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Base Counting and Simple Mathematic Applications for the Special Education Classroom.

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Base Counting and Simple Mathematic Applications for the Special Education Classroom

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by

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ABSTRACT

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This thesis was designed as a self study unit for middle school aged students with special needs. The unit is broken into subunits that specifically cater to each number base. Also included in this plan is a brief history of counting and practical uses for the mathematics of different number bases. This has been designed to be a "fun" unit to study after taking S.O.L. tests or other state standard testing. Included in each unit are worksheets for assessment of understanding.

CONTENTS

Page

ABSTRACT ……………………………………………………………………2

LIST OF FIGURES ……………………………………………………………...8

Chapter

1 PREAMBLE ...…………………………………………………………...10

2 BASE COUNTING INTRODUCTION 11

2.1 "Eight Toes" ………………………………………………………11

3 USEFUL PRACTICES IN MATHEMATICS …………………….....19

3.1 Introduction ………………………………………………………19

3.2 Review Questions ……………………………....................22

3.3 Review Answers …………………………………………………23

4 A BREIF HISTORY OF COUNTING ……………………………….24

4.1 Introduction……………………………………………………… 24

4.2 Unit Vocabulary…………………………………………………. 26

4.3 Review Questions……………………………………………….. 27

4.4 Review Answers………………………………………………… 29

5 NUMBER BASES TODAY……………………………………………… 31

5.1 Introduction……………………………………………………. 31

5.2 Review Questions.…………………………………………… 34

5.3 Review Answers...……………………………...……..... 35

6 DECIMAL SYSTEM…………………………………………………..… 36
6.1 Introduction………………………………………………..…… 36
6.2 Unit Vocabulary………………………………………..……… 44
6.3 Review Questions…………….……………………...………. 45
6.4 Review Answers…………………………………..…………. 46
6.5 Base Ten Addition and Subtraction Exercises……..…………... 47
6.6 Base Ten Addition and Subtraction Answers………………..… 48
6.7 Base Ten Addition and Subtraction Exercises (Carry and Borrow).49
6.8 Base Ten Addition and Subtraction Answers (Carry and Borrow)..50
7 BASE FIVE……………………………………………………….... 51
7.1 Introduction……………………………………………………. 51
7.2 Classroom Activity: Mengenlehreuhr Base Five Clock…….…. 60
7.3 Base Five Manipulatives…………………………………….… 65
7.4 Base Five Exercises…………………………………………..... 67
7.5 Base Five Exercise Answers…………………….…………….. 68
7.6 Base Five Review Questions…………………………………... 69
7.7 Base Five Review Answers……………………………………..... 70
8 BINARY COUNTING SYSTEM.……………………………………… 71
8.1 Introduction…………………………………………………… 71
8.2 Binary Manipulatives……………………………… ……..… 78
8.3 Unit Vocabulary……………………………………… ……..… 79
8.4 Vocabulary Review...…………………………… ………….... 80
8.5 Vocabulary Answers…………………………………………..… 81

8.6 Review Questions…………………………………………………….. 82

8.7 Review Answers……………………………………………………. 83

8.8 Binary Exercises…………………………………………………… 84

8.9 Binary Exercises Answers………………………………………. 85

9 OTHER BASE SYSTEMS……………………………………………. 86

9.1 Introduction..…... 86

9.2 Unit Vocabulary…………………………………………………… 96

9.3 Review Questions………………………………………………… 97

9.4 Review Answers…………………………………………………... 98

9.5 Duodecimal Addition and Subtraction Exercises………………..99

9.6 Duodecimal Addition and Subtraction Answers……………….. 100

9.7 Base Twenty Addition and Subtraction Exercises…….......……. 101

9.8 Base Twenty Addition and Subtraction Answers……….....……. 102

9.9 Base Twelve Manipulatives……………………………...…….. 103

9.10 Base Twenty Manipulatives……………………………......…….. 106

10 MULTIPLYING WITH BASE SYSTEMS..…………………………. 108

10.1 Introduction…………………………………………………….. 108

10.2 Base Five Multiplication……………………………………….. 112

10.3 Base Five Multiplication Answers…………………...………… 113

10.4 Base Two Multiplication…………………………….………… 114

10.5 Base Two Multiplication Answers……………………………... 115

11 CREATING YOUR OWN BASE SYSTEM..………………………... 116

11.1 Introduction…………………………………………………….. 116

11.2 Base Five Cipher………………………………………………….. 118

11.3 Code Breaking……………………… ………………………. 119

11.4 My Secret Cipher…………………… …………………………. 120

BIBLIOGRAPHY …………………………………………………………..121

VITA ……………………………………………………………………….122

LIST OF FIGURES

Figure Page

1 Hands with Ten Fingers……………………………………………………36

2 Example of place value in base ten …………………..………………37

3 Ten bars of ten units equals one block of 100 units ……………… ……38

4 Single digit addition example ……………………………………..……..38

5 Two digit addition example ……………………………………..……..39

6 Three digit addition example …………………………………………40

7 Addition example with carrying ……...………………………………41

8 Single digit subtraction example ………..……………………………42

9 Two digit subtraction example …………………………………………43

10 Single Hand showing 5 digits ………………………………………..51

11 Example of place values in the Base five system ……………………...53

12 Two digit addition example of base five ………………………………54

13 Example of place value in base five …………………………………...55

14 Two digit addition example with carrying ……………………………56

15 Two digit addition example with carrying ……………………………57

16 Two digit subtraction example in base five ……………………...…58

17 Example of base five clock …………………………………………61

18 Base five manipulative hand ………………………………………..65

19 Base five manipulative blocks ………………………………………66
20 Examples of place values in base two system …………………………73
21 Single digit addition example in base two ……………………………74
22 Addition example in base two ………………………………………75
23 Subtraction example in base two ……………………………………77
24 Base two manipulative blocks ………………………………………78
25 Example of how 6+6 =10 in base twelve system ……………………...89
26 Addition example in base twelve …………………………………...90
27 Example showing how 10-9 does not equal 1 in base twelve ………...91
28 Two digit subtraction example in base twelve ………………………...92
29 Addition example in base twenty …………………………………...94
30 Subtraction example in base twenty ………………………………...95
31 Base twelve manipulative blocks ……………………………………104
32 Base twelve manipulative hand ……………………………………105
33 Base twenty manipulative blocks ……………………………………106
34 Base ten multiplication example …………………………………..108
35 Base 5 multiplication example ………………………………………109
36 Base two multiplication example ……………………………………110

1 PREAMBLE

This unit plan has been designed with the student in mind. It is written for a self study lesson involving counting, adding and subtracting, and multiplying numbers in different base counting systems. With this unit, students will receive a better understanding of mathematics and by result be more comfortable with the basic functions involved with everyday computations.

This unit plan was written with special needs children in mind. In practice it could boost these children's confidence about simple mathematics by letting them have a little more fun and a less restrictive environment during the process. Through studies and experience teaching children with special needs I have observed that the children learn better when they feel less pressure to learn.

This plan has been designed as a fun alternative lesson for use after students have taken the Virginia Standards of Learning (SOL) tests. These units teach the students math skills they normally may not learn and let them try old processes from a different perspective. This will allow a fresh start in the minds of many of the children targeted with this lesson.

Chapter 2 introduces the concept of alternative bases by presenting a story about an alien named Eight Toes. After using the story to capture students' attention, teachers will be able to proceed with base counting.

2 BASE COUNTING INTRODUCTION

2.1 "Eight Toes"

Nothin' ever happens in this town. I mean nothin'. Well, there was that one time when Jason Gobble, Steve, and me was down at the old Food Lion. Jason Gobble swallowed three eggs we found in the dumpster. The expiration date on the carton said February and it was August. Besides that the Food Lion had been closed for about 2 years. Steve and I paid Jason $3.18 if he would eat them, so he leaned his head pinched his nose and tossed the eggs back one, two, three. It was awful he turned green and then he turned some kind of weird orangish purple color. I never new anyone could even turn that color, but he did. Any way the Ambulance came and had to pump his stomach on the spot. It was kinda gross but kinda cool. The Ambulance people yelled at Steve and me for laughing at the whole thing and called our parents to tell them what we were doing. I got grounded for a month. I tried to argue my case but mom and dad would just get madder and madder. Anyway, other than that, nothin' cool has ever happened in this town. That was until last week.

Last week, around Monday or Tuesday (I can't remember which), I was sittin' around being bored when I heard a loud explosion. I look around and couldn't find anything until I looked up. This weird looking airplane was falling out of the sky. It was an airplane I guess? I mean I've seen a lot of airplanes and most of them look the same. The standard two wings and long sleek body, but this one didn't look like that. It was round, I swear, round and not a wing to be found. At first I though that the explosion might have been the wings blowing off of it, but then why would it be round? Well I eventually decided that I needed to go and check this thing out. Partly because of my own curiosity and partly because if it was an airplane it could have people still in it. So I took off looking for this thing and you will never guess where I found it. That's right, behind the old Food Lion. Now, I was told to never go back down to the old Food Lion by both Mom and Dad and the police and the E.M.T.s. but I think they would all understand if I saved a bunch of people and became a hero. So, I went on. I walked around the building and there it was. Well, Mom and Dad said there was nothing but trouble behind that old store and, for once, it looked like they might be right.

I was right about one thing. It was round. Other than that, I was way off. If it was an airplane, it was a type I have never seen before. The wings apparently didn't blow off the thing because, from what I could tell, there never were any wings on it. It was a perfectly round disc shaped object. The more I stared at it the less I understood about how it could fly. Maybe that was the problem. That's why it fell. That thing couldn't fly and some idiot shot it into the sky, that's where the explosion came from, and now that poor idiot is lying inside the thing waiting for someone to come and save him.

Well I guess that someone had to be me seeing as nobody else seemed to hear it explode or see it fall. I walk slowly up to this poorly made homemade "airplane" and looked in what I thought was a window. I mean it looked like a window and it was clear, except for the scorch marks on it, but when I looked through it, it was just a piece of the "plane" covered by glass. That was weird enough, but then I looked at the number. The number was about 20 digits long but the odd thing was there were not any numbers over 7. Now, you are probably thinking "so what?" but for some reason it caught my attention. I know that there is a possibility that someone might write a number on their homemade airplane for tracking purposes. I also know that it could be possible that the number had to be 20 digits long. It is also possible that the number could very easily be that long with out having a digit over 7 in it. But altogether it gets a little out there. First why go through all the work of registering an airplane that doesn't have any wings. Second, I'm pretty sure I've read somewhere that the Federal Government assigns short number letter combination to register aircraft in the United States. But enough about the plane, there was somebody inside and they wouldn't be getting out on their own. So I walked around this thing looking for a door, a window, or just anything that would open. After about the third lap around it I decided that "this idiot built himself inside this thing!" There were no doors, windows, small holes, large holes, or anything that looked like an entrance or exit. I was just about to give up on helping and leave when I heard something pop. The whole disc looked like it was being pulled apart top from bottom. And you know, even that didn't prepare me for what happened next.

Well, its hard to describe exactly what I though when saw the person, or thing inside the funny little plane. At first I thought I should run, then I was too scared to move. The guy inside looked like he was in bad shape. He had to have a head trauma or something. His head was all swollen and lumpy kinda like a watermelon dented up by a hammer. I climbed into the plane and reached out to check if the guy was still alive when all of a sudden his eyes opened. He jumped back and screamed the most horrible scream I have ever heard in my life. He jumped back out of his seat, and when he did I jumped back as well. I wasn't quite as graceful as the other guy and hit my head on something.

Well, I guess I must have knocked myself out because the next thing I remember is looking up directly in the face of this guy in the strange airplane. This guy's head was probably not much of a concern for him. He had the strangest face I had ever seen. His eyes were about the size of baseballs. They were completely pitch black and so shiny you could see yourself in them. He didn't have a nose. He just had two small holes in the center of his face. He didn't have any ears either; there weren't even any holes where ears should be. His mouth was small but did kinda look like a mouth. He was wearing this thing around his neck that looked like a speaker box from McDonalds.

When I saw him the first time after waking up I jumped back again. This time I was lucky and didn't hit my head, but, I was trapped in this plane with the weirdest person I have seen in my life. I was looking around trying to figure out where to run when he spoke. He said something I am really not sure what but I know I understood it. It made me feel better about the whole situation. I spoke back to him, well I yelled (no ears), and I asked him if he was okay. He laughed, I think, and said I was the one who was passed out. I told him that when I found him he was passed out as well. He dismissed my concern and said he was in hyper sleep. "Hi bear sleep" I questioned. And after what I guess to be a laugh he corrected me. He told me that he was from a planet millions of light-years away and he was on vacation. He had some friends that just came back from Orlando and they told him it was very nice. So, he decided that he would come down to this Florida place and check it out. But he guessed that he must have gotten his calculations wrong for the auto pilot because this didn't look like any of the pictures of Disney World he had ever seen. I told him it didn't look like Florida because he was in Virginia, about 750 miles north of Orlando. The only thing I understood for the next two or three minutes was "Well I'll" then he turned off his speaker and jabbered nonstop until he finally calmed down. He turned the speaker on again and asked me if I would mind helping him. He told me his wife and 6 kids were in the bedroom and they would be mad if they woke up and wasn't in Disney World. I told him that I would be glad to help him but didn't know anything about aliens or spaceships. He said it was okay, he really just needed me to get airplane tickets for 10 people from the nearest airport to Florida on the next flight. "Ten people?" I asked. I counted everyone up in my head and came up with 8 people. I asked him if there were more people than he had told me about. He said "no" and then listed out everyone. "First there's me" he said, "then my wife, and 6 kids that's 10." As he did this he held up his fingers. I looked at his hands and noticed there were only 4 fingers on each hand. Still thinking he made a mistake I held his hands up in front of his face and showed him he only had 8 fingers. He looked at me and said "Yes, 10, like 10 fingers." Even more confused now I held up 4 fingers on each hand and said "that's only eight fingers. There was a long silence until the little guy finally said "WHAT!?" "Eight," I said as I showed him my hands again. "Look," I counted my fingers as I put them down. "1, 2, 3, 4, 5, 6, 7, 8." "No, no, no," he said and he counted his fingers. "1, 2, 3, 4, 5, 6, 7, 10." He and I looked at each other totally confused and now well aggravated.

After a little time and a lot of thinking I finally asked him to write his numbers down 1-10. When he did I finally saw the problem. His number system didn't have the numbers 8 or 9 in it anywhere. I quickly wrote my numbers 1-10 down underneath his. When I did I showed him the numbers 8 and 9. I pointed at the 8 and told him that on earth this is how many fingers he had. I showed him my hands and counted my fingers out for him to see. He looked at me and told me on his planet I would have 12 fingers. I guess we must have both thought about having two extra or less fingers because we both started laughing hysterically. After we calmed down we called the airport and got 8 tickets on the next flight to Orlando, Florida. After we got the tickets he asked me to help hide his ship until he could get back to Virginia and get it. We rolled it behind the dumpster and I told him that it would be okay there because no one was allowed behind the old Food Lion. After we moved the ship he went back in it and woke up his family. When he came back out with them he introduced them to me. I'm sorry to say that I did not understand any of their names and so I can't remember them. He asked my name so he could find me when he came back to get his ship. I told him and as he was leaving I yelled "Have fun in Orlando Eight Toes."

On a side note, we still communicate every once and a while. Eight Toes decided to use a secret code based on his number system. "To make it easier on me" he said "people might start to catch on if they read letters between a boy and his alien friend." I really didn't have the heart to tell him that people would probably wonder why a huge satellite just got built in my back yard but man you should see the channels I can get on my TV now.

3 USEFUL PRACTICES IN MATHEMATICS

3.1 Introduction

Since counting was first discovered, it has been one of the most important things in a society. Along with spoken and written language, counting and mathematics have helped develop and destroy civilizations. Numbers have been used to count, tell time, set monetary value, and measure things.[1] Later we will discuss number systems and how they have helped simplify life, but for right now we will discuss the basic practices of mathematics and why they are useful.

The most important practice involving numbers is the act of counting. Without it, no other arithmetic operations could be exercised. The act of counting is believed to be first used as a concrete system. The first counters used it to count specific things like animals or members of a tribe. These early counters are believed to use tick marks to total what they were counting [1]. We still use tick marks today as an early stage counting tool and in statistics and probability. These early tick marks are thought to mean 1 and 1 more and 1 more until all items were accounted for [1]. Eventually numbers became abstract and could be used as just a number.

After counting there is the concept of addition and its opposite, subtraction. These two concepts were brought about to group two things together. Addition is the act of counting the total of two or more groups of things. This is used with money and counting large groups of things. For example we could add the total number of students in each grade at your school.

The concept of addition is coupled with the concept of subtraction. Subtraction is used to split one large group into two or more groups. This is useful for counting change when buying something or when splitting groups of things into more manageable groups. If you go to the store and buy a new CD that cost $15 with a $20 bill, subtraction tells us that you would get $5 in change.

After addition and subtraction come the concepts of multiplication and division. The concept of multiplication is a fast method of counting used for adding together a large amount of groups. These groups are all the same size. For example 4x9 means to add 9 groups of 4, or 4+4+4+4+4+4+4+4+4. This concept can be used if you are measuring volume inside a room. If you measure each side and multiply them together you can find the amount of space inside the room. Another use for multiplication is figuring out taxes and discounts when shopping.

Division is the opposite action of multiplication. In other words, if multiplication is fast addition then division would be fast subtraction. With division you can split one large group into many small ones. For example, if you would like to split a group of 36 into groups of 6 you could divide 36 by 6 and you would find out that there would be 6 groups of this size.

With these four basic functions there are a lot of things that can be done with numbers and counting. These are just a few practices used in mathematics. However, they are the most frequently used. Addition, subtraction, multiplication, and division are all very important in day to day life in our country and all around the world. Think about this the next time you or your parents get paid. If someone offers you $6.25 an hour to work for them and you work five hours, you do not want to get paid less than $31.25.

3.2 Review Questions

1. ___________________ is one of the most important discoveries in the history of the world.
2. The first counters probably used a _____________ counting style to count animals.
3. _____________ ____________ were and are used to count things one at a time.
4. __________________ is the concept of totaling two groups of things into one large group.
5. __________________ is the concept of splitting one large group into 2 or more groups.
6. __________________ is a type of fast addition.
7. __________________ is a type of fast subtraction.
8. __________________ would be the easiest way to count the total number of students in each grade of your school.
9. __________________ would be the easiest way to measure how many square feet of space is inside your classroom.
10. __________________ is the easiest way to figure sales tax on an item you want to buy.

3.3 Review Answers

1. ___Counting__ is one of the most important discoveries in the history of the world.

2. The first counters probably used a ___concrete_ counting style to count animals.

3. ___tick_____ ____marks__ were and are used to count things one at a time.

4. ___addition______ is the concept of totaling two groups of things into one large group.

5. __subtraction_____ is the concept of splitting one large group into 2 or more groups.

6. ___multiplication___ is a type of fast addition.

7. ___division________ is a type of fast subtraction.

8. ___Addition_______ would be the easiest way to count the total number of students in each grade of your school.

9. ____Multiplication__ would be the easiest way to measure how many square feet of space is inside your classroom.

10. ____Multiplication__ is the easiest way to figure sales tax on an item you want to buy.

4 A BRIEF HISTORY OF COUNTING

4.1 Introduction

Before we really can get into our lesson on counting in number bases we first must learn several things. What is the point of base counting, what base do we count in and why, and what differences could occur when changing bases?

To explain counting in number bases we will look at the history of counting and of numbers in general. The system most of the world uses today is called the decimal system. It is the number system that we know and use daily. The numbers used are 0,1,2,3,4,5,6,7,8,9. To count beyond the number 9 we combine two numbers together (i.e. 10). We will learn more about that later in the lesson though. For now we want to know where the decimal system came from and why. While neither is known for sure, it is thought that the number ten is significant because we have ten fingers [1]. This makes sense because when first learning to count young children use their hands. By counting using fingers we associate the number of items counted to how many fingers we pushed aside. Where the decimal system came from is a question that is still not exactly answered. It is believed to have been around since around 300 BC [2].

The 10 digit base system is not the only counting system used in the world. Certain civilizations and communities have used other bases for counting. There was a small group of people in Sri Lanka who used only 4 words for counting.

These words: single, couple, another one, and many, are all that were needed for this tribe to discuss amounts. They did not need any more detail. On the other hand, the Sumerians used a number system based on 60. They used this system because the number 60 is easily divided by many numbers. The number 60 is divisible by 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60. This is why we have 60 minutes in an hour and 60 seconds in a minute [1].

There are still other styles and ways to count. The Paiela tribe in New Guinea count by pointing to certain body parts.[1] They have no words for any number. Their largest number is 28 and is shown by clinching both fists. In Russia, the Gilyaks have 24 different sets of numbers. If they count long thin things they would use a different set of words than when they count textured things or round things. These methods are known as counting without numbers (Paiela) and concrete counting (Gilyaks) [1].

Unlike the Gilyaks, the majority of counting numbers used today are abstract. Abstract numbers can be used to count anything. It is thought that abstract counting took a long time to develop, coming only after counting without numbers and concrete counting. It takes the ability to use and understand abstract counting to be able to count in different bases.

4.2 Unit Vocabulary

Number- A word expressing how many items there are in a group.

Counting- To recite numbers in order. Computing how many numbers are in a group.

Base- The unit in a number system that is multiplied by itself in order to create a higher number. Can be determined by counting the amount of single digit numbers in the system.

Decimal system- A number system that uses a notation in which each number is expressed in base 10 by using one of the first nine integers or 0 in each place and letting each place value be a power of 10.

Concrete Counting- Numbers that combine the idea of counting and the things that are counted.

