G(2, k, k\text{-}2)$.

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Chromatic Number of the Alphabet Overlap Graph, $G(2, k, k-2)$

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by

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ABSTRACT

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A graph $G(a, k, t)$ is called an alphabet overlap graph where a, k, and t are positive integers such that $0 \leq t < k$ and the vertex set V of G is defined as, $V = \{v : v = (v_1 v_2 ... v_k);\ v_i \,\epsilon\, \{1, 2, ..., a\},\ (1 \leq i \leq k)\}$. That is, each vertex, v, is a word of length k over an alphabet of size a. There exists an edge between two vertices u, v if and only if the last t letters in u equal the first t letters in v or the first t letters in u equal the last t letters in v. We determine the chromatic number of $G(a, k, t)$ for all $k \geq 3$, $t = k - 2$, and $a = 2$; except when $k = 7$, 8, 9, and 11.

DEDICATION

I would like to dedicate this thesis to my parents, Saul and Judy Noble, for being supportive in my decisions to further my education and in my pursuit of happiness. I would also like to thank my remaining family for their love and belief in me. Finally, I would like to thank everyone that I call my friend, you all have added significant value to my life and have had a part in molding me into the person I am today and too that I am thankful.

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1 INTRODUCTION TO ALPHABET OVERLAP GRAPHS

The alphabet overlap graph, $G(a,k,t)$ is an underlying simple graph of the de Bruijn graph where $t = k-1$. The de Bruijn graph, $\beta(a,k)$, is a directed graph with order $n = a^k$, where each vertex is a word of length k over an alphabet of size a. There is an arc from vertex $u = u_1u_2...u_k$ to vertex $v = v_1v_2...v_k$ if and only if $u_i = v_{i-1}$ for $i = 2,...,k$ [2]. Therefore, the underlying simple graph is $G(a,k,k-1)$ where $k-1$ is the overlap.

The *alphabet graph* is denoted by $G(a,k,t)$. Let a, k, and t be positive integers with $0 \leq t \leq k$. The vertex set of G is the set of all k-letter words over an alphabet of size a. *Adjacency* between two words, say u and v, occurs if and only if the first t letters of u equal the last t letters of v or if the last t letters of u equal the first t letters of v. More specifically, we will be dealing with the cases of $a = 2$ and $t = k-2$, that is $G(2,k,k-2)$.

In this paper, we emphasize coloring $G(2,k,k-2)$ and determining the chromatic number for $G(2,k,k-2)$. Now we define a *coloring* of a graph G as an assignment of colors to the vertices of G, where each vertex gets exactly one color. A coloring is known as *proper* if no two adjacent vertices are assigned the same color. A set of vertices consisting of all vertices assigned the same color is known as a *color class.* If a graph can be colored using k or fewer colors, then G is said to be k-colorable. The minimum number k for which a vertex is k-colorable is the *chromatic number* of G, denoted $\chi(G)$ [1].

Other notations appearing are those of "$*$", "$**$", and "#". So, "$*$" means any symbols from {0,1,2,...,$a-1$} can appear, "$**$" simply means we have two placeholders for the same symbols, and "#" will be seen as $\#ii...i$ where "#" means any symbol except i.

2 FIRST 7 GRAPHS OF $G(2, k, k-2)$

In this chapter, we will disuss the first 7 specific cases in our $G(2, k, k-2)$ graph. In general, it can be shown that as k increases the chromatic number of our graph decreases and eventually becomes constant at a value of 3. We will try to find the first value of k that allows for this chromatic number of 3. We begin with the first case, that is when $k = 3$.

2.1 $k = 3...6$

Theorem 2.1 $\chi(G(2, 3, 1)) = 4$

Proof:

$G(2, 3, 1)$ contains eight vertices. Next, we partition these vertices into color classes making sure not to violate adjacency:

A: 000, 111

B: 010, 101

C: 110, 100

D: 011, 001

Thus, $\chi(G(2, 3, 1)) \leq 4$ by construction.