Abstract Counting- Numbers that are separated from the items they are counting.

4.3 Review Questions

1. The ____________ system is used for counting by the majority of the world today.
2. To count beyond the number 9 in the decimal system we combine _____ numbers.
3. The decimal system is thought to have been around since around ____________.
4. The _____________ used a number system based on 60.
5. They used the number 60 base because it is _____________ by 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.
6. The Paiela tribe in New Guinea count by ________________________, using parts of their body to represent numbers.
7. Counting certain things with a special set of numbers is called ____________________.
8. The _______________ use different words to count round, straight, and textured items.
9. ____________________ can be used to count anything.
10. ____________________ can be determined by counting single digits of the system used.

Match the vocabulary term with the correct definition.

1. ____Number
2. ____Counting
3. ____Base
4. ____Decimal System
5. ____Concrete Counting
6. ____Abstract Counting

a. Numbers that combine the idea of counting and the things that are counted.
b. To recite numbers in order. Computing how many numbers are in a group.
c. A number system that uses a notation in which each number is expressed in base 10.
d. Numbers that are separated from the items they are counting.
e. The unit in a number system that is multiplied by itself in order to create a higher number.
f. A word expressing how many items there are in a group.

4.4 Review Answers

1. The _Decimal_ system is used for counting by the majority of the world today.

2. To count beyond the number 9 in the decimal system we combine _2__ numbers.

3. The decimal system is thought to have been around since around __300 BC__.

4. The _Sumerians_ used a number system based on 60.

5. They used the number 60 base because it is __divisible__ by 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.

6. The Paiela tribe in New Guinea count by ___Body Counting___, using parts of their body to represent numbers.

7. Counting certain things with a special set of numbers is called __Concrete Counting__.

8. The __Gilyaks__ use different words to count round, straight, and textured items.

9. ___Abstract Numbers___ can be used to count anything.

10. ___Base___ can be determined by counting single digits of the system used.

1. __F_Number
2. __B_Counting
3. __E_Base
4. __C_Decimal System
5. __A_Concrete Counting
6. __D_Abstract Counting

a. Numbers that combine the idea of counting and the things that are counted.

b. To recite numbers in order. Computing how many numbers are in a group.

c. A number system that uses a notation in which each number is expressed in base 10.

d. Numbers that are separated from the items they are counting.

e. The unit in a number system that is multiplied by itself in order to create a higher number.

f. A word expressing how many items there are in a group.

5 NUMBER BASES TODAY

5.1 Introduction

Today, number bases are used for many different things. Some bases have been used for special things, such as time, calendars, and computers. Other bases are used more freely but have just as many, if not more uses.

The first base we will discuss is the decimal system. This is the most widely used counting system in the world [1]. It is also known as the base ten system meaning it has ten single digit numbers. We use this system more than any other counting system. The base ten system is used in everyday math for counting. Our money system is also based on the decimal system, one dollar equals ten dimes which equals one hundred pennies.

Besides money, we also use the decimal system when we describe or count things or groups. For example, if I wanted to tell someone how many people are in my family I would tell them 3, or 1 and 1 more and 1 more. If we wanted to say how many people were in this class we would say 15, or 1 group of ten and 5 groups of one.

We use the base ten system because it is easily divisible and multiplies very easy as well. For example, half of ten is five, ¼ of ten is 2.5. When multiplying ten you add a zero to the number you multiplied by ten. For example, 2x10 equals 20.

The base 12 system is also used by a lot of people in the world today. The duodecimal system is used for time and calendars. The reason for using the base 12 system is because it divides by 2, 4, and 6 evenly. This makes it easy to split time into quarter and half days. When using this system with a calendar it splits the year into four seasons or two halves of a year, the warm seasons and the cold ones [1].

The binary system is probably the most specialized counting system. It is used for computers and encryption. This system is designed to be used to answer series of yes and no questions. For this reason the system works really well to give computers instructions. It tells the computers to open programs or to change options by a series of yes and no questions. The answer to these questions would look like this 10001010110 [1].

Another use for the binary system is for encryption. Encryption is used to encode messages so someone can secretly send information to another person with no one else being able to read the information.

Encryption is used mostly with computers and credit cards. Credit cards use the binary system combined with prime numbers. The prime numbers are usually very large numbers and then converted into an even larger binary number. These numbers are very hard to decipher and this has to be done by special computer programs that are designed to do number conversions very quickly.

Because of the discovery of the binary counting system we are making new technology everyday. We are in what is being called the digital age. Almost everything being made uses computers and the binary code, from your alarm clock to your i-pod. New cars and trucks all have computer systems that keep vital information stored in them that helps the cars and trucks run. If a vehicle breaks down its computer system can be connected to another computer that will diagnose the problem and tell the mechanic what is wrong with the vehicle. Without the binary code many of our conveniences of life would not be possible. So the next time you turn up your i-pod or blast away some aliens in the newest sequel to Halo, thank the binary code and all it has done for you.

5.2 Review Questions

1. Base number systems have been used for many different things throughout history. Among these are ____________, _____________, and _____________.
2. The _______________ system of counting is used today with money systems as well as most every day computations.
3. The _______________ system of counting has been used world wide for time and calendars.
4. The _______________ system of counting has been used with computer electronics and with encryption.
5. The _______________ base system is used for time because it is easily divisible by 2, 4, and 6.
6. The _______________ base system is used because it is fairly easy to divide into halves and quarters.
7. ________________ is used for computers by answering a series of yes or no questions.
8. ________________ has given rise to what is being called the digital age.
9. The binary system is used to power things like ___________ and _______________.

5.3 Review Answers

1. Base number systems have been used for many different things throughout history. Among these are __time_____, ___Calendars_, and ___computers__.
2. The __Decimal_____ system of counting is used today with money systems as well as most every day computations.
3. The ___Base 12_____ system of counting has been used world wide for time and calendars.
4. The ___Binary______ system of counting has been used with computer electronics and with encryption.
5. The ___Base 12______ base system is used for time because it is easily divisible by 2, 4, and 6.
6. The _Base 10/ Decimal_ base system is used because it is fairly easy to divide into halves and quarters.
7. __Binary___________ is used for computers by answering a series of yes or no questions.
8. __Binary Code _____ has given rise to what is being called the digital age.
9. The binary system is used to power things like __i-pods____ and ___video games____.

6 DECIMAL SYSTEM

6.1 Introduction

The decimal system is where we are going to start learning to count in bases. We are starting here because we have all the materials we need on hand, no pun intended. Seriously though, we do have everything we need; our fingers! It is no coincidence that we have a total of ten fingers, five on each hand, and that there are ten single digit numbers in our counting system, 0,1,2,3,4,5,6,7,8,9. Using a concrete style of counting we can see that each number matches up with a corresponding finger.

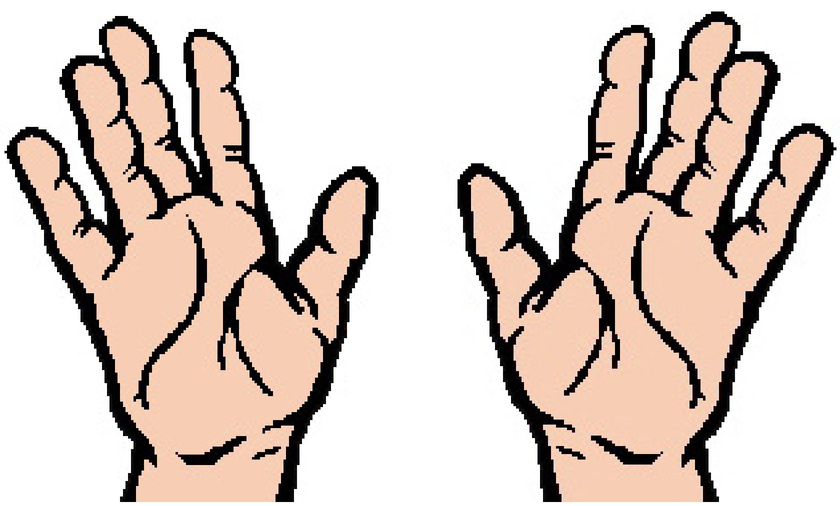


Figure 1: Hands with Ten Fingers [3]

Look at the picture of the hands above. Notice I counted 0-9 on the fingers. That is because the 0 is a single digit in our modern base ten system. Also notice that I stopped before I counted 10. That is because 10 is a two digit number. Ten is the first two digit number in this system (as well as in others.) The 1 in the number 10 represents the ten's position. This means there is 1 group of ten things. The 0 in the number 10 represents the one's position. The 0 means that there are no groups of "one" things. This is called place value. The number 100 is bigger than the number 010 and both are bigger than the number 001. The place value is what makes the difference in these numbers. They, all three, have the same digits just put in different places. For the number 001 it has a 1 one the ones place, a 0 in the tens place, and a 0 in the hundreds place. This gives it a value of 1 "one". The number 010 has a 0 in the ones place, a 1 in the tens place, and a 0 in the hundreds place. This gives it a value of 1 "ten". The number 100 has a 0 in the ones place a 0 in the tens place and a 1 in the hundreds place. This gives it a value of 1 "hundred". Place value can continue infinitely with thousands, ten thousands, hundred thousand, and so on.

Another example would be the number 24. The digit 2 represents 2 groups of ten, or 20. The digit 4 represents 4 groups of 1, or 4. 20+4=24.

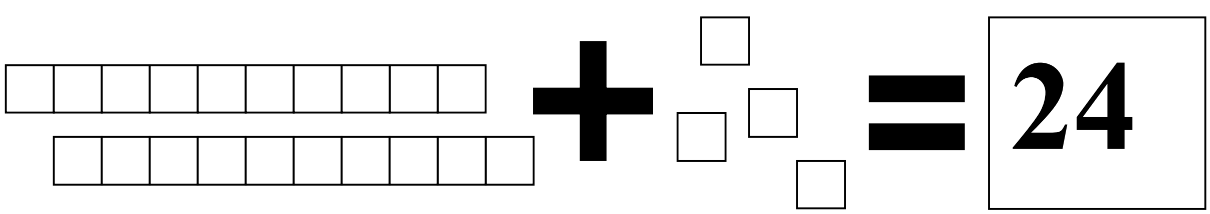


Figure 2: Example of place value in base ten

This system can continue infinitely. Ten groups of "tens" become 1 group of "hundreds"; ten groups of "hundreds" become 1 group of "thousands", and so on.

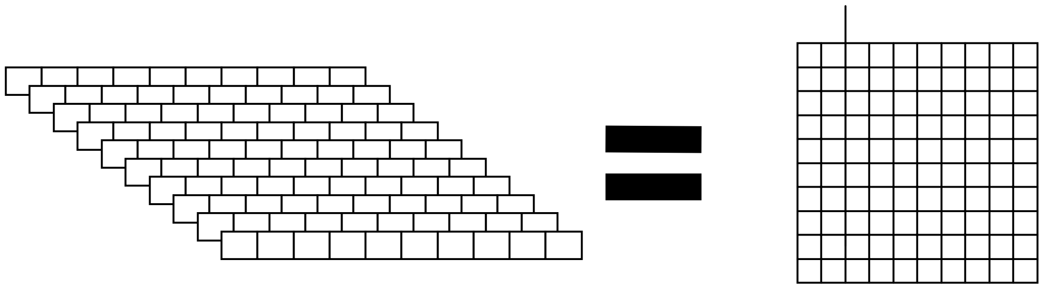


Figure 3: 10 bars of 10 units equals 1 block of 100 units

When adding in base ten we count all the numbers that we are adding together. For example, if we are adding 5+3, we count to 5 and then we count three more digits giving us eight.

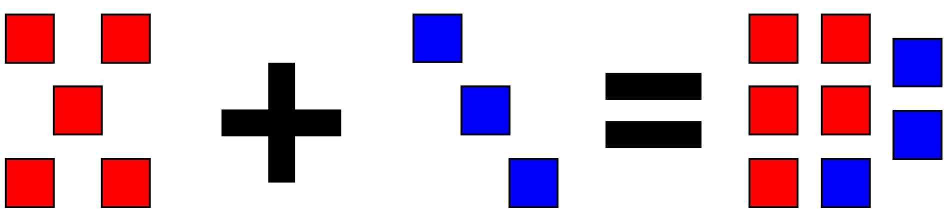


Figure 4: Single digit addition example

When adding two digit numbers in the decimal system you add each group separately. Add the ones group first then add the tens group. Given the numbers 31 and 12, we will add the ones group (1+2), and after getting our answer for the ones group we add 3+1 from the tens group. Adding these numbers this way will give us our answers of 4 groups of ten and 3 groups of ones or 43.

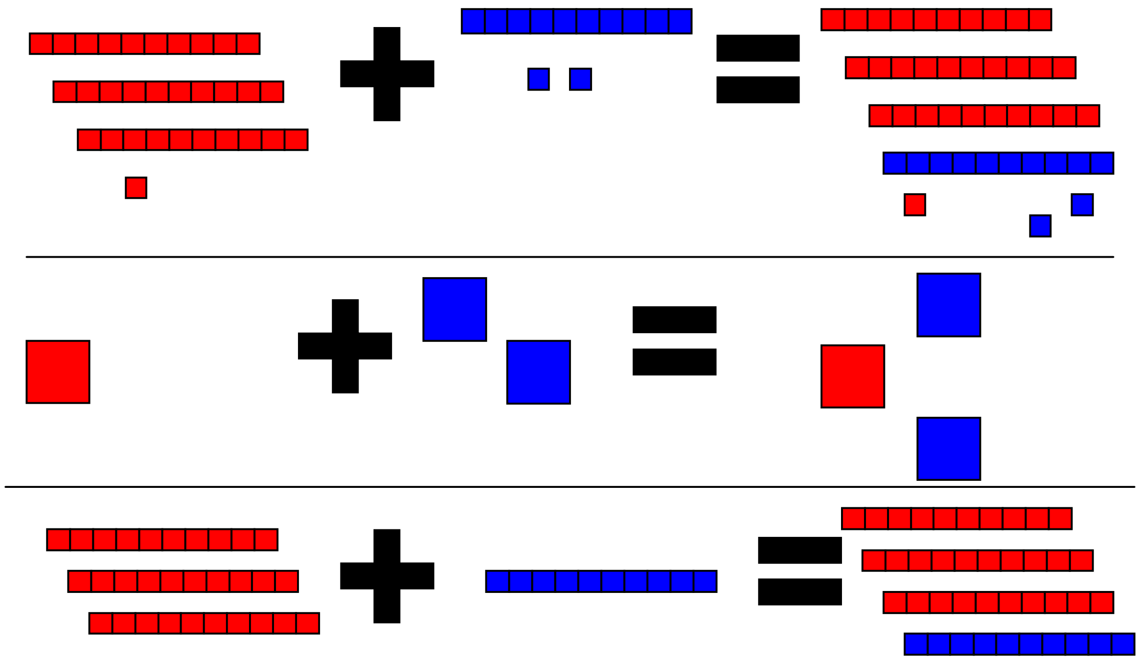


Figure 5: Two digit addition example

If adding 3 digit numbers, you will add the same way you did with single digit or two digit numbers. Start adding with the ones position, continue with adding the tens position, and finish by adding the hundreds position. For example, if adding 142+257, we will add 7+2, then 4+5, and finish by adding 1+2. This gives us the answers 9 ones, 9 tens, and 3 hundreds, or 399.

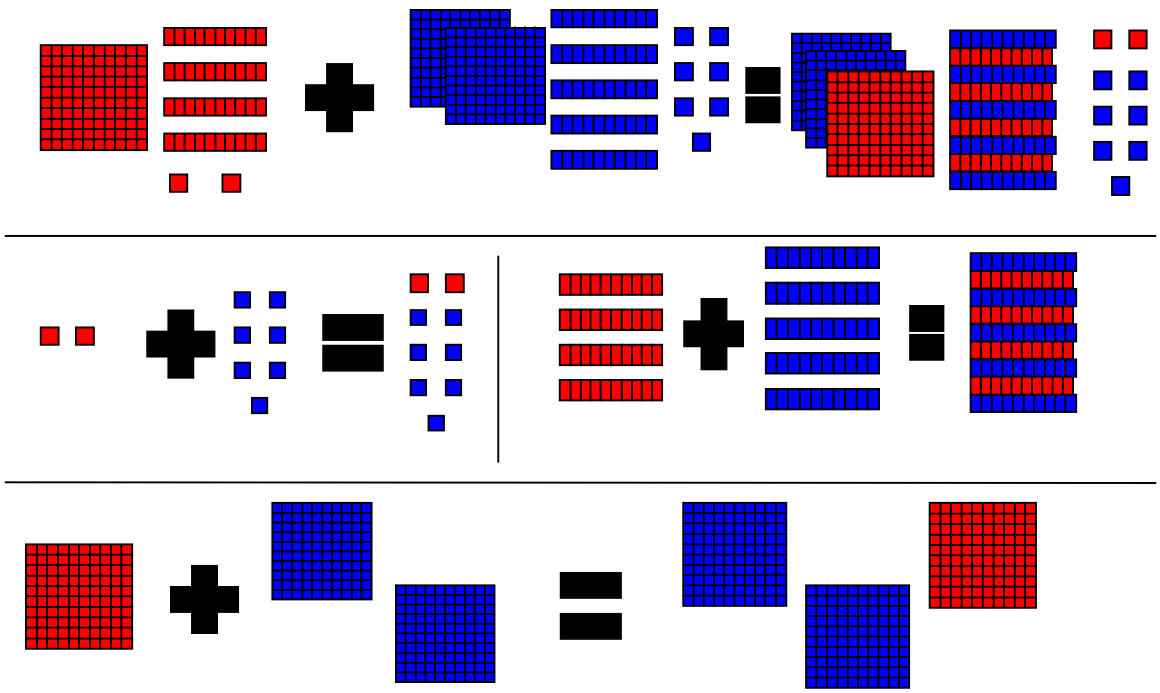


Figure 6: Three digit addition example

When adding two numbers in any position that "count up" past 9 take ten units from that group and place them as one unit in the next highest group. This method is known as carrying over places. An example of this is adding 5+8. Counting 5 ones and 8 ones gives us 13 ones or 1 ten and 3 ones.

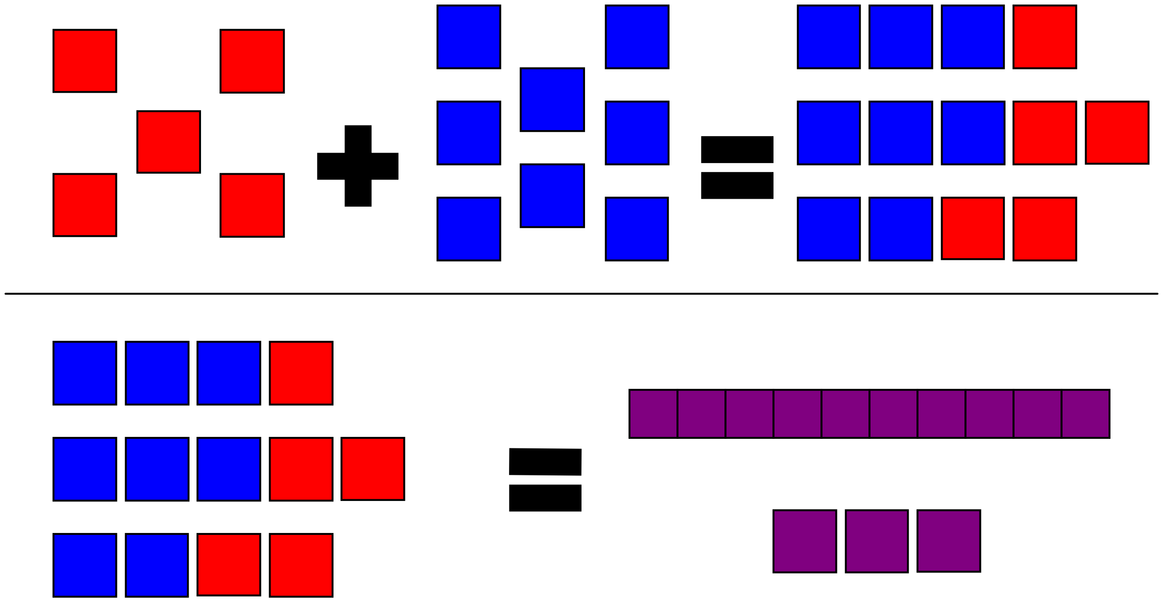


Figure 7: Addition example with carrying

The opposite of adding numbers is subtracting them or taking them away. This has to be done with a little bit more discipline than adding numbers. Although it doesn’t matter which number you start with when adding (first or second), you must start with the first number when subtracting. Take the first number and count down from it the value of the second number. If given the problem 8-3, start with the number 8 and count backwards 3 digits; 8,7,6,5. This makes 5 the answer to the subtraction problem.

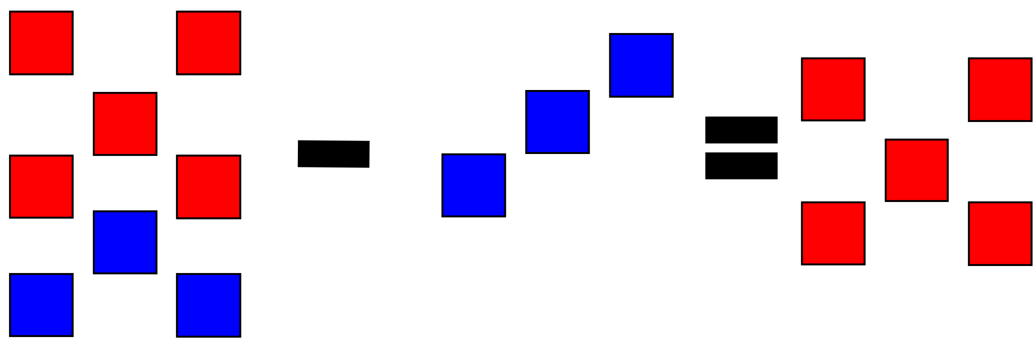


Figure 8: Single digit subtraction example

When subtracting numbers that are more than 1 digit (13, 54, 745) start from the right and work your way left. 54-13 would be subtracted by first subtracting 4-3 in the ones group and 5-1 in the tens group. This gives you the answers of 1 one and 4 tens or 41. If subtracting a number that is larger than the one you are subtracting it from, you will have to move units over from the next highest group. This is called borrowing. When subtracting 63-28 we must borrow from the tens group. This is because 8 doesn't subtract from 3 very easily. Borrowing from the 6 tens in 63 we will take one group of ten and add it to the 3 giving us 13 ones and 5 groups of tens. We will then take the group of 13 ones and subtract eight from them, giving us 5 ones. We take the remaining 5 tens and subtract 2 tens from them. This gives us 3 groups of ten or 30. 30+5=35. Our answer is 35 or 3 groups of tens and 5 groups of ones.

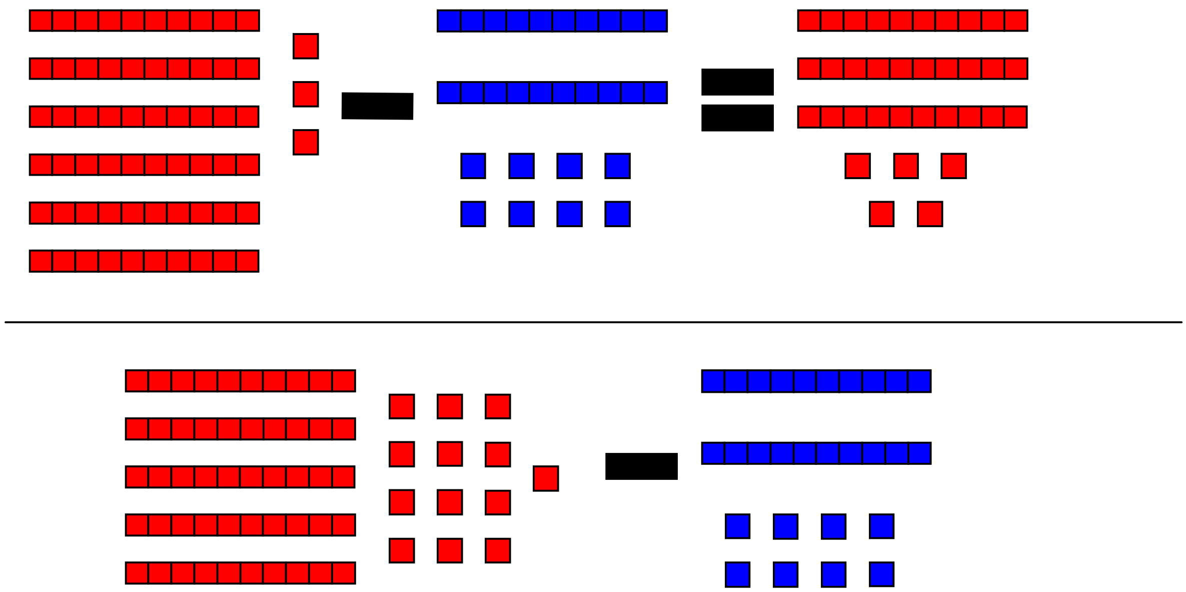


Figure 9: Two digit subtraction example

This process can be practiced with any subtraction problem where there is a higher group from which to borrow numbers. It works because of our base number system. The tens groups is 10 times larger than the ones group, the hundreds group is ten times larger than the tens group, the thousands group is ten times larger than the hundreds group, and this continues forever.

This finishes the lesson on the base ten counting system and how to use it when adding and subtracting. Our next lesson will be on the base 5 counting system, how it works, and how to add and subtract with it.

6.2 Unit Vocabulary

1. Place value**-** The worth of a digit based on its position in a number.

2. Infinite- Continuing forever with no end.

3. Addition (add, adding)- An arithmetic operation of combining two or more numbers to find a total.

4. Carry (carrying over places)- To transfer from one place or group to another while adding. Do this when having ten or more items in one group.

5. Subtraction (subtract, subtracting)- The arithmetic operation of taking the amount of one number away from another.

6. Borrow**-** To take a digit from one group in order to add ten units to the next smallest group while subtracting.

6.3 Review Questions

1. The ____________________ is thought to be used for counting because we have a total of ten fingers.
2. The value of a digit based on its position in a number is its _____________________.
3. ____________________ is taking away the amount of one number from the amount of another number.
4. If something continues forever without stopping it is considered ____________________.
5. ____________________ is to take ten units from one group and add them as one unit in the next highest group.
6. ____________________ is to take one unit from one group and add it as ten units to the next smallest group.
7. ____________________ is the concept of combining two or more numbers to find a total amount.
8. Our ____________ are probably the most important tool we can use when counting with the decimal system.

6.4 Review Answers

1. The ___decimal system___ is believed to be used for counting because we have a total of ten fingers.
2. The value of a digit based on its position in a number is its ____place value____.
3. ___subtracting____ is taking away the amount of one number from the amount of another number.
4. If something continues forever without stopping it is considered ____infinite________.
5. ____carrying________ is to take ten units from one group and add them as one unit in the next highest group.
6. _____borrowing______ is to take one unit from one group and add it as ten units to the next smallest group.
7. _____adding_________ is the concept of combining two or more numbers to find a total amount.
8. Our ___hands____ are probably the most important tool we can use when counting with the decimal system.

6.5 Base Ten Addition and Subtraction Exercises

1. 7 + 2	2. 5 + 2	3. 1 + 3
4. 6 + 3	5. 2 + 3	6. 13 +35
7. 52 +24	8. 79 + 20	9. 14 + 5
10. 9 - 3	11. 4 - 1	12. 8 - 6
13. 2 - 1	14. 6 - 3	15. 7 - 4
16. 36 - 17	17. 29 - 14	18. 41 - 30

6.6 Base Ten Addition and Subtraction Answers

1. $\begin{array}{r} 7 \\ +\ 2 \\ \hline 9 \end{array}$

2. $\begin{array}{r} 5 \\ +\ 2 \\ \hline 7 \end{array}$

3. $\begin{array}{r} 1 \\ +\ 3 \\ \hline 4 \end{array}$

4. $\begin{array}{r} 6 \\ +\ 3 \\ \hline 9 \end{array}$

5. $\begin{array}{r} 2 \\ +\ 3 \\ \hline 5 \end{array}$

6. $\begin{array}{r} 13 \\ +35 \\ \hline 48 \end{array}$

7. $\begin{array}{r} 52 \\ +24 \\ \hline 76 \end{array}$

8. $\begin{array}{r} 79 \\ +\ 20 \\ \hline 99 \end{array}$

9. $\begin{array}{r} 14 \\ +\ 5 \\ \hline 19 \end{array}$

10. $\begin{array}{r} 9 \\ -\ 3 \\ \hline 6 \end{array}$

11. $\begin{array}{r} 4 \\ -\ 1 \\ \hline 3 \end{array}$

12. $\begin{array}{r} 8 \\ -\ 6 \\ \hline 2 \end{array}$

13. $\begin{array}{r} 2 \\ -\ 1 \\ \hline 1 \end{array}$

14. $\begin{array}{r} 6 \\ -\ 3 \\ \hline 3 \end{array}$

15. $\begin{array}{r} 7 \\ -\ 4 \\ \hline 3 \end{array}$

16. $\begin{array}{r} 36 \\ -\ 15 \\ \hline 21 \end{array}$

17. $\begin{array}{r} 29 \\ -\ 14 \\ \hline 15 \end{array}$

18. $\begin{array}{r} 41 \\ -\ 30 \\ \hline 11 \end{array}$

6.7 Base Ten Addition and Subtraction Exercises

(Carry and Borrow)

1. $\begin{array}{r} 5 \\ \underline{+\ 7} \end{array}$	2. $\begin{array}{r} 9 \\ \underline{+\ 4} \end{array}$	3. $\begin{array}{r} 3 \\ \underline{+\ 9} \end{array}$
4. $\begin{array}{r} 18 \\ \underline{+59} \end{array}$	5. $\begin{array}{r} 77 \\ \underline{+18} \end{array}$	6. $\begin{array}{r} 39 \\ \underline{+51} \end{array}$
7. $\begin{array}{r} 16 \\ \underline{+87} \end{array}$	8. $\begin{array}{r} 53 \\ \underline{+62} \end{array}$	9. $\begin{array}{r} 76 \\ \underline{+45} \end{array}$
10. $\begin{array}{r} 10 \\ \underline{-\ 7} \end{array}$	11. $\begin{array}{r} 24 \\ \underline{-\ 8} \end{array}$	12. $\begin{array}{r} 31 \\ \underline{-\ 6} \end{array}$
13. $\begin{array}{r} 54 \\ \underline{-37} \end{array}$	14. $\begin{array}{r} 71 \\ \underline{-14} \end{array}$	15. $\begin{array}{r} 63 \\ \underline{-49} \end{array}$
16. $\begin{array}{r} 46 \\ \underline{-27} \end{array}$	17. $\begin{array}{r} 17 \\ \underline{-\ 9} \end{array}$	18. $\begin{array}{r} 90 \\ \underline{-81} \end{array}$

6.8 Base Ten Addition and Subtraction Answers

(Carry and Borrow)

1. 5 + 7 12	2. 9 + 4 13	3. 3 + 9 12
4. 18 +59 77	5. 77 +18 95	6. 39 +51 90
7. 16 +87 193	8. 53 +62 115	9. 76 +45 121
10. 10 - 7 3	11. 24 - 8 16	12. 31 - 6 25
13. 54 -37 17	14. 71 -14 57	15. 63 -49 14
16. 46 -27 19	17. 17 - 9 8	18. 90 -81 9

7 BASE FIVE

7.1 Introduction

Suppose that people were born with only one hand. On that hand everybody has five fingers. Do you think that we would still count with base ten system? The answer is probably not. We would more than likely only have a base five (quinary) system. We would still have the numbers 10 and 100 but they would mean something completely different. We will learn those differences as well as how to count, add, and subtract in base five in this lesson.

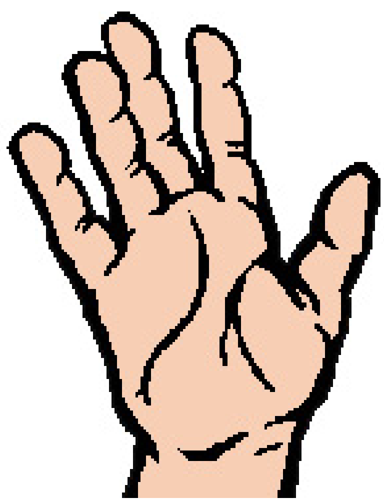


Figure 10: Single Hand showing 5 digits [3]

The base five system has been used multiple times through history and is still found in use in rare occasion today. The Romans used the quinary system as a sub-base to their decimal system. The Mayans used a base five system as part of their counting system as well. Also, until very recently, the Luo tribe in East Africa and the Yoruba tribe in Nigeria were using the base five system [1]. Today you can see the quinary system in the Muslim religion. The Muslim religion has 5 pillars of faith and prays 5 times a day to Allah. The amount of prayers was talked down from 50, which is 200 in the base five system [1].

The base 5 system of counting is similar to the decimal system. With this system you can still count, add, and subtract. In the base 5 system you have 5 single digit numbers (0, 1, 2, 3, 4.) That's it, that is all you get. That is all you need for this base system though.

When counting in the base five system, 4 is the largest single digit number. It acts like the number 9 in the decimal system. After the number 4 comes the first two digit number which is 10. The number ten does not mean the same in base 5 as it does in base ten. In the base ten system the number 10 means 0 groups of ones and 1 group of tens. In the base 5 system the number 10 represents the same 0 groups of ones but the 1 stands for how many groups of fives there are. If you are given the number 32 in the base 5 system, you would dissect it to 2 groups of ones and 3 groups of fives. In the decimal counting system 100 mean 0 groups of ones, 0 groups of tens, and 1 group of hundreds, or 10 x 10. To understand what 100 means in base 5 you need to remember how 100 equals 10 x 10 in base ten. In base 5, 100 equals 5 x 5 (or 25 in our base ten system). So 100 dissected in base 5 means 0 groups of ones, 0 groups of fives, and 1 group of twenty-fives. The number 413 stands for 3 groups of ones, 1 group of fives, and 4 groups of twenty-fives.

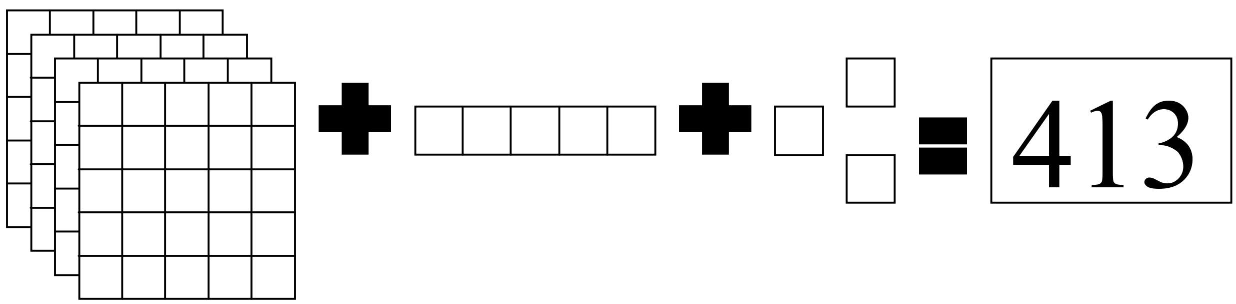


Figure 11: Example of place values in the Base five system

The place value system is still just as important in the base 5 system as it is in the base ten system. The places are just renamed as you can see from earlier. Place value starts with the ones place in base 5 but then continues with multiples of 5. The second place is the fives place or 5 x 1, the third is the twenty-fives place or 5 x 5, the fourth is the one hundred twenty-fives place or 25 x 5, and so on.

When counting in the base five system, it is important to remember not to count past four to the number 5. The number five is still there it is just written as the number 10. After the number 14, you will continue with the number 20. This is one of the more difficult things to remember when counting with base five because we are accustomed to counting 4, 5, 6.

Adding in the base five system is similar to adding in the base ten system. When given a set of two or more numbers, take one of the numbers and count the value of the next number higher until all numbers are counted. For example 1 + 2 will equal 3. Adding two digit, three digit, or multiple digit numbers works the same way as well, unless the total is five or above. If given the problem 12 + 11 you first add the ones position, then add the fives position, giving you the answer of

23.

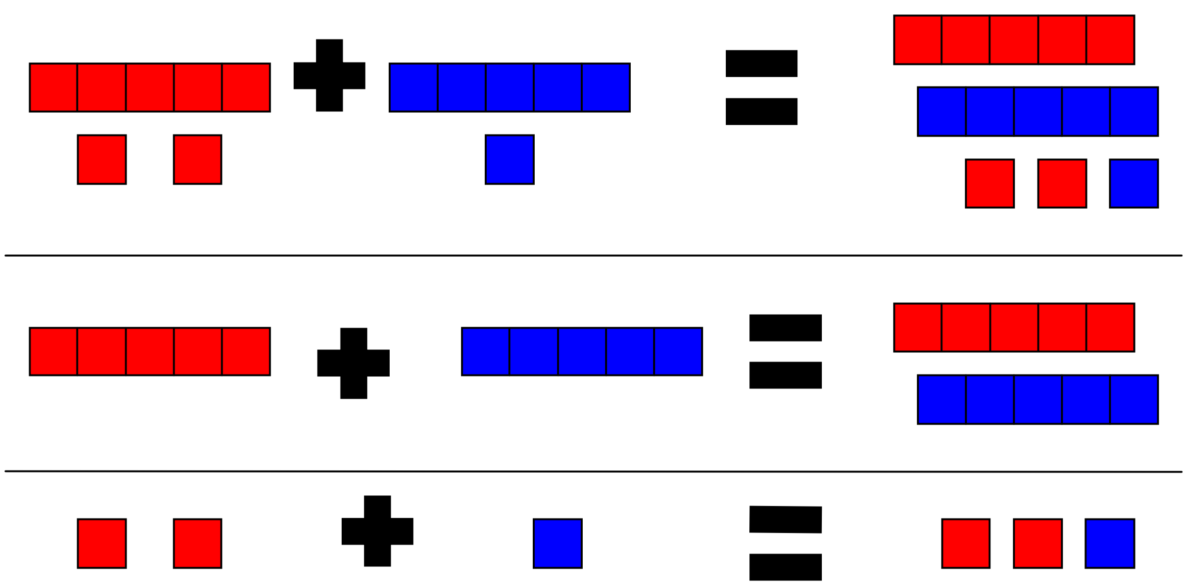


Figure 12: Two digit addition example of base five

When adding numbers that would normally equal 5 or above, we will carry over to the fives group. This is just like carrying over in the base ten system. Convert groups of 5 ones to 1 group of fives, or 5 groups of fives in to 1 group of twenty-fives.

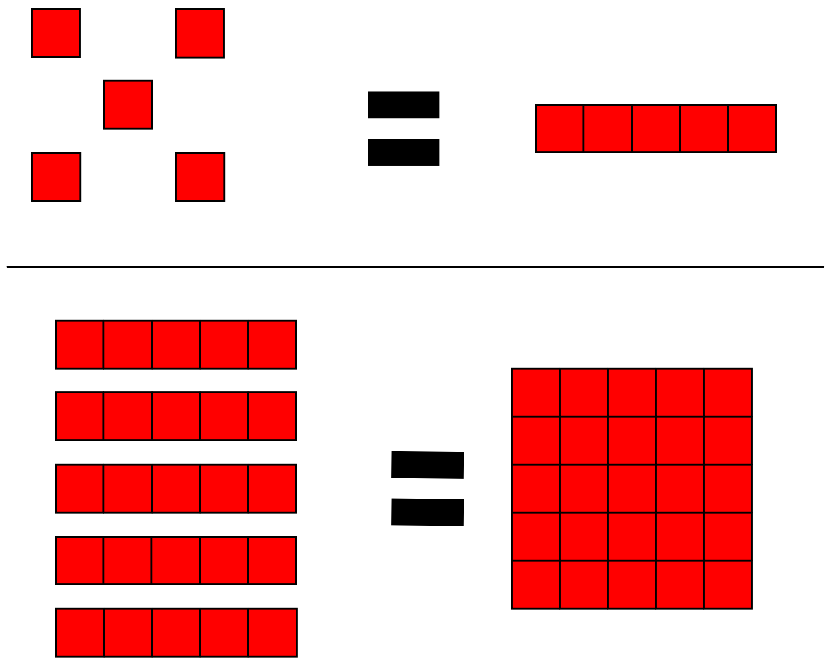


Figure 13: Example of place value in base five

Adding 13 + 23 is a problem in which you will carry over in base five. The first step of adding this problem is to add the ones group 3+3. After add these two numbers together you get a total of 6. You are not allowed 6 groups of ones in base five so you must carry one group of five to the next highest place, the fives group. This gives you 1 group of ones. You will then take the 1 group of fives that you carried and add it to the other two groups of five that you already have. This gives you 1+1+2, giving you the answer of 4 groups of fives. 4 groups of fives plus 1 group of ones gives you your answer of 41.

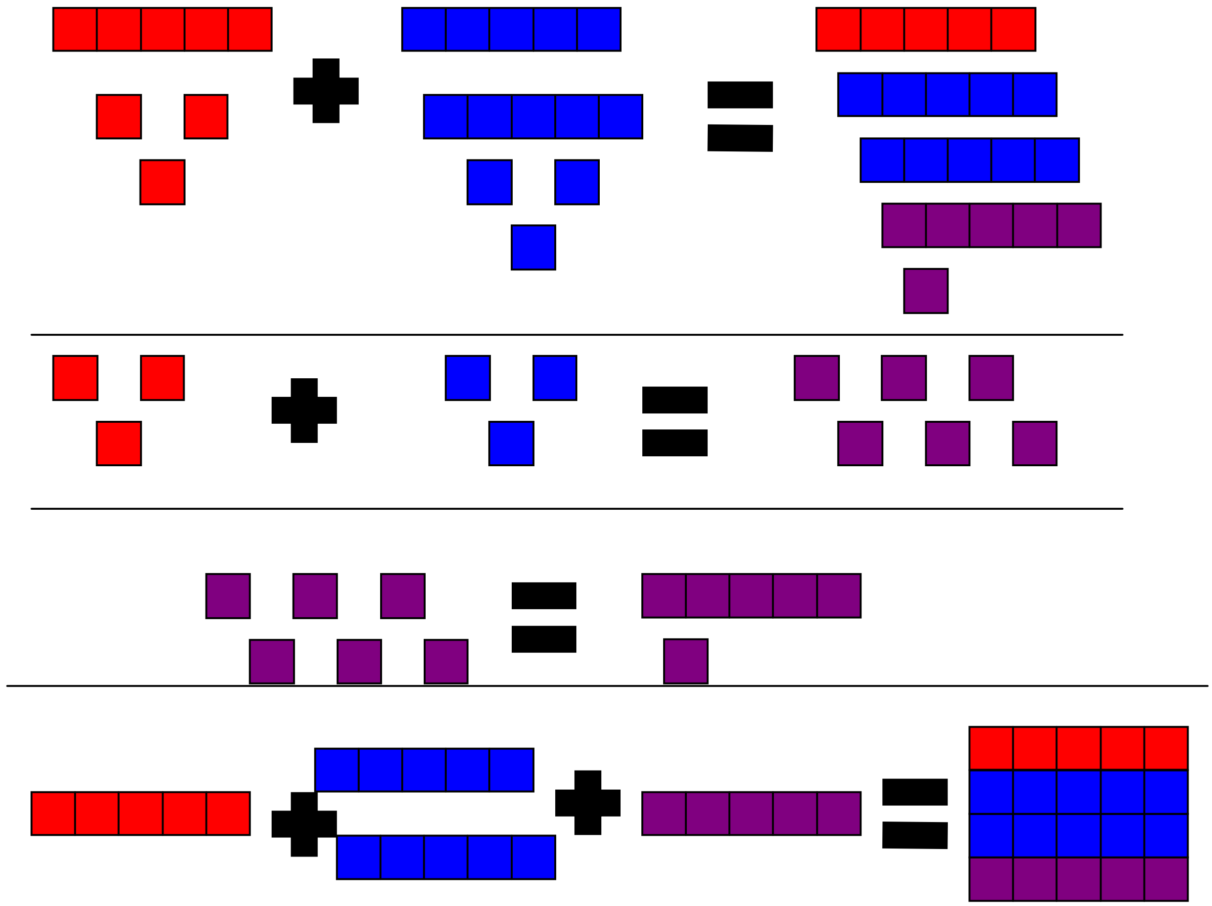


Figure 14: Two digit addition example with carrying

When adding two numbers from the fives group that add up to over 4, you will carry one group of five units over to the next highest group, the twenty-fives. Take the problem 40+21, first add the ones group 0+1, and then add the fives group 4+2. Adding 4+2 equals 6 which is impossible in base five. Carry one group of five units to the next highest group, leaving you with the answer 111.