Now, the graph of $G(2, 3, 1)$ contains a K_4 sub-graph, refer to Figure 1. K_4 is the largest clique in $G(2, 3, 1)$ so we have $\chi(G(2, 3, 1)) \geq 4$.

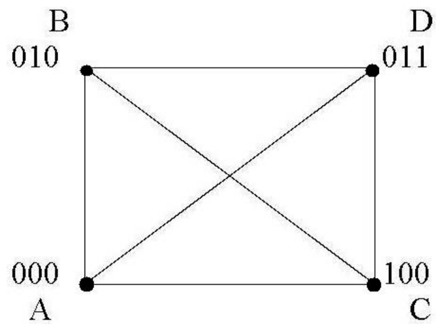

Figure 1: K_4 Sub-Graph of (2,3,1)

Therefore, $\chi(G(2,3,1)) = 4$. $\square$

Let us look at a few examples of how quickly alphabet graphs can become complex to draw, even though they have but only a few vertices.

Figure 2 is a simple graph with 4 vertices and only 5 edges.

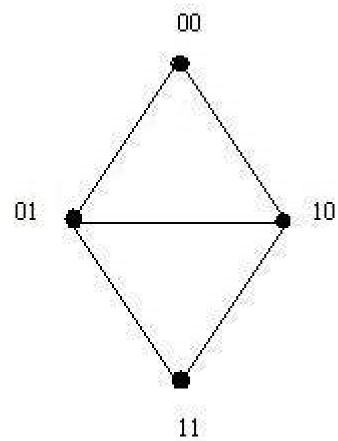

Figure 2: Graph of $G(2, 2, 1)$

Now notice in Figure 3 we have increased the letter length by 1 and the overlap stays the same.

Now we proceed into the next case, that of $k = 4$. In solving the chromatic number of $G(2, 4, 2)$ we will use an Isomophism Lemma.

Lemma 2.2 Isomorphism Mapping: $G(a, 2m, 2m - 2) \cong G(a^2, m, m - 1)$.

Proof:

Take the graph $G(2, 2m, 2m - 2)$ where $a = 2$, let v be an arbitrary vertex in $G(2, 2m, 2m - 2)$ so, $v : a_1 a_2 a_3 a_{(2m-1)} a_{2m}$. In $G(4, m, m - 1)$, we associate this arbitrary vertex $v = (a_1 a_2)(a_3 a_4)(a_{(2m-1)} a_{2m})$. Notice that $a_i = \{0,\ 1\}$ for $i = \{1,$ 2,...$2m\}$. That is exactly two symbols in $G(2, 2m, 2m - 2)$. Now in $G(4, m, m - 1)$, we have exactly four symbols, {00, 01, 10, 11} and we re-label these as {0, 1, 2, 3}, respectively. Next, if u and v are adjacent vertices in $G(2, 2m, 2m - 2)$, then the same

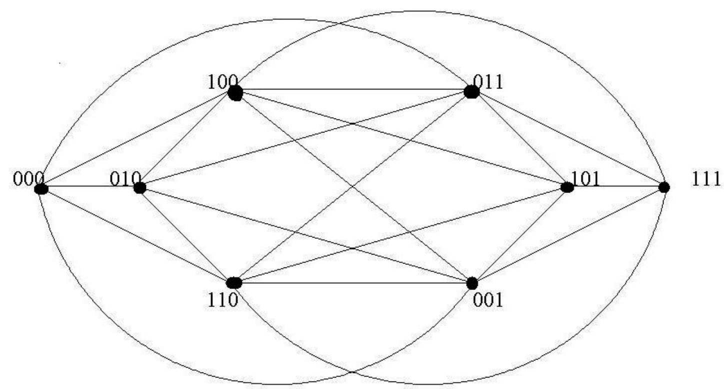

Figure 3: Graph of $G(2,3,1)$

vertices are adjacent in $G(4,m,m-1)$, the same is true for the opposite direction. □

Theorem 2.3 $\chi(G(2,4,2)) = 5$

Proof:

From Figure 4 notice that there is a wheel subgraph on 6 vertices, denoted W_5. From the W_5 subgraph we know that $\chi(W_5) \geq 4$. Next we show that $G(2,4,2)$ is not 4 colorable, we do so by assuming 4 colorability and reach a contradiction.

Assume $\chi(G(2,4,2)) = 4$. We know that $G(2,4,2) \cong G(4,2,1)$ by Isomorphism Lemma. $G(4,2,1)$ has an alphabet consisting of four symbols, $a = \{0,\ 1,\ 2,\ 3\}$. We will begin with a subgraph, $G(3,2,1)$, which has an alphabet consisting of $a = \{0,\ 1,\ 2\}$.

Now we will color $G(3,2,1)$, refer to Figure 4 for coloring. We begin by coloring the wheel subgraph W_5 with hub $\{01\}$ and W_5 has a unique 4-coloring. Then the remaining vertices $\{21\}$, $\{22\}$, and $\{02\}$ have a forced coloring. With the sub-graph

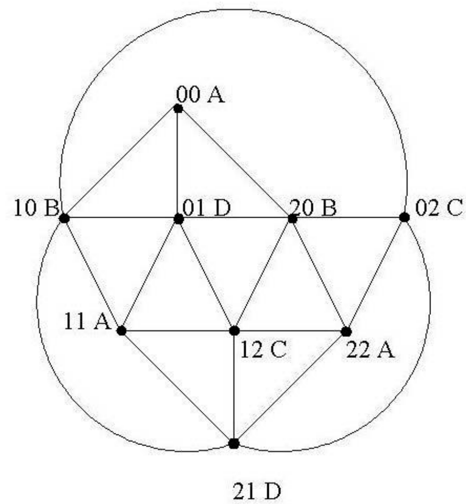

Figure 4: (3,2,1) Subgraph

colored we need to focus now on the remaining vertices of $G(4,2,1)$. Let us call the graph of these remaining vertices G^*3, refer to Figure 5. So, G^*3 consists of all vertices with the form {3*} and {*3}, note that G^*3 contains 7 vertices since each set contains the vertex {33} so we need to only count that vertex once. Notice that each vertex in G^*3 is adjacent to at least one vertex in our $G(3,2,1)$ subgraph. So when we begin coloring the vertices of G^*3, we must take into account the colors that each vertex is adjacent to in the $G(3,2,1)$ subgraph.

Now let's color G^*3. To do so we choose vertices and see what colors they are adjacent to and determine what color they are forced to be colored. We start with {03}, which is adjacent to {00, 10, 20} $\Rightarrow$ {A, B, B}, so the choice of color C is given to vertex {03}. (We could have given {03} color D, it wouldn't change the following result.)

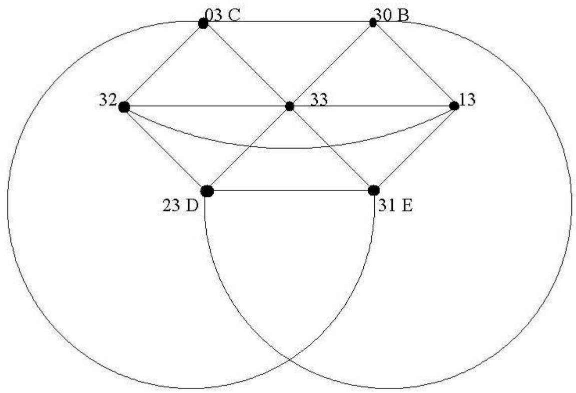

Figure 5: Remaining vertices of $G(4,2,1)$

Next we color vertex {30}, which is adjacent to {00, 01, 02, 03} $\Rightarrow$ {A, D, C, C}, so this forces {30} to be color B since we do not want more than four colors.

Next we color vertex {23}, which is adjacent to {02, 12, 22, 30} $\Rightarrow$ {C, C, A, B}, so this forces {23} to be color D.