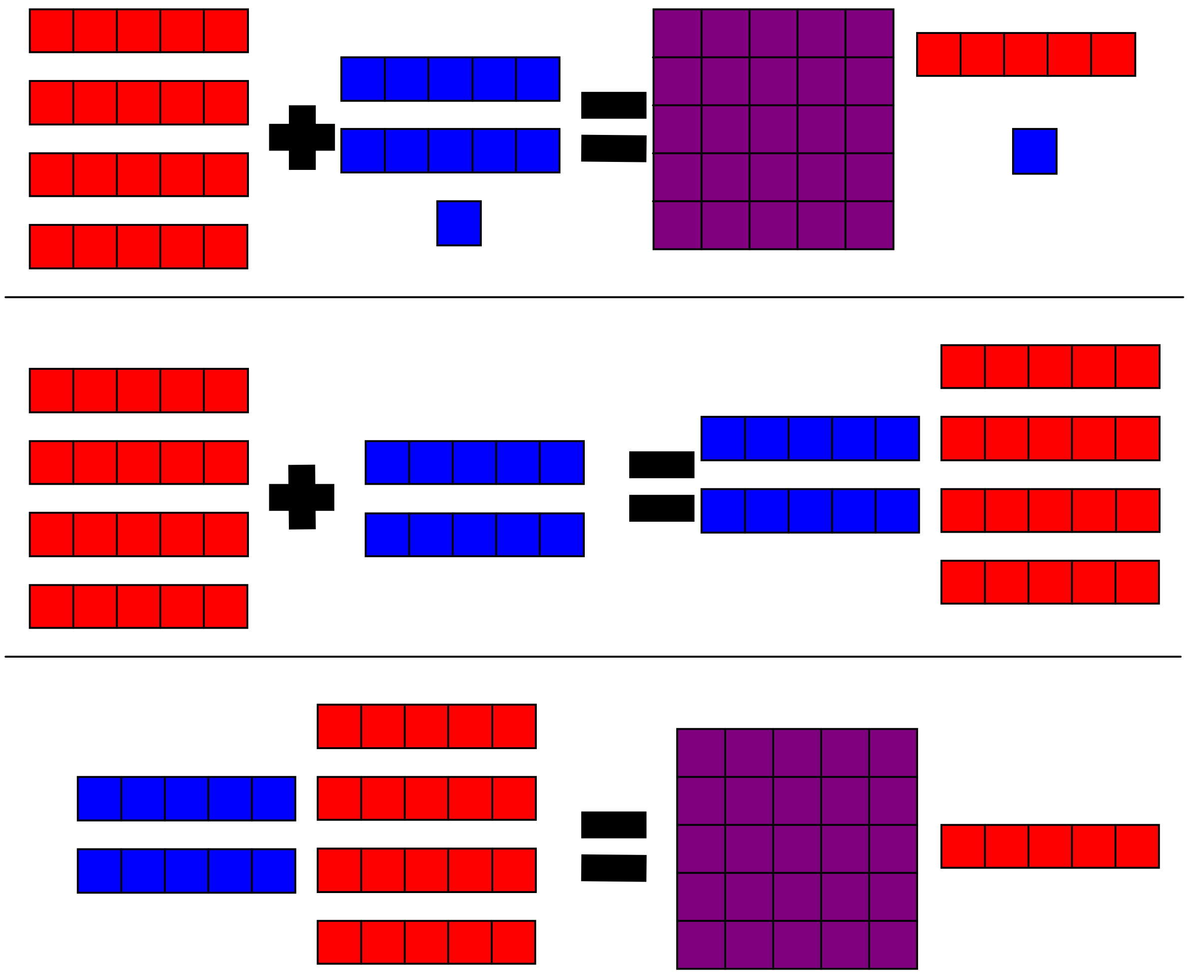


Figure 15: Two digit addition example with carrying

Subtraction problems are a little more complicated in the quinary system. Borrowing and subtracting in general can get a little confusing. It is very important to remember that 4 is the largest a single digit can be. I can not express this enough. When subtracting single digit numbers, you will only deal the numbers 4 and below. For example, subtracting 4-3 will give you the answer 1. If you subtract 10-4 you will also get the answer 1. This is because 10 means five in the quinary system.

When subtracting multiple digit numbers you first need to subtract the ones place. If the number you are subtracting is larger than the number you are subtracting from, you will need to borrow. As we saw in the base ten system, borrowing in the base five system means to bring over 1 unit from the next highest group. The difference is that this time instead of bringing a unit of ten over, you will bring 1 unit from the fives group. Although the concept behind borrowing changes, the practice remains the same. If subtracting 34 from 42, we will write out the problem 42-34. The next step is to subtract the ones place, 2-4. However, we can not subtract 2-4. We must borrow from the fives group. That would make the ones group 12-4 and the fives group becomes 3-3. Subtracting 12-4 equals 3 and 3-3 equals 0 this gives us an answer of 3.

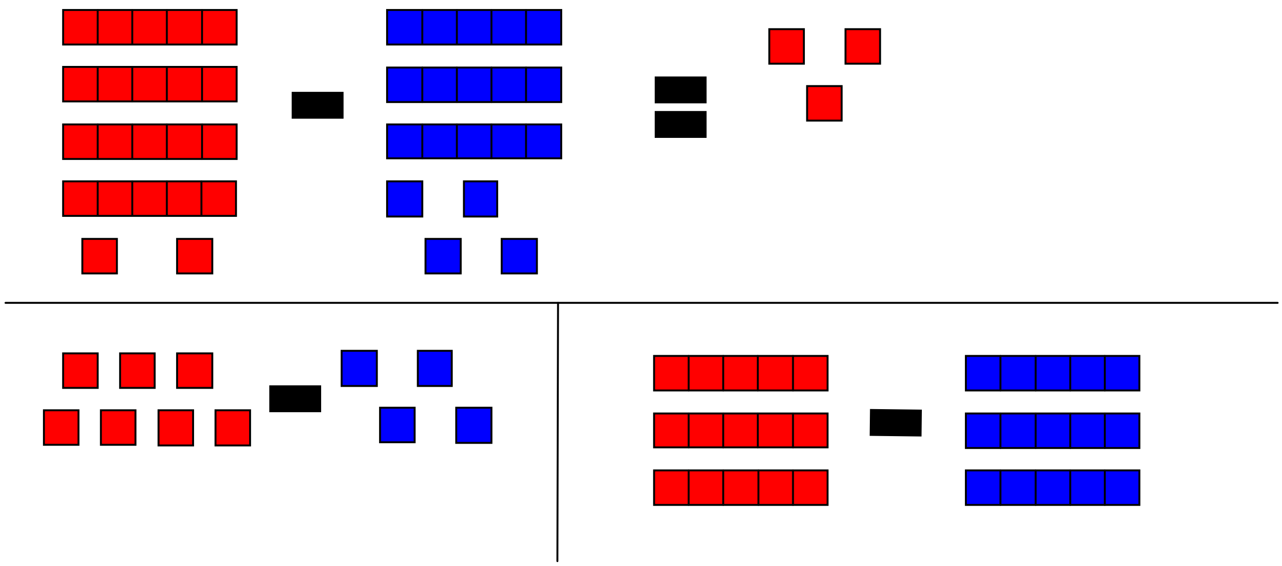


Figure 16: Two digit subtraction example in base five

That concludes this lesson on base five. In our next lesson we will look at the binary counting system, its uses, how to add and subtract, and how to convert the numbers to the base ten system.

7.2 Classroom Activity: Mengenlehreuhr

Base Five Clock [4]

In Germany, there is a clock that is set up to tell time in a base five system. It uses our modern 24 hour clock only it is split into blocks of fives and ones. During class today, I will give you eight base five clocks to read. Here is what to do on the first four clocks. First, tell me what time it is in base five. Second, tell me what time it is normally. Third, tell me what you would be doing at school during that time (reading in English or Playing in Gym). On the next four clocks, tell me by coloring the clocks when it is 1) Time for first period, 2) Time for lunch, 3) Time for study hall, and 4) Time to leave.

On the clock, the top row each light counts a group of five hours. In the second row each light counts 1 hour between the top row's five hour periods. The third row counts five minute periods in each hour, and the bottom row counts single minutes in between the third row's five minute periods.

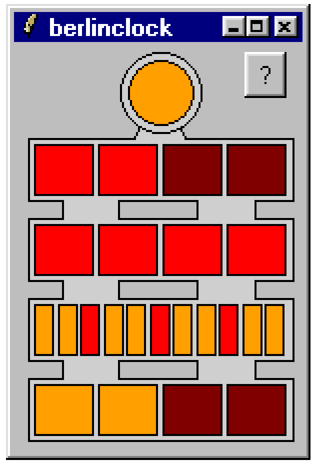
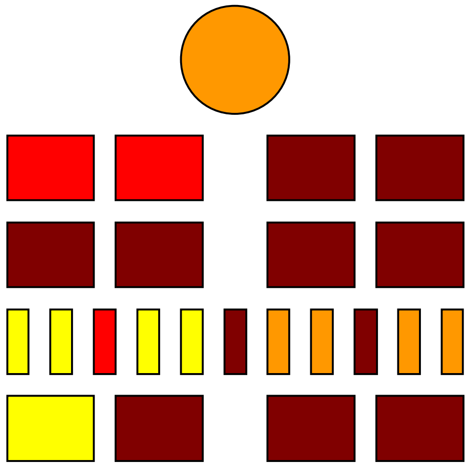


Figure 17: Example of base five clock [4]

For example, this clock shows the time of 14:57 or 2:57 pm. The top row shows 2 five hour blocks lit, or 10 hours. The second row shows 4 one hour blocks or 4 hours. The third row shows 11 five minute blocks or 55 minutes and the last row shows 2 one minute blocks or 2 minutes. Therefore, the time on the clock is 10 hours +4 hours +55 minutes +2 minutes which equals 14:57.



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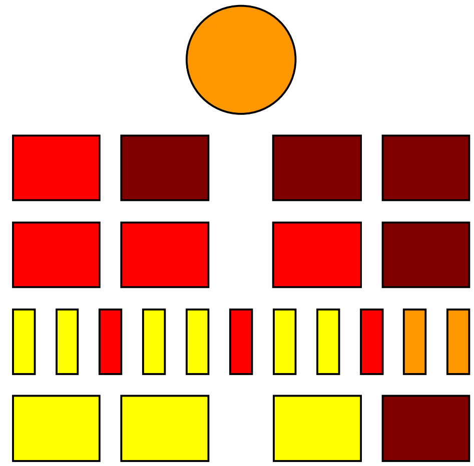

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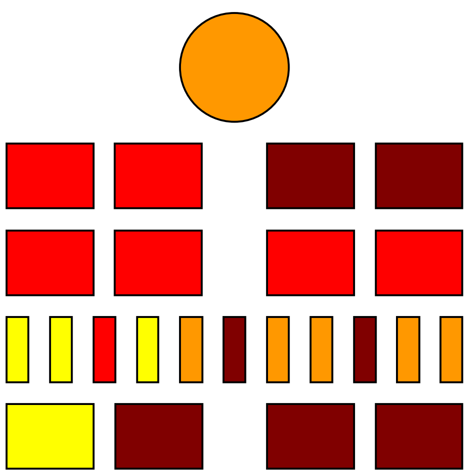

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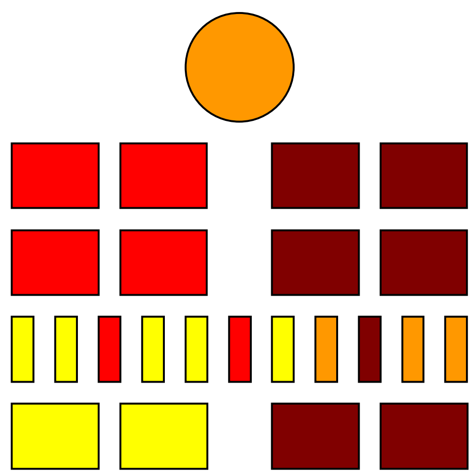

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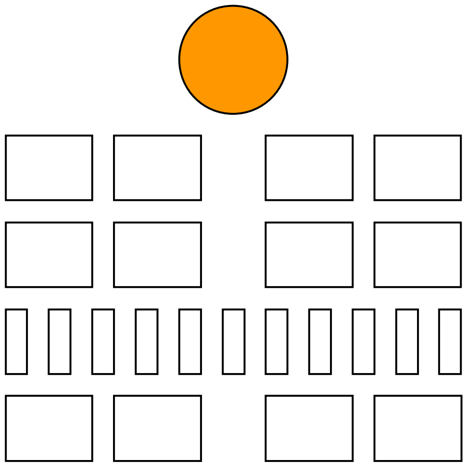
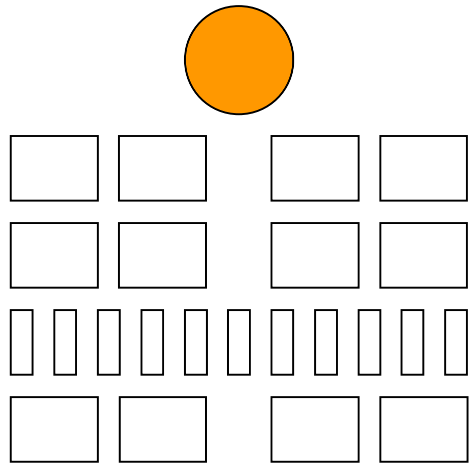
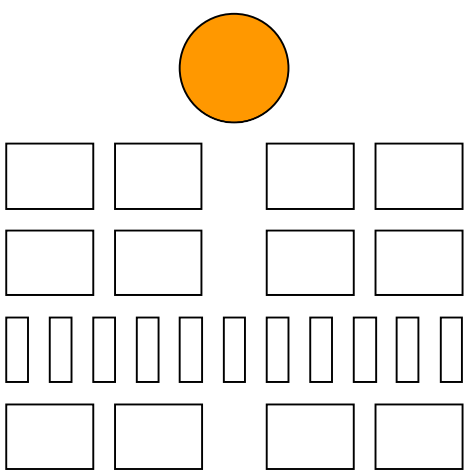


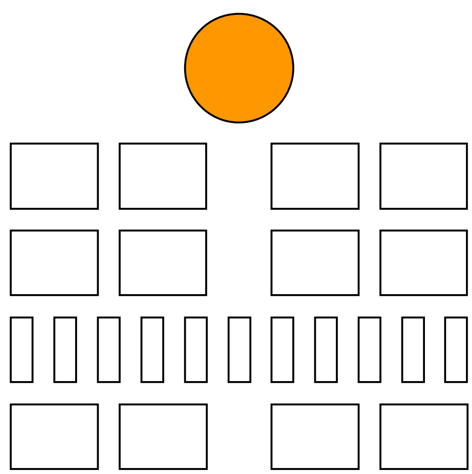
1. Time for 1st period



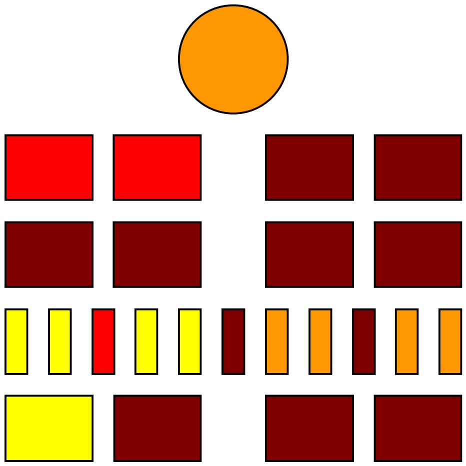
2. Time for lunch



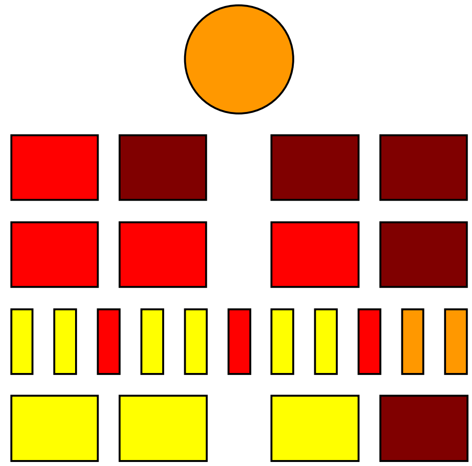
3. Time for study hall



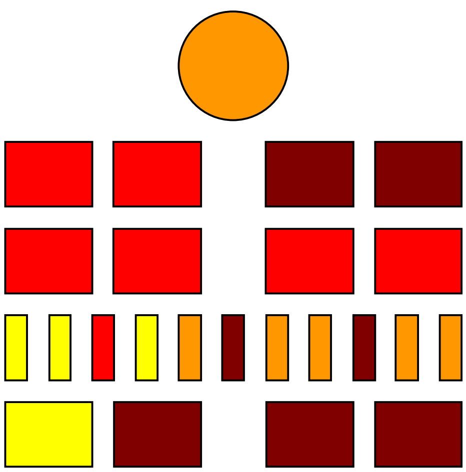
4. Time to leave



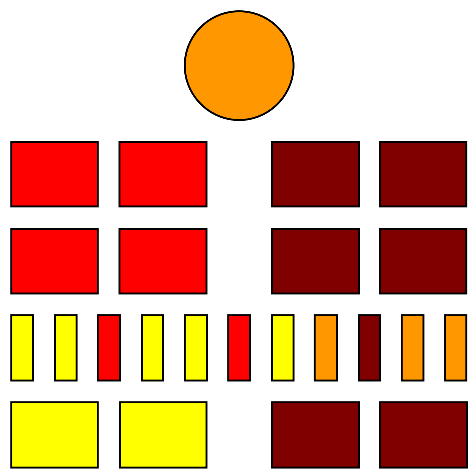
1. 20:51
2. 10:26
3. Answer Varies



1. 13:93
2. 8:48
3. Answer Varies



1. 24:41
2. 2:21
3. Answer Varies



1. 22:72
2. 12:37
3. Answer Varies

7.3 Base Five Manipulatives

Cut out the manipulatives on the following pages. Use in class to demonstrate and solve base five problems.