Now, {31} is adjacent to {10, 11, 12, 03, 23} $\Rightarrow$ {B, A, C, C, D}, so this means {31} is adjacent to all four colors. Thus, {31} must be colored with color E, a fifth color.

Contradiction, we assumed $G(4,2,1)$ was 4 chromatic. Therefore, $G(4,2,1)$ has a minimum coloring of 5. Thus, $\chi(G(2,4,2)) \cong \chi(G(4,2,1)) = 5.\square$

Theorem 2.4 $\chi(G(2,5,3)) = 4$

Proof:

Since $\chi(G(2,3,1) = 4$, then we know $\chi(G(2,5,3) \leq 4$. We will assume $G(2,5,3)$ is 3 colorable and reach a contradiction. First, we will separate the vertices of $G(2,5,3)$ into two subgroups, one starting with 0's, label it G*0, and 1's, label it G*1. Since $G(2,5,3)$ is 3-colorable we have a triangle in G*0 and G*1. Now in G*0, we have the vertices {00000, 00010, 01000}, each with a different color, (A, B, C) respectively. Next, take vertex {01010}, it's adjacent to {00010, 01000}, so {01010} is forced to be the same as {0000}, A.

Now, look at {00001, 00011}. These two vertices are adjacent to {00000}, A, and {01000}, C, so this forces {00001, 00011} to be colored B.

Next, look at {01001, 01011}. These two are forced to be colored C since they are both adjacent to {01010}, A, and {00010}, B. Thus far we have the following coloring:

A: {0000, 01010}

B: {00010, 00001, 00011}

C: {01000, 01001, 01011}

Finally, we have {00100, 00101, 00110, 00111} and let's call these four vertices subgroup $G3$. All of these vertices are adjacent to a vertex from color classes B and C. This forces these vertices to be colored A.

Now look at G*1. We have a triangle in G*1 between the three vertices {11111, 11110, 10111}. Here we need to notice that {00111, 00101} are adjacent to the triangle in G*1. Recall that {00111, 00101} are color A. Since these vertices are adjacent to the triangle in G*1, no vertices of the triangle can have color A. So, we are forced to have a fourth color, which is a contradiction, refer to Figure 6 for illustration. Thus

$G(2,5,3)$ has a minimum coloring of 4. Therefore, $\chi(G(2,5,3)) = 4.\square$

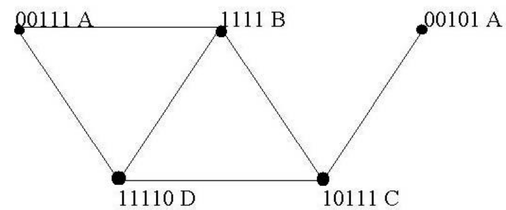

Figure 6: Subgraph of (2,5,3) 4-coloring

Theorem 2.5 $\chi(G(2,6,4)) = 4$

Proof:

The Isomorphism Lemma gives, $G(2,6,4) \cong G(4,3,2)$. Now, $G(3,3,2)$ is a subgraph of $G(4,3,2)$. It is easy to verify $\chi(G(3,3,2)) \neq 3$, thus $\chi(G(3,3,2)) = 4$. Table 1 is the 4 coloring of $G(3,3,2)$, it is straight forward and left to the reader to verify. Now, we need to add the remaining vertices of $G(4,3,2)$ to finish the coloring. We

Table 1: 4-coloring of $G(3,3,2)$.

A	B	C	D
0*0	1*1	2*2	11#
#11	00#	#00	021
#22	22#	012	201
102	120		

need to add vertices containing the symbol "3", refer to Table 2 for the 4-coloring of $G(4,3,2)$.

Table 2: 4-coloring of $G(4,3,2)$.

A	B	C	D
0*0	1*1	2*2	11#
#11	00#	#00	021
#22	22#	012	201
102	120		
#33	33#	032	3*3
320	132	031	012
310	123	201	013
210	102	213	021
230	103	231	023
120		301	203
130		302	312
			321

□

2.2 $k = 7, 8, 9$

The next case is when $k = 7$. This problem is quite difficult and we could not reach a solution. The smallest coloring of $G(2,7,5)$ obtained was a 4-coloring. This doesn't mean the chromatic number could not be 4, but it also means the chromatic number could be 3. Thus we can make the following claim on the bounds of the chromatic number of $G(2,7,5)$:

Claim: $3 \leq \chi(G(2,7,5)) \leq 4$

We run into another unsuccessful attempt in finding the chromatic number when $k = 8$. In attempting the coloring of $G(2,8,6)$, we use the Isomorphism Lemma, $G(2,8,6) \cong G(4,4,3)$. We now use a lemma to define a function, f, to map $G(4,4,3)$ to $G(3,3,2)$.

Lemma 2.6 Homomorphism Mapping 1 [6]: *If $a \leq k$ and $k \geq 3$ then there exists a homomorphism f: $G(a, k, k-1) \rightarrow G(3, k-1, k-2)$.*

Proof:

Let $\mu(a_1 a_2) = sign(a_2 - a_1)$ mod 3, that is:

$$\mu(a_1 a_2) = \begin{cases} 0, & \text{if } a_2 = a_1 \\ 1, & \text{if } a_2 > a_1 \\ 2, & \text{if } a_2 < a_1 \end{cases}$$

If $\phi(w_1) = \phi(w_2) = 0...0$ then both words are constant so if they are overlapping then they are the same. If $\phi(w_1) = \phi(w_2) = 1...1$ then $w_1 = a_1...a_k$ and $a_1 < a_2 < ... < a_k$. Therefore $a_k - a_1 = k - 1$. The assumption of the lemma is that $a \leq k$. If $a < k$ than this is impossible so no such word w_1 exists such that $\phi(w_1)$ is the constant 1 word. If $a = k$ then the only possibility is that $w_1 = 12...k$ but the same is true for w_2 so $w_1 = w_2$. Similar arguments hold if $\phi(w_1) = \phi(w_2)$ is the constant 2 word.□

We are able to map $G(4, 4, 3) \rightarrow G(3, 3, 2)$. We can show $\chi(G(3, 3, 2) = 4$, refer to the proof of $\chi(G(2, 6, 4))$. So, we can get a 4-coloring of $G(2, 8, 6)$. However, we did not find a 3-coloring of $G(2, 8, 6)$. We were unable to prove that $\chi \neq 3$ so once again we can make a claim on the bounds of the chromatic number:

Claim: $3 \leq \chi(G(2, 8, 6)) \leq 4$

Finally, the last case is when $k = 9$. Remember that we are attempting to find the first value of k that will give a chromatic number of 3. The problem of finding $\chi(G(2, 9, 7)$ is a more difficult problem than $G(2, 7, 5)$. We are again able to make a claim about the bounds of the graph:

Claim: $3 \leq \chi(G(2, 9, 7)) \leq 4$

Thus, in our attempt to find a value for k to give us $\chi = 3$ we were unsuccessful in the first seven cases. Thus, we have three probems that still remain open and waiting to be solved; $G(2,7,5)$, $G(2,8,6)$, and $G(2,9,7)$. It might be possible that one of these k values is the first value to result in $\chi = 3$.

3 REMAINING SIGNIFICANT GRAPHS OF $G(2, k, k-2)$

In this chapter we will discuss the importance of three graphs in determining $\chi(G(2, k, k-2))$. We will look at $G(2, 10, 8)$, $G(2, 19, 17)$ and $G(2, 13, 11)$.