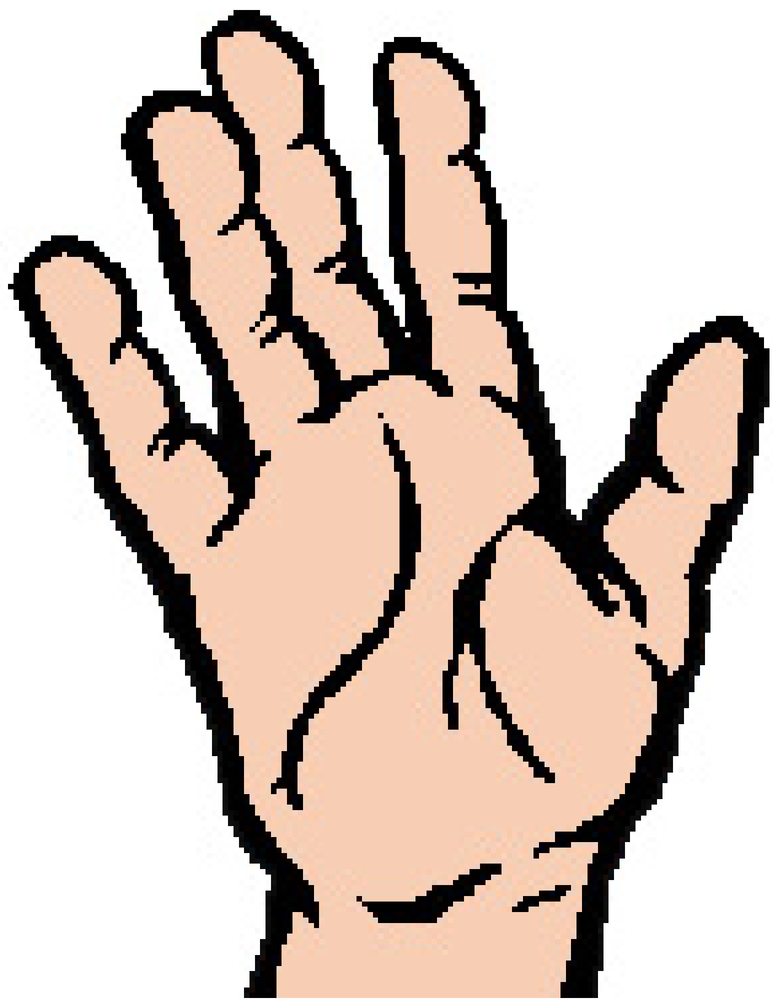


Figure 18: Base five manipulative hand [3]

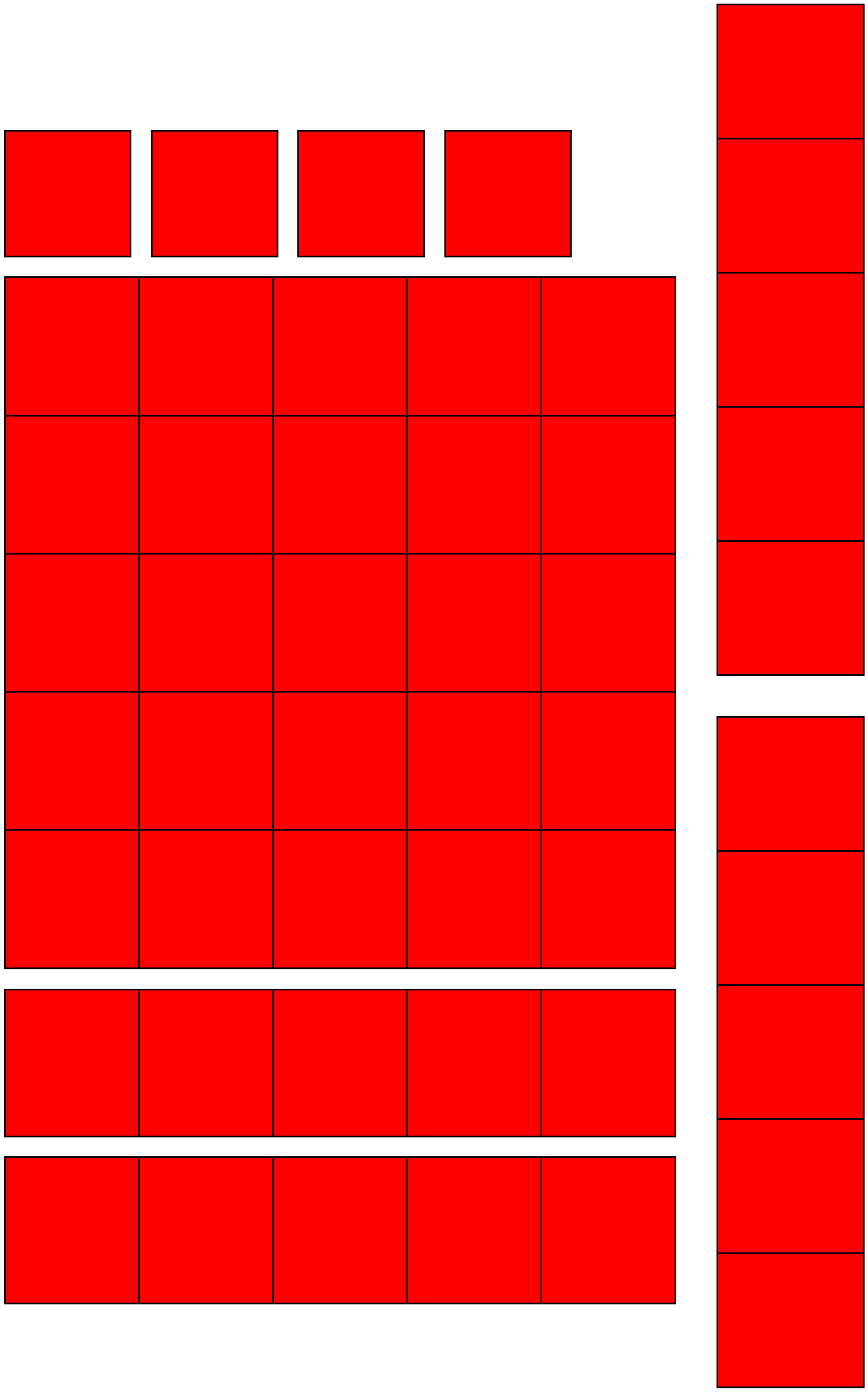


Figure 19: Base five manipulative blocks

7.4 Base Five Exercises

1. 2 +1	2. 1 +3	3. 3 +0
4. 4 +1	5. 2 +5	6. 3 +4
7. 13 +11	8. 21 +32	9. 44 + 1
10. 4 - 1	11. 2 - 1	12. 3 - 2
13. 12 - 4	14. 21 - 12	15. 42 - 24

7.5 Base Five Exercise Answers

1. $\begin{array}{r} 2 \\ \underline{+1} \\ 3 \end{array}$

2. $\begin{array}{r} 1 \\ \underline{+3} \\ 4 \end{array}$

3. $\begin{array}{r} 3 \\ \underline{+0} \\ 3 \end{array}$

4. $\begin{array}{r} 4 \\ \underline{+1} \\ 10 \end{array}$

5. $\begin{array}{r} 2 \\ \underline{+3} \\ 10 \end{array}$

6. $\begin{array}{r} 3 \\ \underline{+4} \\ 12 \end{array}$

7. $\begin{array}{r} 13 \\ \underline{+11} \\ 30 \end{array}$

8. $\begin{array}{r} 21 \\ \underline{+32} \\ 103 \end{array}$

9. $\begin{array}{r} 44 \\ \underline{+\ \ 1} \\ 100 \end{array}$

10. $\begin{array}{r} 4 \\ \underline{-1} \\ 3 \end{array}$

11. $\begin{array}{r} 2 \\ \underline{-1} \\ 1 \end{array}$

12. $\begin{array}{r} 3 \\ \underline{-2} \\ 1 \end{array}$

13. $\begin{array}{r} 12 \\ \underline{-\ \ 4} \\ 3 \end{array}$

14. $\begin{array}{r} 21 \\ \underline{-12} \\ 3 \end{array}$

15. $\begin{array}{r} 42 \\ \underline{-24} \\ 13 \end{array}$

7.6 Base Five Review Questions

1. The _____________ system is counting with only using the numbers 0, 1, 2, 3, 4.
2. The ____________ and the ____________ are 2 ancient civilizations that used the quinary system with their counting and math.
3. The _________ and ____________ tribes have been known to use the base five system up until very recently.
4. The _____________ religion used the base five system when founding the practices of the religion.
5. The number ____ is the largest single digit used in the quinary system.
6. The number ____ represents the number 5 from the decimal system.
7. The base ten number ____ is written as 100 in the base five system.
8. The base-5 clock, Mengenlehreuhr, is located in _____________.
9. The base five system of counting is set up to count assuming you only have ______ fingers.

7.7 Base Five Review Answers

1. The __quinary/base 5__ system is counting with only using the numbers 0, 1, 2, 3, 4.

2. The __Romans__ and the ___Mayans__ are 2 ancient civilizations that used the quinary system with their counting and math.

3. The __Luo_______ and __Yoruba__ tribes have been known to use the base five system up until very recently.

4. The __Muslim__ religion used the base five system when founding the practices of the religion.

5. The number __4__ is the largest single digit used in the quinary system.

6. The number __10__ represents the number 5 from the decimal system.

7. The base ten number __25__ is written as 100 in the base five system.

8. The base-5 clock, Mengenlehreuhr, is located in __Germany__.

9. The base five system of counting is set up to count assuming you only have __5____ fingers.

8 BINARY COUNTING SYSTEM

8.1 Introduction

The binary counting system was first discovered around the same time as the number 0. The modern version of the system came about sometime in the 17^{th} century [1]. The system itself uses two numbers, 0 and 1, and therefore is recognized as the base two counting system.

Binary counting has been used for many things throughout history. Most recently though, it has been one of the main forces in making computers and electronic devices work. The binary system sets up a simple code for turning on and off switches. The electronic switches, in turn tell the device what needs or does not need to be done by using the number 0 to represent off and the number 1 to represent on. For example, if we had a very simple device that had a light and a bell we could tell it what to do by using the binary code. If we wanted to use the light and bell both we would tell the device the number 11. If we didn't want to use either we use the code of 00. Now, the great thing about using the binary code is if we want to turn on the light but not the bell we would simply send the number 10 to the device. For each 1 or 0, the device has what is called a bit. A bit is an acronym for binary digit; this is one unit of information that can give two possible outcomes. The device above has two bits. Computers and other technology we use today have what are called gigabytes. Considering that a byte is 8 bits and a gigabyte is roughly 1billion bytes, you can kind of understand the power of today's electronics. For example, a small i-Pod has 2 gigabytes of storage. That is 2 billion bytes, or around 16 billion bits. We can not even begin to comprehend the size of that number, and that is what is found in a handheld device. After this lesson you will understand why.

The binary code is also used for encryption. Encryption is the process of encoding a number to make it harder or, hopefully, impossible to steal information. Encryption uses prime numbers, very large prime numbers, around 30 to 40 digits long. These numbers are translated to binary code to be understood by the computer. When these numbers are sent to another computer, it decodes the information and translates the message back to its original form.

The binary code is the base two counting system. This means that you are using 2 single digit numbers, the 0 and the 1. The numbers above 1 can not be used. So the number 2 would now be written as 10 because it is the first number above the number 1. The binary system continues on 11, 100, 101, 110, 111, and so on.

Compared to the decimal system of counting, the binary system only has two numbers with the same meaning. The number 2 is written as the first two digit number 10. The number 4 is the first 3 digit number being written as 100.

The place value system is still a matter of concern when using the binary system. The first place still represents the ones group. The next place represents how many groups of two there are. The third place represents how many groups of 4 there are. The fourth place would represent how many groups of eight there are and so on.

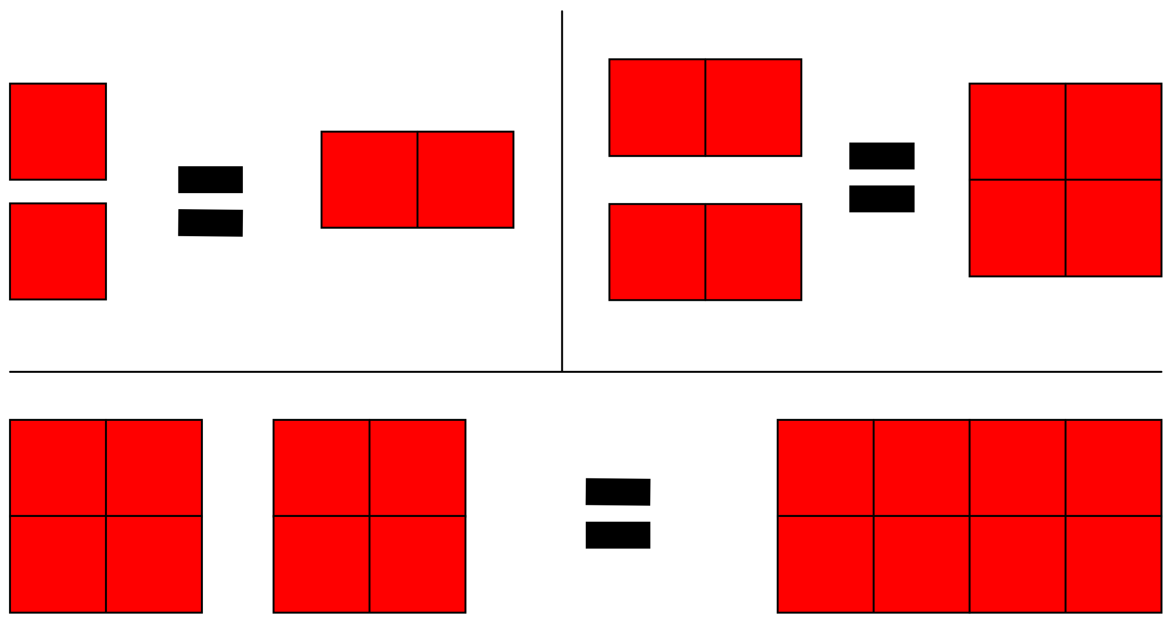


Figure 20: Examples of place values in base two system

When using base 2, the numbers can get long and complicated. It can be very easy to start with small numbers and end with numbers that drag on. The key to adding in the binary system is to remember that we are only dealing with two numbers, 0 and 1. So, 1+1 can not give you the answer 2. Your answer will be 10 or 1 group of twos and 0 groups of ones. This is because 1 is the highest single digit number.

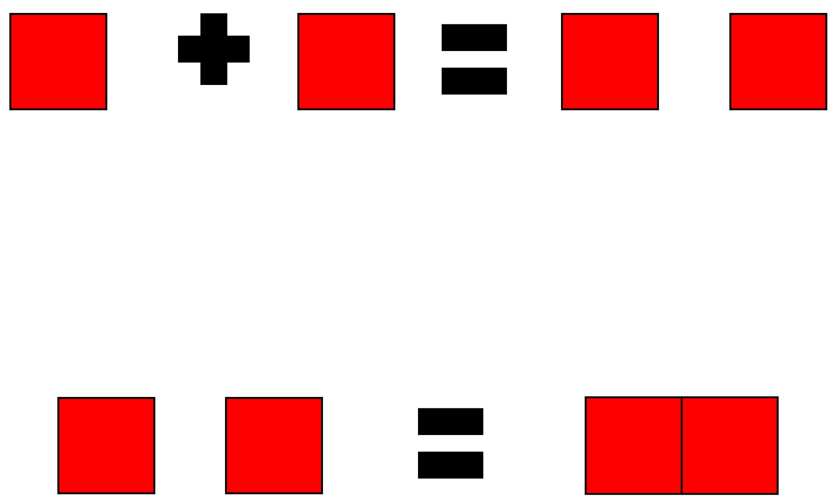


Figure 21: Single digit addition example in base two

When you get up to three or four digit numbers or higher, you do get a little more to work with but there is still a lot of carrying involved. If you add the numbers 10110+1010 your will first add your ones place 0+0 which gives you the answer of 0 ones. Next add the twos place 1+1. This gives you the answer of 10 groups of 2. We will carry the 1 to the next highest place, the fours. Next we will add the fours place plus the one you carried over before giving us 1+0+1. Adding these three together gives us the answer of 10 groups of four. We carry the 1 again and that gives us the answer of 0 groups of four. For the next place we will add 0+1+1. We will again get the answer of 10 this time for the eights place. Again we need to carry the one giving us 0 eights. Finally we will add the sixteens place 1+0+1. This again gives us an answer of 10. So, we carry the 1 one last time giving us 1 group of 32 and 0 groups of 16. Adding 1 thirty-twos +0 sixteens +0 eights +0 fours +0 twos +0 ones equals 100000.

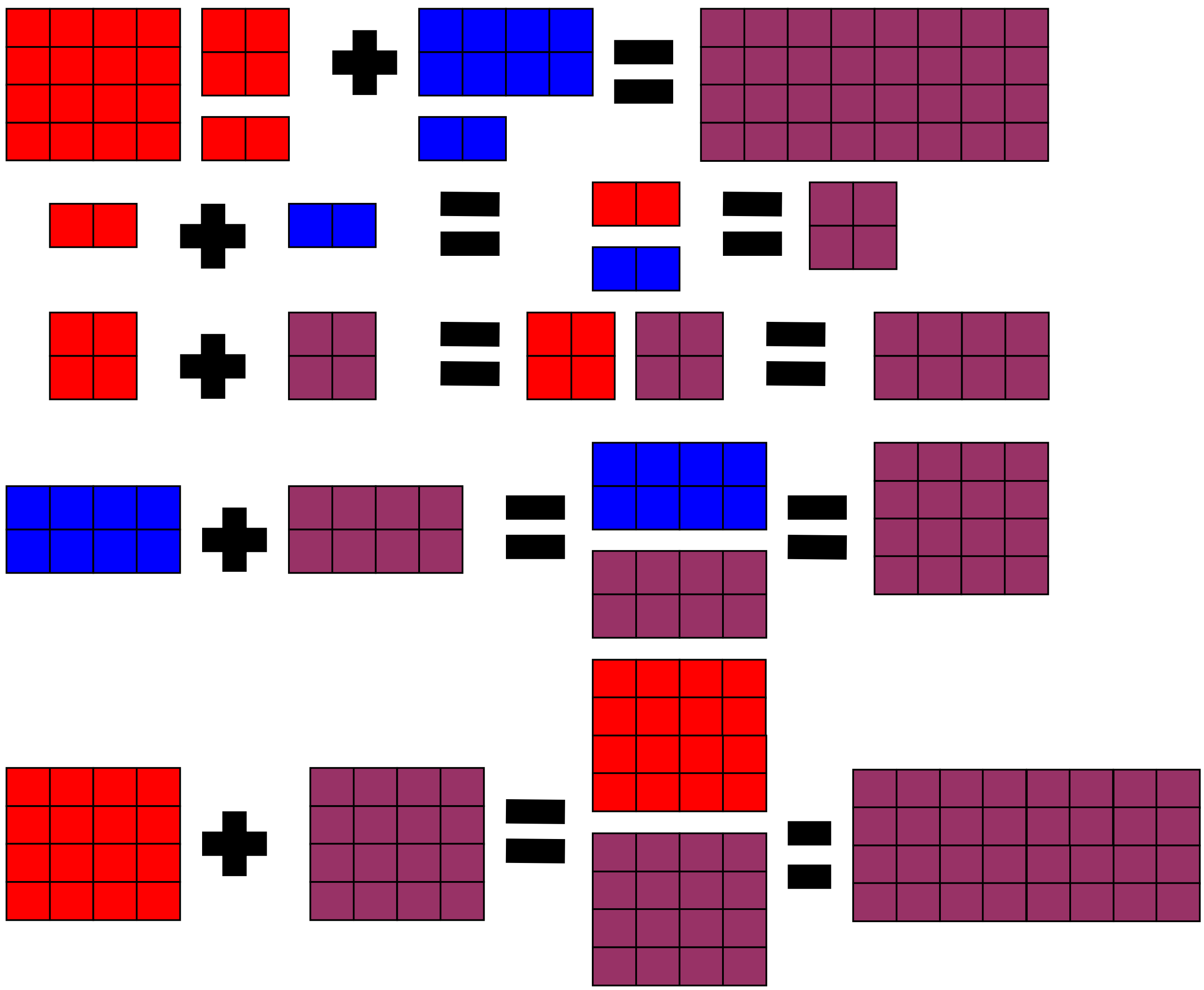


Figure 22: Addition example in base two

Subtracting is just as complicating as adding base two if not more difficult. Where we continuously carried in addition problems, we will be borrowing a lot in the subtraction problems. Subtracting 1-0 equals 1, 0-0, and 1-1 equals 0; they are the only single digit subtraction problems there are (that give you positive answers). There are a few more two digit subtraction problems and the amount grows pretty fast after that as long as you can borrow. Take the problem 10101-1111, we will subtract the ones place first, 1-1. The answer for the ones place is 0. Next, we will subtract the twos place, 0-1. We can't do this unless we borrow from the fours place. So now we have 10-1 giving us the answer of 1 group of twos. This left 0 in the fours place so we will subtract 0-1. Again, we need to borrow. Look at the next place, the eights. It has a 0 in its place so we will need to borrow from the sixteens place. This gives us 10 groups of eights. If we borrow from those groups of eights it will leave us with 1 group of sixteen and it will give us 10 groups of fours. So now we can subtract the fours place 10-1. This gives us the answer of 1. Since we borrowed the only group in the sixteens place our last step is to subtract the eights place which is now 1-1. So the answers are 0 groups of sixteens, 0 groups of eights, 1 group of fours, 1 group of twos, and 0 groups of 1, or 0 eights +1 four+1 two +0 ones, or 110.

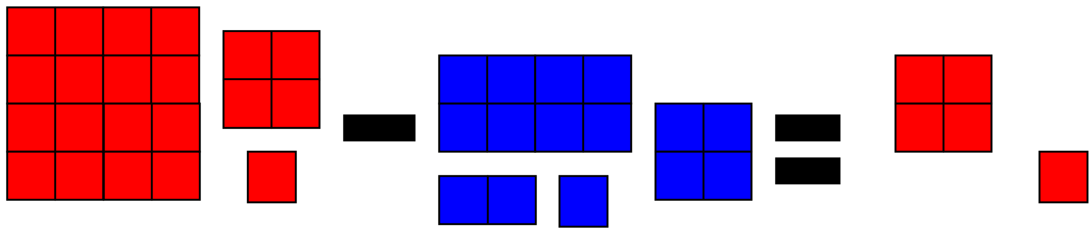


Figure 23: Subtraction example in base two

It is confusing, but with a little time and work the binary system is easy to get used to. The numbers are simple and this can be a great practice for carrying and borrowing, whichever base you are using. This concludes our lesson on the binary counting system, and with that I will tell you another bad joke. There are only 10 types of people in the world, those that understand binary code and those who don't. In our next lesson we will be looking at various other base counting systems including a few above ten, such as base twelve and base twenty.

8.2 Binary Manipulatives

Cut out the manipulatives on this page. Use in class to demonstrate and solve base two problems.