3.1 $G(2, 10, 8)$ and $G(2, 19, 17)$

We will first begin with a discussion of why $G(2, 10, 8)$ and $G(2, 19, 17)$ are important in finding the chromatic number of $G(2, k, k-2)$. We start first with $G(2, 10, 8)$. It has been shown by Knisley, Nigussie, and Pór that $\chi(G(2, 10, 8)) = 3$ [6]. In proving this theorem, the use of the Isomorphism Lemma was needed, along with the Homomorphism Mapping 1. So, we have $G(2, 10, 8) \cong G(4, 5, 4)$ by the Isomorphism Lemma. Then $G(4, 5, 4) \to G(3, 4, 3)$ by Homomorphism Mapping 1. Now $\chi(G(3, 4, 3)) = 3$. The coloring is shown in table 3[6].

Table 3: 3-coloring of $G(3, 4, 3)$.

A	B	C
01	*21*	*10*
02	*20*	*12*
#000	000#	0000
1111	111#	#111
222#	#222	2222
2112	1001	2002
1221	0110	0220
2110	1002	2001
1220	0112	0221

Thus, $\chi(G(2, 10, 8) \cong \chi(G(4, 5, 4) \to \chi(3, 4, 3) = 3$. The value $k = 10$ holds importance when coloring $G(2, k, k-2)$ for any even number greater than 10, and k

= 10 serves as the base case for 3-coloring. So for any value of $k \geq 10$ that is even, we can begin with the coloring of $G(2, 10, 8)$ which is 3-colorable and then we simply add multiples of "$**$" to the end of each vertex in $G(2, 10, 8)$ until the desired word length is reached, $v = a_1 a_2 ... a_{10}(*)(*)...(*)(*)$. We need to leave out the vertices of constant 0's and 1's until last and place them in color sets where adjacency isn't violated.

The next question we need to answer is for what odd value of k will we have χ = 3? In applying the lemmas, we have that when $k = 19$ there exists a 3-coloring of the graph. So, $\chi(G(2, 19, 17)) = 3$. The coloring $G(2, 19, 17)$ begins by using Homomorphism Mapping 2. This lemma is as follows;

Lemma 3.1 Homomorphism Mapping 2: *If* $a \leq k$ *and* $k \geq 3$ *then there exists a homomorphism* f*:* $G(a, k, k-2) \rightarrow G(3, k-1, k-3)$.

Thus, $G(2, 19, 17) \rightarrow G(3, 18, 16)$ by Homomorphism Lemma 2. By Isomorphism Lemma we have $G(3, 18, 16) \cong G(9, 9, 8)$. From Homomorphism Lemma 1, $G(9, 9, 8) \rightarrow G(3, 8, 7)$. Now, by deleting "$*$" from each word we can map $G(3, 8, 7) \rightarrow G(3, 7, 6) \rightarrow G(3, 6, 5) \rightarrow G(3, 5, 4) \rightarrow G(3, 4, 3)$. And $\chi(G(3, 4, 3) = 3$. The reason we are able to delete "$*$" from these vertices is similar to the argument above about adding "$**$" to the end of vertices. We know that $\chi(G(3, 4, 3)) = 3$ and by adding "$*$" we will still have a 3-coloring, thus deleting "$*$" allows us to eventually map back down to $G(3, 4, 3)$.

Now we have a lower bound on what value of k will give us the chromatic number of our graph $G(2, k, k-2)$. Therefore, when $k \geq 19$ we can map any graph to a 3-coloring based o the cases when $k = 10$ or $k = 19$, as shown above.

What happens for the cases between $k = 10$ and $k = 19$? These cases will be looked at in the next section.

3.2 $\chi(G(2,13,11))$

We begin first by introducing a new lemma.