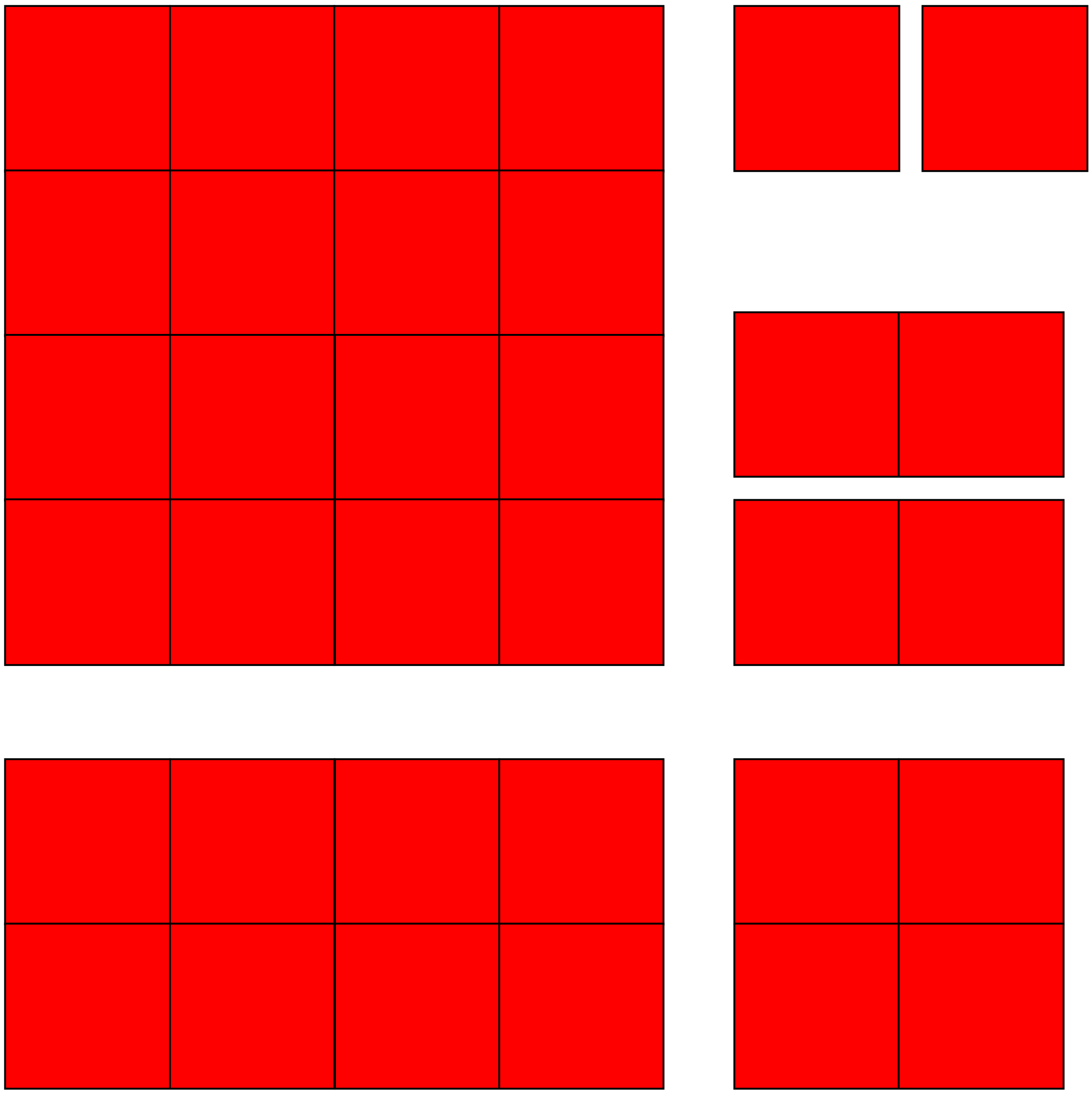


Figure 24: Base two manipulative blocks

8.3 Unit Vocabulary

Binary- Counting system using a base of 2; uses the numbers 0 and 1 to represent all numbers.

Bit- Acronym for binary digit

Binary Digit- A peace of information used in computers and electronic devices to store and share information by using the numbers 0 and 1.

Byte- A group of eight binary digits (bit).

Gigabyte- A group of 1 billion bytes.

Encryption- used to encode information so as to make it harder or impossible to steal or read the information.

Encoding- Translating information into a secret language known only to the people involved.

Decoding- Translating information back from an encoded state.

Prime Numbers- Number that can only be divided by itself and the number 1. The set of prime numbers does not include the number 1.

8.4 Vocabulary Review

Prime numbers, Bit, Binary, Encoding, Gigabyte, Encryption, Binary digit, Decoding, Byte

1. __________________ A group of eight binary digits.
2. __________________ Translating information back from an encoded state.
3. __________________ Acronym for binary digit.
4. __________________ Used to encode information so as to make it harder or impossible to steal or read the information.
5. __________________ Information back from an encoded state.
6. __________________Number that can only be divided by itself and the number 1. Does not include the number 1.
7. __________________ Translating information into a secret language known only to the people involved.
8. __________________ Counting system using a base of 2; uses the numbers 0 and 1 to represent all numbers.
9. __________________ A group of 1 billion bytes.

8.5 Vocabulary Answers

Prime numbers, Bit, Binary, Encoding, Gigabyte, Encryption,
Binary digit, Decoding, Byte

1. ____Byte_____ A group of eight binary digits.
2. ____Decode______ Translating information back from an encoded state.
3. ____Bit_____ Acronym for binary digit.
4. _____Encryption____ Used to encode information so as to make it harder or impossible to steal or read the information.
5. ____Decoding ___ translating information back from an encoded state.
6. _____Prime Number____Number that can only be divided by itself and the number 1. Does not include the number 1.
7. ____Encoding__ Translating information into a secret language known only to the people involved.
8. ____Binary____ Counting system using a base of 2; uses the numbers 0 and 1 to represent all numbers.
9. _____Gigabyte_____ A group of 1 billion bytes.

8.6 Review Questions

1. The modern version of the binary counting system was first used in the ________ century.
2. The system uses only two numbers ____ and ____ to represent the amount of whatever you are counting.
3. The main use of the binary system today is to give instructions to _____________ and _____________ _____________.
4. A ______ or ___________ ___________ is one unit of stored information on a computer or electronic device.
5. Eight bits of information on a computer can be grouped together to make 1 ___________.
6. One billion bytes equal 1 __________________.
7. ________________ is another use for the binary code translating messages to make them harder to read.
8. ________________ is the process of making a message hard to read and safe to send over unsecured connections.
9. To translate a message back from its coded form is called __________.
10. The base two number 100 translated into a base ten number would equal ____________.

8.7 Review Answers

1. The modern version of the binary counting system was first used in the __17th______ century.
2. The system uses only two numbers __0__ and __1__ to represent the amount of whatever you are counting.
3. The main use of the binary system today is to give instructions to __computers__ and ___electronic__ ____devices__.
4. A __bit_ or _Binary_ __Digit__ is one unit of stored information on a computer or electronic device.
5. Eight bits of information on a computer can be grouped together to make 1 __Byte__.
6. One billion bytes equal 1 ___gigabyte___.
7. __Encryption___ is another use for the binary code translating messages to make them harder to read.
8. ____encoding__ is the process of making a message hard to read and safe to send over unsecured connections.
9. To translate a message back from its coded form is called _decoding_.
10. The base two number 100 translated into a base ten number would equal ___4______.

8.8 Binary Exercises

1. 1 +1	2. 110 +11	3. 1011 + 100
4. 111 +101	5. 1001 +11111	6. 1100 +1101
7. 110 - 10	8. 11 - 1	9. 1111 - 111
10. 1001 - 101	11. 1110 -1000	12. 11110 - 1111
13. 10001 - 1011	14. 10011 - 101	15. 11011 - 1111

8.9 Binary Exercises Answers

1. 1 +1 10	2. 110 +11 1001	3. 1011 + 100 1111
4. 111 +101 1100	5. 1001 +11111 101000	6. 1100 +1101 11001
7. 110 - 10 100	8. 11 - 1 10	9. 1111 - 111 1000
10. 1001 - 101 100	11. 1110 -1000 110	12. 11110 - 1111 1111
13. 10001 - 1011 110	14. 10011 - 101 1110	15. 11011 - 1111 1100

9 OTHER BASE SYSTEMS

9.1 Introduction

We have covered some of the more known base systems, including our own base ten system, so now we are going to look at a few other systems. We will focus on three more bases especially, but we will look at how there are a countless number of bases that can be used.

There are an infinite number of base counting systems. All systems can be used, but most are not practical. For example, we could use a base counting system of 173 but it would be very long, drawn out, and confusing. A base of 200 is also very long and drawn out, but it is much more simple than a base of 173.

All of the systems we have talked about so far have been smaller than our base ten counting system. We are going to talk about some systems that go past the number 10 now. There are three major systems that do this, they are the duodecimal (base 12), the vigesimal (base 20), and the sexagesimal (base 60). All of these have been used in history by major civilizations.

The sexagesimal system was used by the Sumerians, one of the first known civilizations [1]. It is believed that the base sixty system was used because of its being easily divisible by many numbers. It can be divided by 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. This makes it fairly easy to use fractions. This system can be seen today by checking the time or by measuring angles and geographic coordinates [1].

The base 20 or vigesimal counting system was believed to first be used by the Mayan civilization. The base 20 system is believed to be connected to the amount of fingers and toes people have. The Mayans used this system in their calendar as well as with everyday computations [1].

The base 12 or duodecimal system has been used by the Romans, Chinese, Europe, and in the United States [1]. The base 12 system is very useful because it is easily divisible by many of the numbers smaller than it is. The system has been used for calendars, for time, and with compasses to tell more specific directions [1].

When using any of these systems for counting, the number 10 will represent the first two digit number of the system. Therefore, in the base 12 system the number 10 means twelve. The number ten could be written as the letter a. The number eleven would be written as the number b. So counting in base 12, we would say 8, 9, a, b, 10. The base twenty system, would use letters a, b, c, d, e, f, g, h, i, and j before we would reach the number 10. In a base sixty system, we would use all single digit numbers, all letters, and then some other various symbols.

The base 12 system is the easiest of the three systems we are talking about and we will use it to get used to adding and subtracting. The numbers 0-9 all represent the same amount that we are used to with the decimal system. The letters a and b represent the numbers ten and eleven, and the number 10 represents the amount twelve. So when counting we would have the numbers 9, a, b, and then 10. With two digit numbers such as teen's and twenty's we would count 19, 1a, 1b, 20… 29, 2a, 2b, 30. This continues until we reach a point where we would use the number 100 in the decimal system. In base twelve, we continue past 99 before reaching 100. We have the units a and b to place in the 12's position. After 99 we count 9a, 9b, a0… a9, aa, ab, b0… b9, ba, bb, 100. I stress again the importance of place value and the positions of the digits written in a number. The third place in the base 12 system is the 144's place.

When adding in the base twelve system, the numbers 1 through 9 add the same way as in the decimal system. Putting the numbers a and b in the system makes it a little tricky though. The problem 5+5 does not equal 10, it equals a. Adding 6+6 equals 10 in the duodecimal system. It is a bit complicated but with a little practice it can be as easy as adding in the base ten system.

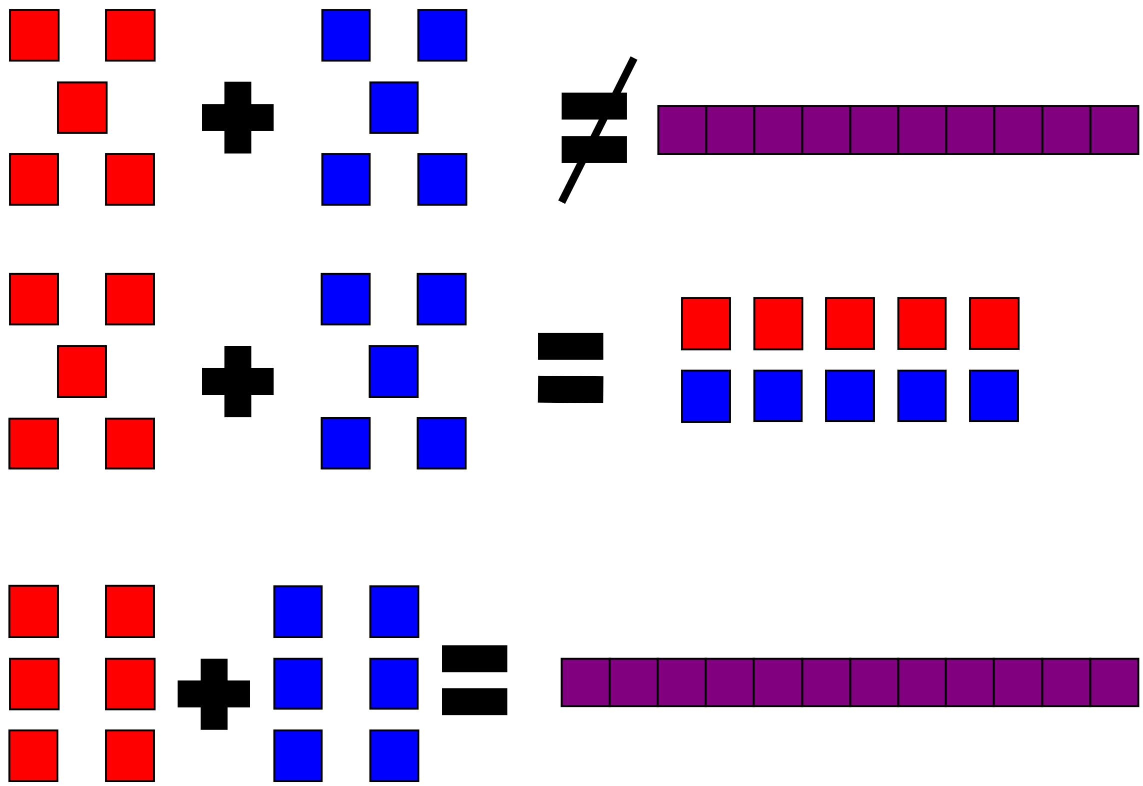


Figure 25: Example of how 6+6 =10 in base twelve system

When adding a or b to another number, remember their value. The number a has a value of ten units, the number b has a value of eleven units. Notice I didn't put a=10 because it doesn't. The number 10 has a value of twelve. When adding a+3 you will start at the number a and count 3 units, b, 10, 11. The answer is 11. If you are adding 4+b start at the number 4 and count on b units 5, 6, 7, 8, 9, a, b, 10, 11, 12, 13.

When carrying in the base 12 system, you apply the same rules as you would with the base ten system. If the numbers you add total 10 or higher you take 1 group of 10 and place it in the next highest group. For example, if we add 16+26, the addition of 6+6 equals 10. You will take 1 group of 10 and place it in the next highest group, the twelve's. That leaves us with 0 groups of ones. The next step is to add the 12's place. That will be 1+2 plus the 1 we carried or 1+2+1. These total up to 4 groups of 12's. We will add the 4 groups of 12 + the 0 groups left over after carrying from the ones group. This gives us the answer 40.

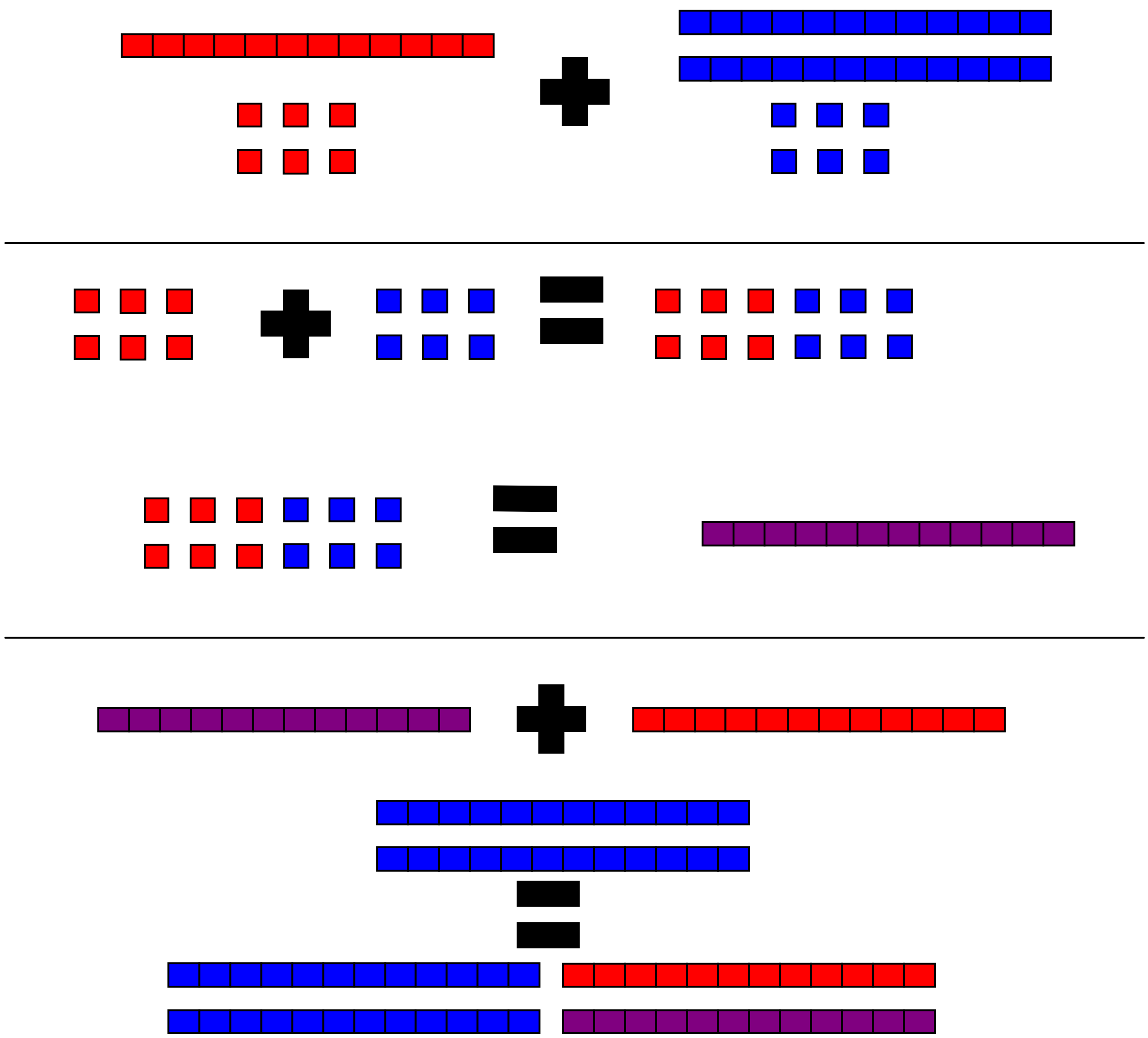


Figure 26: Addition example in base twelve

If subtracting in base twelve you must remember that 10 equals twelve. Subtracting 10-9 does not give you 1, the answer to that problem is 3. You have the numbers a and b that changes values of numbers from the base ten system. All the numbers from 9 to 0 stay the same. The number a represents ten and the number b represents the value eleven. So 10-b will give you the answer of 1.

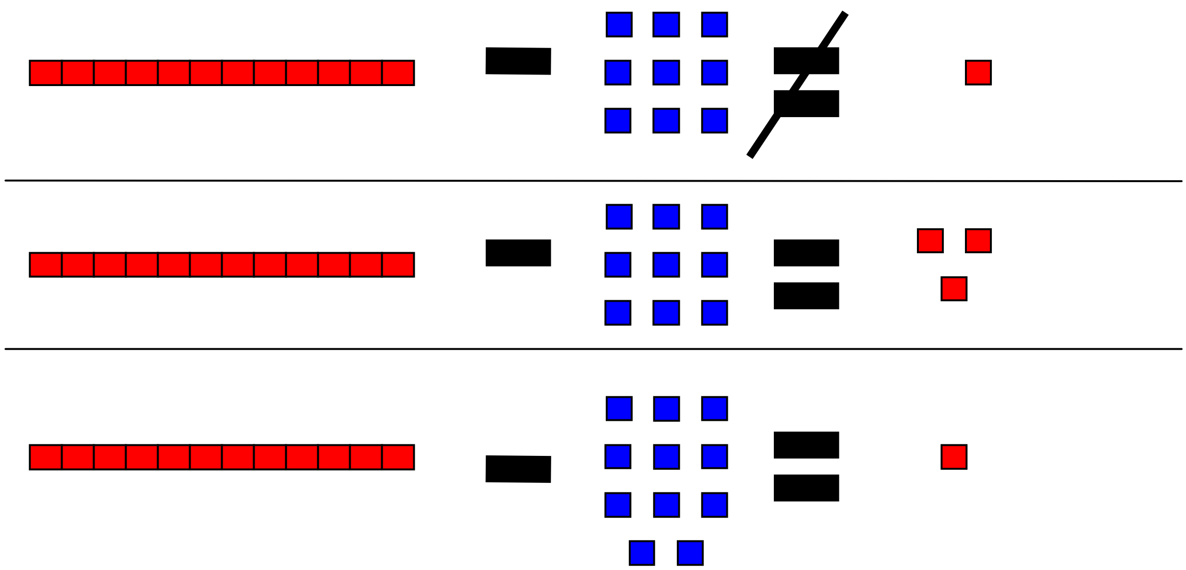


Figure 27: Example showing how 10-9 does not equal 1 in base twelve

Borrowing in the base 12 system works the opposite from carrying. The rules are the same as borrowing in the decimal system. If the number being subtracted is smaller than the number being subtracted from it, take 1 unit from the next highest place and add it as 10 (twelve) units to the place you are subtracting. So, if you subtract 23-16 you will subtract the ones place first, 3-6. Since 3-6 does not subtract easily we will borrow from the twelve's place making the 2 a 1 and adding twelve units to the ones place. So the answer would equal 9 ones and, since the twelve's place was changed to 1-1, the answer will be 9.

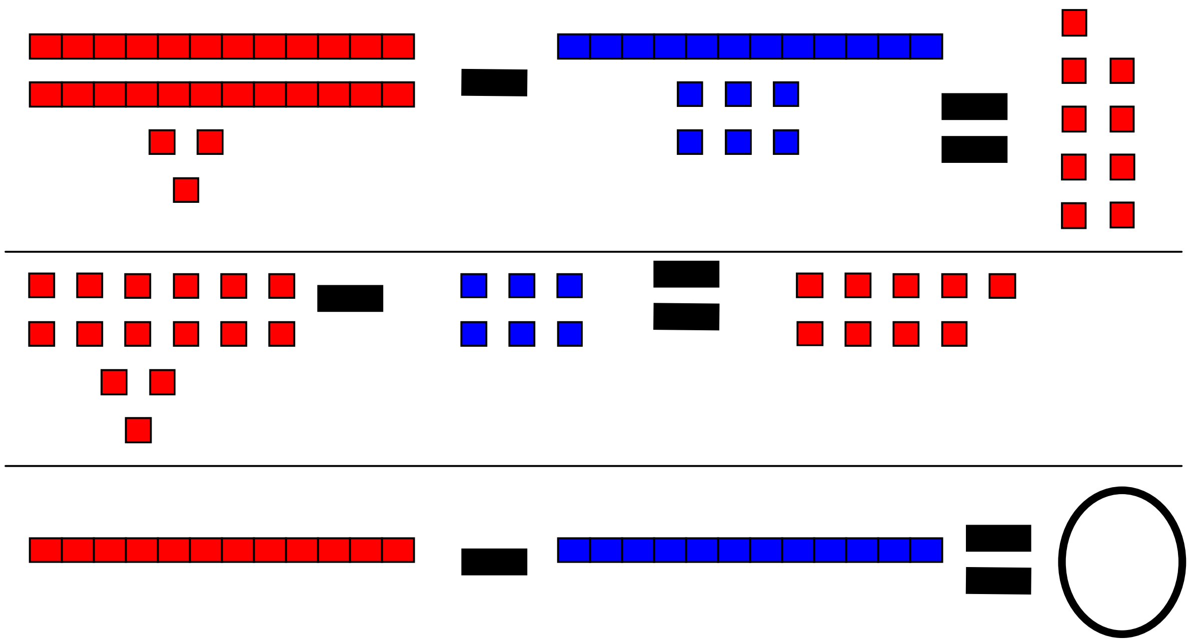


Figure 28: Two digit subtraction example in base twelve

The vigesimal system is a counting system using the base of 20. The base 20 system uses twenty single digit numbers 0 through 9 and a through j. The place values for base twenty are ones, twenty's, four hundred's and so on.

Adding in the vigesimal system is a lot like adding the duodecimal system. The numbers 0 through 9 add the same as in the decimal system. The letters that represent numbers change the values of anything past 9. The letter a has a value of ten, b the value of eleven, c the value of twelve, and so on. If adding the problem b+3 you will start with the number b and count on 3 units, c, d, e. So, b+3 equals c. With the problem 2+d, start at the number 2 and count on d units, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f. 2+d=f.

If adding numbers in the base 20 system that total 10 or above carrying may be involved. For example, if we add 1a+b, a+b will add to 11. We must carry the 1 to the next highest place the 20's place. We will then add 1+1 in the 20's place giving us the answer of 2 groups of 20's + 1 group of ones, or 21.

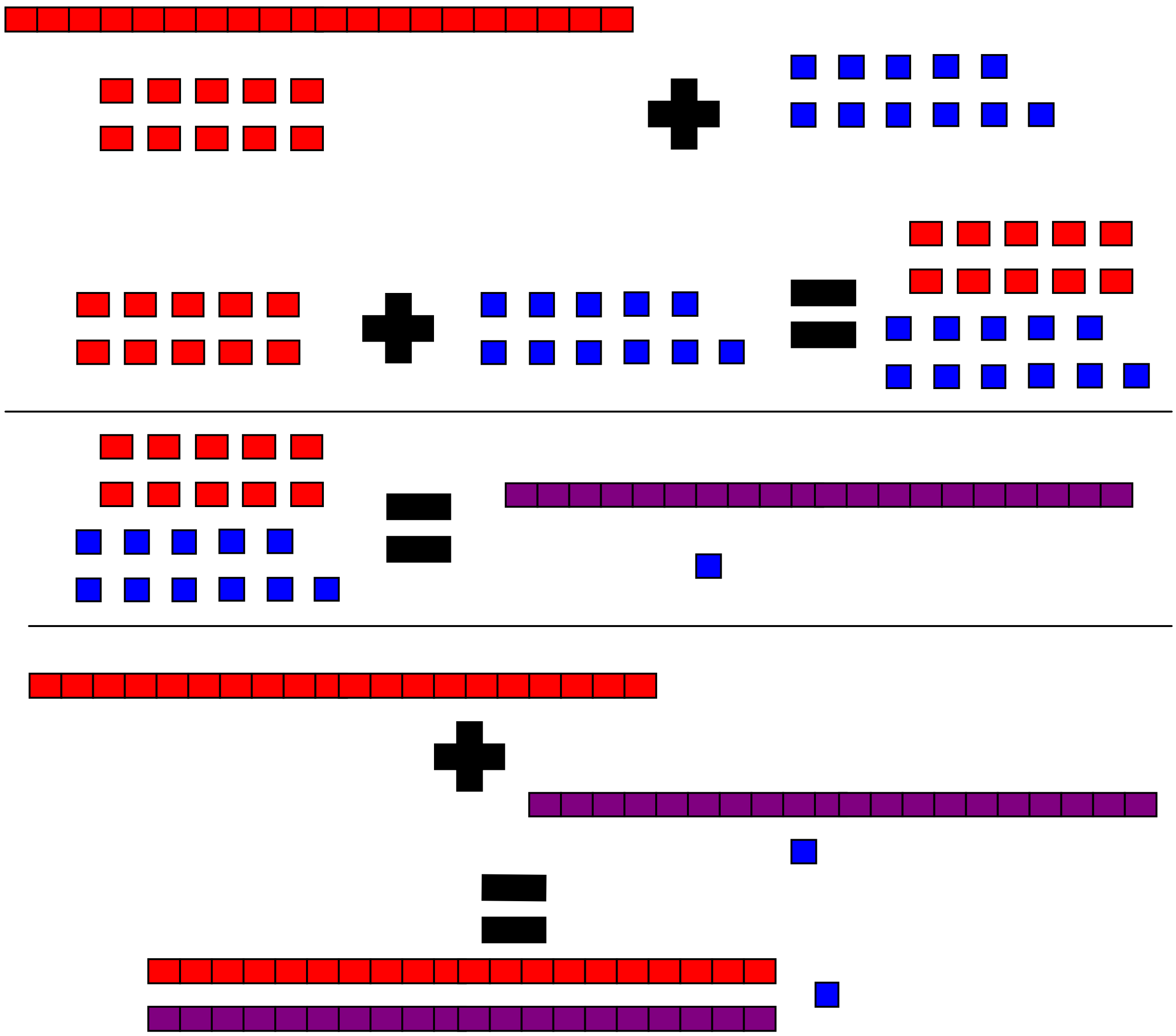


Figure 29: Addition example in base twenty

If subtracting, remember the values of the letters and numbers past the number 9. If borrowing, you will use the same rules as with the duodecimal system. Take the problem 24-15; we will subtract the ones position first. Since 4-5 doesn't subtract easily we will borrow form the next highest position, the twenty's. Borrowing from the twenty's converts the 2 to a 1 and the 4 to 14 units. 14-5 will give us the answer of i ones. Since we borrowed 1 from the twenty's group our next step is 1-1 giving us 0 groups of twenty, and our answer i.

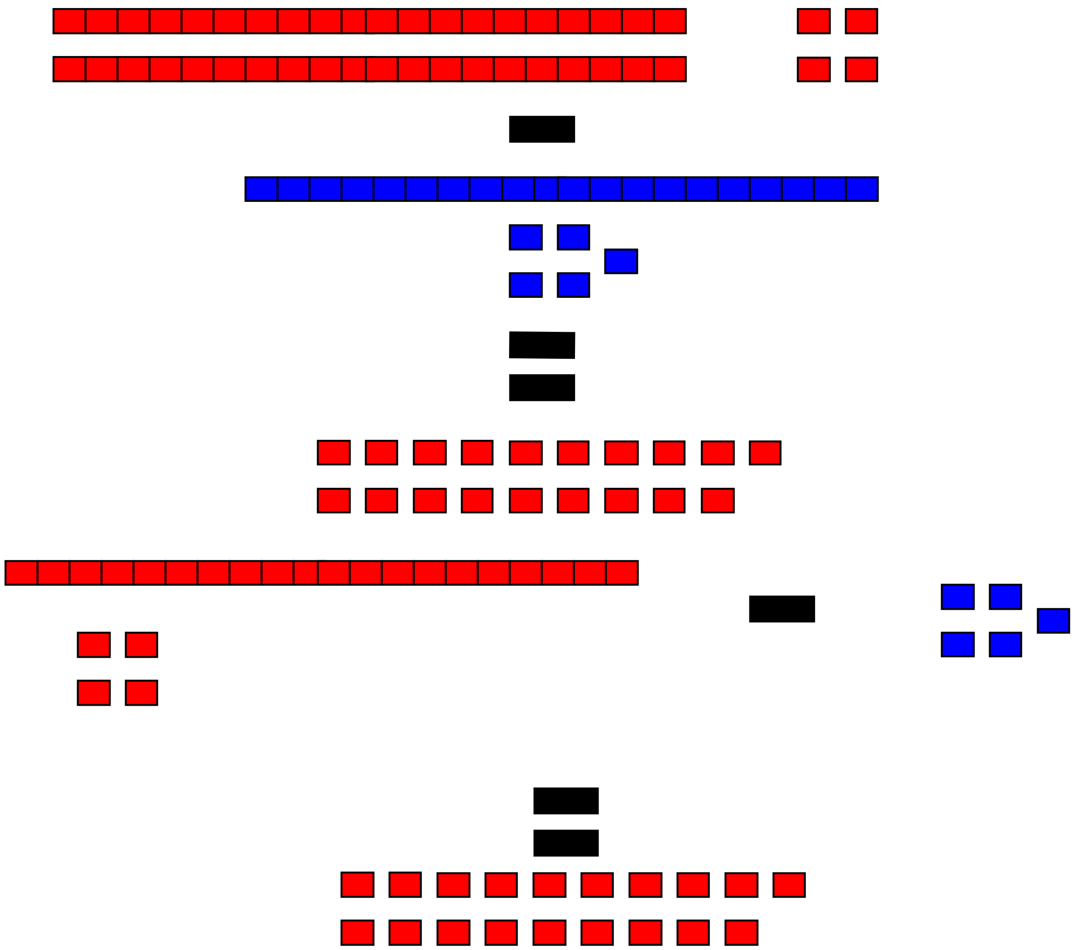


Figure 30: Subtraction example in base twenty

The sexagesimal system works in the same way as the duodecimal and vigesimal systems. Counting, adding, and subtracting will all use the same methods with the number and letter combinations. Just remember that the number 10 will equal more than ten and that the value ten will usually be represented by the letter a.

9.2 Unit Vocabulary

Duodecimal**-** A counting system using a base of 12. In a base 12 system there are a total of 12 single digit numbers, 0 through b. The duodecimal system is used in time, calendars, and compass directions.

Vigesimal**-** A counting system using a base of 20. In a base 20 system there are a total of 20 single digit numbers, 0 through i. The Mayans used the vigesimal system in their calendar.

Sexagesimal**-** A counting system using a base of 60. In a base 60 system there are a total of 60 single digit numbers using numbers and various other symbols. The sexagesimal system is used with time and with measuring angles.

Sumerians**-** A classic civilization that used the base 60 system of counting in their everyday life.

Mayans**-** A classic civilization that used the base 20 system of counting in their everyday life.

9.3 Review Questions

1. 1. There are an infinite number of _____________ that can be used.
2. _________________ is the system of counting in a base of 12.
3. _________________ was used by the Sumerians for basic everyday computations.
4. The vigesimal sytem was used by the _____________ civilization to make calendars and do basic computations.
5. _________________ can be divided by 1, 2, 3, 4, 5, 6, 10, 12, 15, and 30. This makes it fairly easy to use fractions.
6. _________________ is used to tell time, make calendars, and give directions on a compass.
7. _________________ uses the letters a through j as numbers in its counting system.
8. _________________ uses the entire alphabet plus other various symbols as numbers in its counting system.
9. ________________ uses the letters a and b as numbers in its counting system.
10. The letter a represents the number value _____________ in any number system above base ten.

9.4 Review Answers

1. There are an infinite number of ___Number systems__ that can be used.
2. ___Duodecimal__ is the system of counting in a base of 12.
3. ___Sexagesimal___ was used by the Sumerians for basic everyday computations.
4. The vigesimal system was used by the ___Mayans___ civilization to make calendars and do basic computations.
5. ___Sexagesimal System___ can be divided by 1, 2, 3, 4, 5, 6, 10, 12, 15, and 30. This makes it fairly easy to use fractions.
6. ___Duodecimal__ is used to tell time, make calendars, and give directions on a compass.
7. ___vigesimal__ uses the letters a through j as numbers in its counting system.
8. ____sexagesimal___ uses the entire alphabet plus other various symbols as numbers in its counting system.
9. ____Duodecimal__ uses the letters a and b as numbers in its counting system.
10. The letter a represents the number value ___ten__ in any number system above base ten.

9.5 Duodecimal Addition and Subtraction Exercises

1. 6 + 4	2. a + 1	3. 3 + 5
4. 5 + 5	5. 2 + 7	6. 8 + 3
7. 6 + 6	8. a + b	9. 8 - 5
10. a - 5	11. 19 - b	12. 24 -15
13. 31 - 1a	14. 40 - 22	15. 28 - b

9.6 Duodecimal Addition and Subtraction Answers

1. $\begin{array}{r} 6 \\ \underline{+\ 4} \\ a \end{array}$

2. $\begin{array}{r} a \\ \underline{+\ 1} \\ b \end{array}$

3. $\begin{array}{r} 3 \\ \underline{+\ 5} \\ 8 \end{array}$

4. $\begin{array}{r} 5 \\ \underline{+\ 5} \\ a \end{array}$

5. $\begin{array}{r} 2 \\ \underline{+\ 7} \\ 9 \end{array}$

6. $\begin{array}{r} 8 \\ \underline{+\ 3} \\ b \end{array}$

7. $\begin{array}{r} 6 \\ \underline{+\ 6} \\ 10 \end{array}$

8. $\begin{array}{r} a \\ \underline{+\ b} \\ 19 \end{array}$

9. $\begin{array}{r} 8 \\ \underline{-\ 5} \\ 3 \end{array}$

10. $\begin{array}{r} a \\ \underline{-\ 5} \\ 5 \end{array}$

11. $\begin{array}{r} 19 \\ \underline{-\ \ b} \\ a \end{array}$

12. $\begin{array}{r} 24 \\ \underline{-15} \\ b \end{array}$

13. $\begin{array}{r} 31 \\ \underline{-\ 1a} \\ 13 \end{array}$

14. $\begin{array}{r} 40 \\ \underline{-\ 22} \\ 1a \end{array}$

15. $\begin{array}{r} 28 \\ \underline{-\ \ b} \\ 19 \end{array}$

9.7 Base 20 Addition and Subtraction Exercises

1. 8 + 6	2. b + 5	3. 6 + a
4. f + 3	5. 7 + 7	6. 4 + i
7. h + g	8. 10 - a	9. e - 7
10. b - 9	11. 1a - 19	12. 36 - 1h
13. 5a - 3c	14. 2f - 1b	15. 42 - 27

9.8 Base 20 Addition and Subtraction Answers

1. $$\begin{array}{r} 8 \\ \underline{+\ 6} \\ e \end{array}$$

2. $$\begin{array}{r} b \\ \underline{+\ 5} \\ g \end{array}$$

3. $$\begin{array}{r} 6 \\ \underline{+\ a} \\ g \end{array}$$

4. $$\begin{array}{r} f \\ \underline{+\ 3} \\ i \end{array}$$

5. $$\begin{array}{r} 7 \\ \underline{+\ 7} \\ e \end{array}$$

6. $$\begin{array}{r} 4 \\ \underline{+\ i} \\ 13 \end{array}$$

7. $$\begin{array}{r} h \\ \underline{+\ g} \\ 1g \end{array}$$

8. $$\begin{array}{r} 10 \\ \underline{-\ \ a} \\ 9 \end{array}$$

9. $$\begin{array}{r} e \\ \underline{-\ 7} \\ 6 \end{array}$$

10. $$\begin{array}{r} b \\ \underline{-\ 9} \\ 2 \end{array}$$

11. $$\begin{array}{r} 1a \\ \underline{-\ 19} \\ 1 \end{array}$$

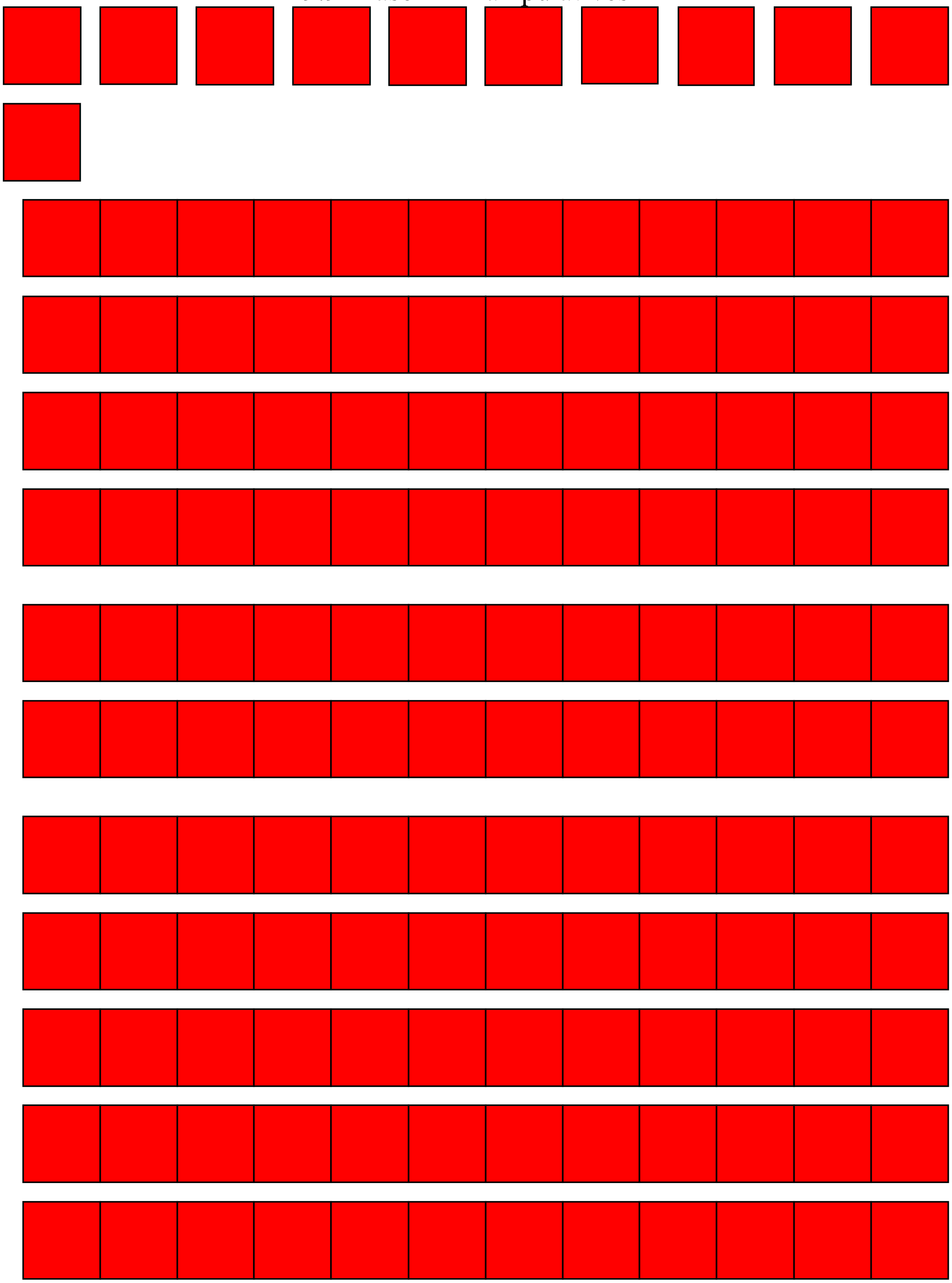
12. $$\begin{array}{r} 36 \\ \underline{-\ 1h} \\ 19 \end{array}$$