Lemma 3.2 Homomorphism Mapping 3 [6]: *If $a \leq 3k$ and $k \geq 3$ then there exists a homomorphism $\phi : G(a,k,k-2) \rightarrow G(5,k-1,k-2)$.*

Proof:

Let

$$\mu(a_1a_2) = \begin{cases} 0, & \text{if } |a_2 - a_1| \leq 2 \text{ and } (a_1 \bmod 3)\text{=}0 \\ 1, & \text{if } |a_2 - a_1| \leq 2 \text{ and } (a_1 \bmod 3)\text{=}1 \\ 2, & \text{if } |a_2 - a_1| \leq 2 \text{ and } (a_1 \bmod 3)\text{=}2 \\ 3, & \text{if } a_2 \geq a_1 + 3 \\ 4, & \text{if } a_2 \leq a_1 + 3 \end{cases}$$

Let $\phi(w_1)$ be a constant word. Obviously it cannot be a 1 or 2 word. If it is constant 0 then w_1 has to be a constant word as well. If it is a constant 3 word then let's say $w_1 = a_1a_2...a_k$ and $w_2 = a_2a_3...a_{(k+1)}$. In this case we know that $a_{(k+1)} - a_1 \geq 3k$ but $a \leq 3k$ so that cannot be. Similarly if $\phi(w_1)$ is a constant 4 word, this cannot be also. □

Previously, we've looked at cases when $k = 3...10$. What about when $k = 11$, 13, 15, and 17? If any of these remaining cases of k give us a 3-coloring then we've found a new lowest bound for k when coloring $G(2,k,k-2)$. Beginning with $k = 11$, $G(2,11,9) \rightarrow G(3,10,8)$ by Homomorphism Lemma 2. $G(3,10,8) \cong G(9,5,4)$ by Isomorphism Lemma. Now using Homomorphism Lemma 3 we have, $G(9,5,4) \rightarrow$

$G(5,4,3)$. The chromatic number of $G(2,11,9)$ is unknown using this method. So, we have an open problem, $\chi(G(2,11,9) = ?$.

The next case, $k = 13$ gives rise to an interesting result. We begin by mapping $G(2,13,11) \rightarrow G(3,12,10)$ by Homomorphism Lemma 2. $G(3,12,10) \cong G(9,6,5)$ by Isomorphism Lemma. Now from Homomorphism Lemma 3 we have, $G(9,6,5) \rightarrow G(5,5,4)$. Finally, $G(5,5,4) \rightarrow G(3,4,3)$ by Homomorphism Lemma 1. And previously, we've shown that $\chi(G(3,4,3) = 3$. Therefore, $\chi(G(2,13,11)) \rightarrow \chi(G(3,4,3))$ $= 3$. Thus, we have found a new lower value of k that will give a 3-coloring of $G(2,k,k-2)$ and a new theorem.

Theorem 3.3 *$\chi(G(2,13,11) = 3$.*

4 CONCLUSION

In this paper, we studied a special case of alphabet overlap graphs, that being $G(2, k, k-2)$ where the alphabet is of size 2, word length k, and overlap size $k-2$. We have found that there is no clear cut method in solving the chromatic number for each k-case. But when using homomorphism and isomorphism lemmas, the colorings, yet still difficult, become easier to work with. We discovered that when $k = 10$, we have a 3-coloring. And for any even number greater than 10 we can map the 3-coloring of $G(2, 10, 8)$ to that particular graph and again have a 3-coloring. We initally found that when $k = 19$ we have a 3-coloring and again can have a 3-coloring of any $k \geq 19$ based off of the coloring of $G(2, 19, 17)$. It was found that we have a 3-coloring when $k = 13$, meaning that we now have a lower k-case from which we can base our 3-color mappings. So, whenever $k \geq 13$ we are able to color that graph based on the colorings of $G(2, 10, 8)$ and $G(2, 13, 11)$. These two results combined tell us that $\chi(G(2, k, k-2)) = 3$, when $k \geq 13$. Thus our main result:

$$\chi(G(2, k, k-2)) = 3.$$

Recalling all open problems from the research, if any of these graphs can be shown to have chromatic number equal to 3 then we would have a new lower k-case. The following are my conjectures on the chromatic number of the graphs.

(1) $\chi(G(2, 7, 5)) = 4$?

(2) $\chi(G(2, 8, 6)) = 4$?

(3) $\chi(G(2, 9, 7)) = 4$?

(4) $\chi(G(2, 11, 9)) = 4$?

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VITA

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