13. $$\begin{array}{r} 5a \\ \underline{-\ 3c} \\ 1i \end{array}$$

14. $$\begin{array}{r} 2f \\ \underline{-\ 1b} \\ 14 \end{array}$$

15. $$\begin{array}{r} 42 \\ \underline{-\ 27} \\ 1f \end{array}$$

9.9 Base 12 Manipulatives



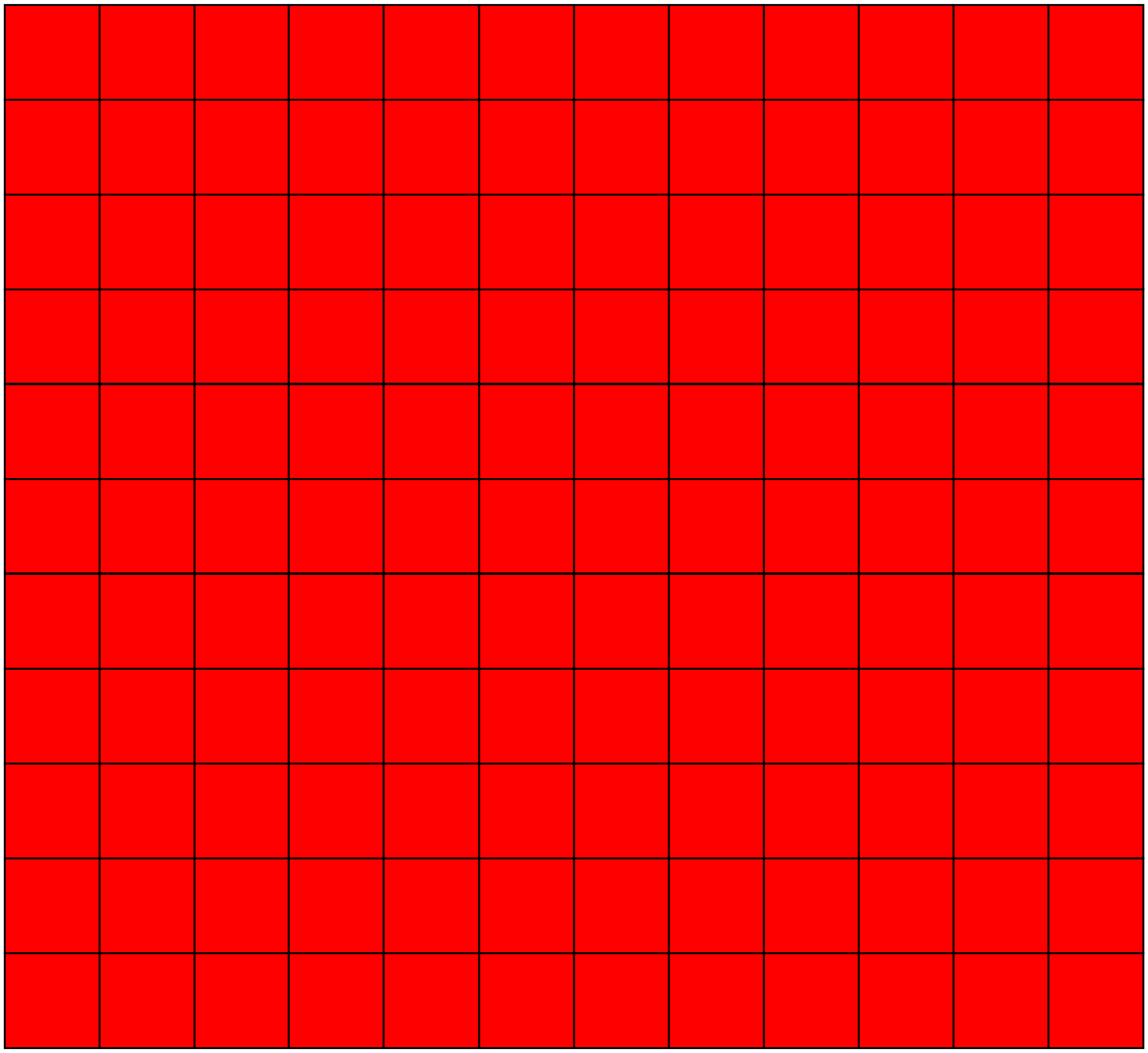


Figure 31: Base twelve manipulative blocks

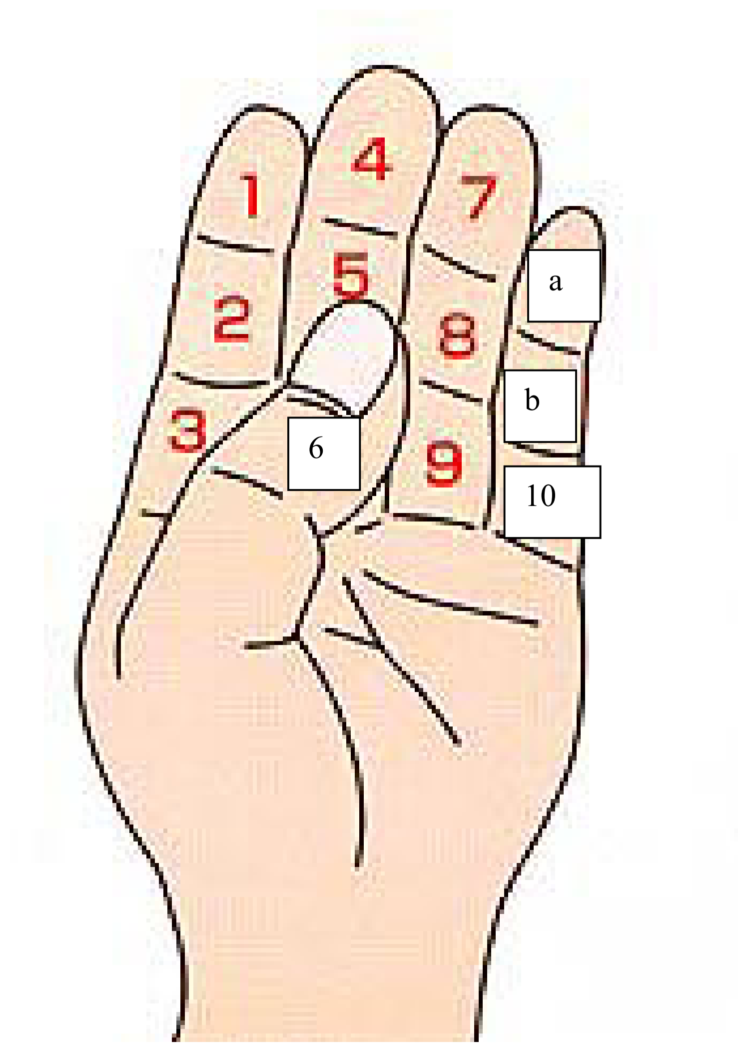


Figure 32: Base twelve manipulative hand [3]

9.10 Base 20 Manipulatives

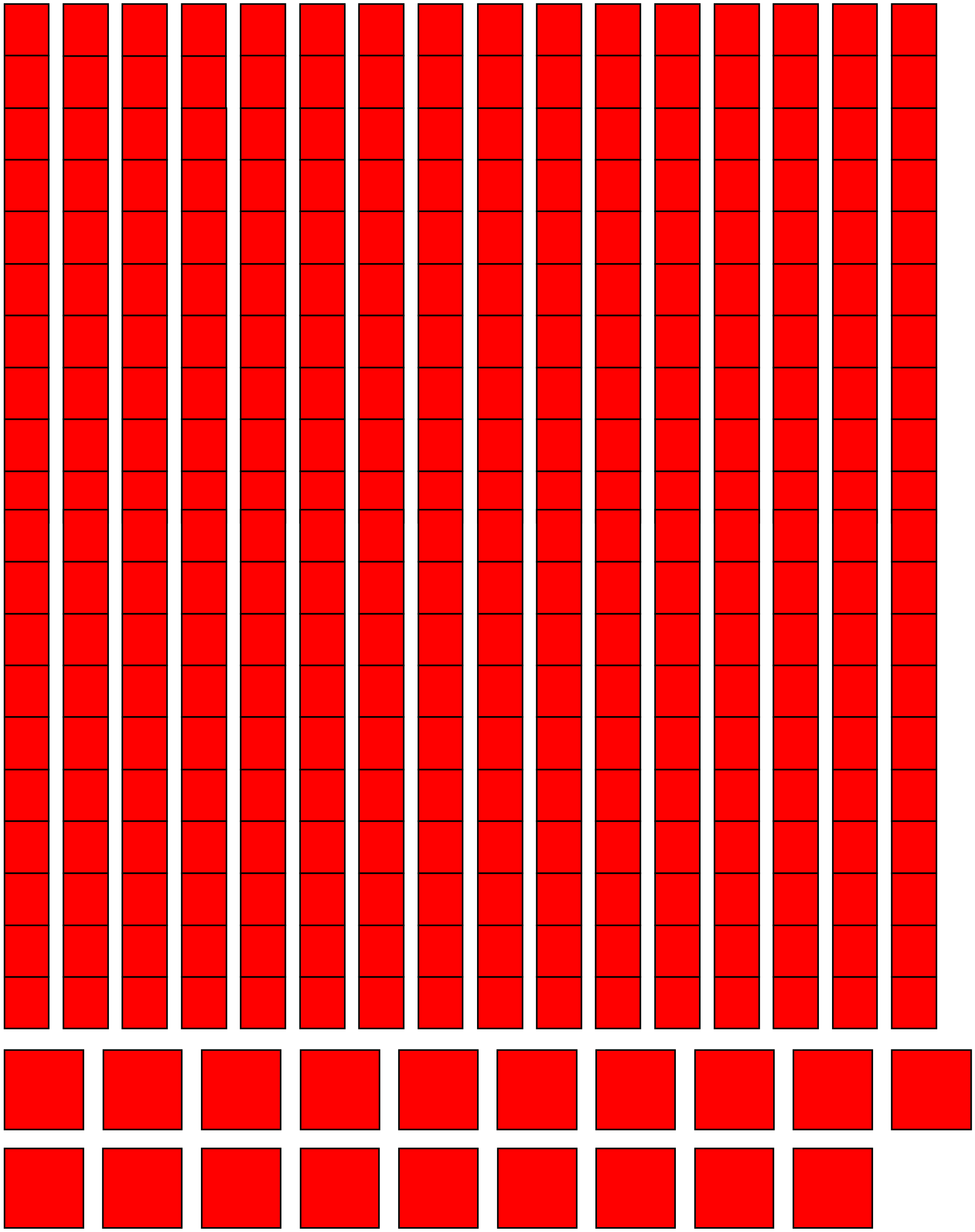


Figure 33: Base twenty manipulative blocks

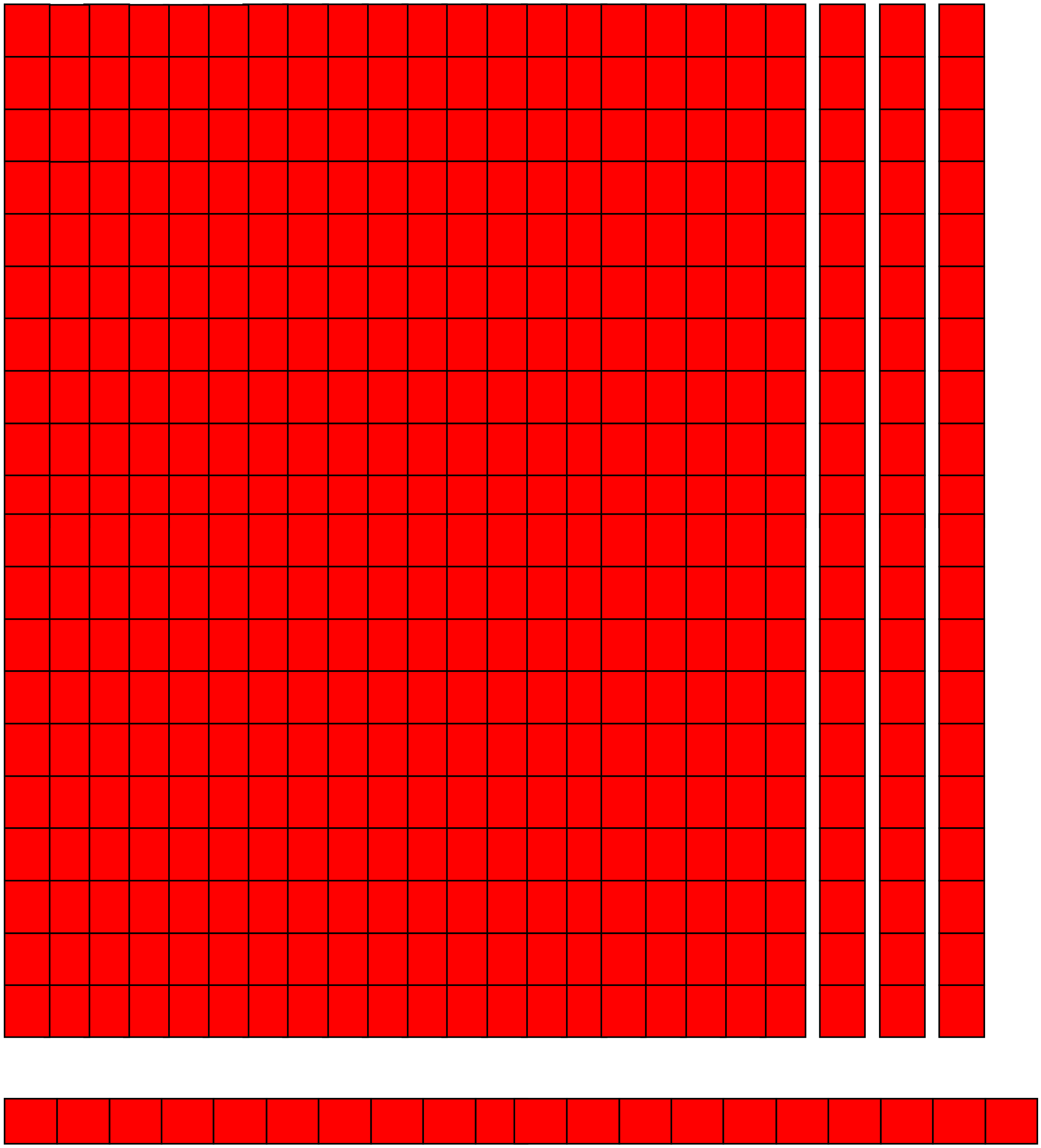


Figure 33 cont.

10 MULTIPLYING WITH BASE SYSTEMS

10.1 Introduction

Now that we have learned to count, add, and subtract with certain base systems, we are going to look at multiplying with them. Multiplying is a quick way to add large groups of numbers. When using multiplication we will use terms like "times". When saying 6 multiply 2, or 6 times (x or ·), it means 6 two times or 6+6. Using addition or multiplication, will give us the answer 12 in the decimal system. If we multiply 5 x 9 it would be the same as adding 5 nine times or 5+5+5+5+5+5+5+5+5.

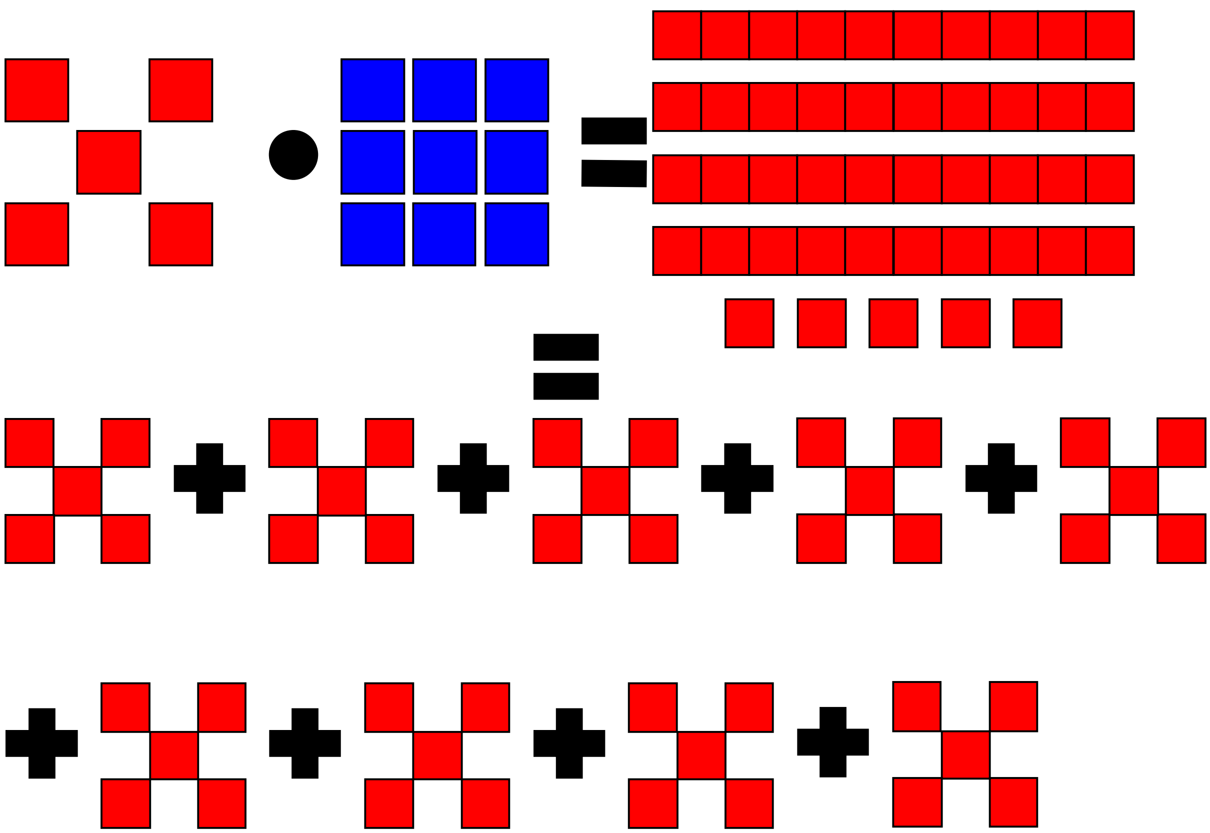


Figure 34: Base ten multiplication example

Multiplying in different bases works the same way as if you were using it in base ten. The only difference, as with adding and subtracting, is the numbers being used. All of the basic rules of multiplication still apply. A number times 0 will still give you the answer 0. A number times 1 will still give you the number you multiplied with as an answer.

With that said lets look at multiplying the base five system. Remember with the base five system we will use only the numbers 0, 1, 2, 3, and 4. Keeping that and our basic multiplication rules in mind, let's try the problem 2x3. When working this problem in base ten, our answer would equal 6. In base 5 however, the number 6 does not exist. Knowing that multiplication if a fast addition method we can add 2 three times, or 2+2+2. In base five our answer will equal 11.

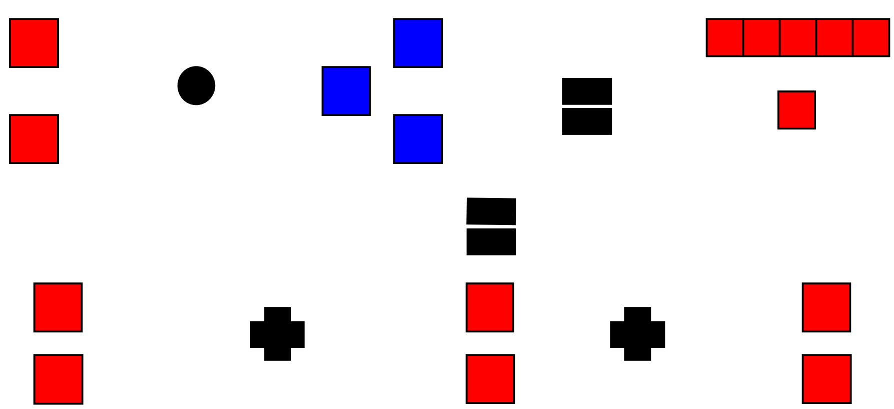


Figure 35: Base 5 multiplication example

If we wanted to multiply in base two, we would only use the numbers 0 and 1. I know you are probably thinking, "if we can only use 0 and 1 wouldn't all the answers be 0?" The answer is, of course, no. If that was true why would I be talking about it right now? I would just say "if you multiply with base two your answer will be 0." If you look at the bigger picture you will remember that 0 and 1 are the only single digit numbers. We still have 2, 3, 4… digit numbers. Let's look at the problem 10x11. We can use the regular steps of multiplication. We will first multiply the ones position, 1x0 and 1x1 this gives us an answer of 10 ones. Next we multiply the twos position, 1x0 and 1x1 giving us the answer 10 twos. We add a 0 on the end of the twos position's answer to fill in the ones position, changing that answer to 100. Finally, we will add the two answers together, 100+10. This gives us our final answer of 110. Another way to solve this problem is by using the fast addition reasoning adding 10 (two), 11(three) times.

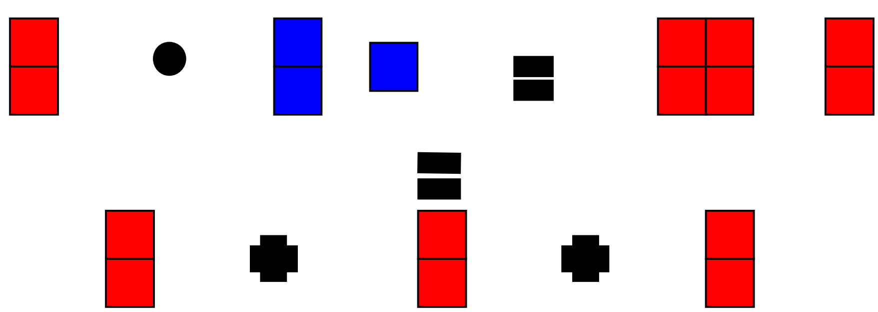


Figure 36: Base two multiplication example

The fast addition reasoning will help us multiply different bases easier. Using this reasoning will keep us from making new rules for bases above base ten, and keep the lower bases easier to work with. Eventually, after you are more comfortable with different bases, you can work on the memorization of your new "times tables".

10.2 Base Five Multiplication

1. 2 x 4	2. 4 x 3	3. 3 x 3
4. 4 x 4	5. 12 x 2	6. 31 x 4
7. 22 x 11	8. 40 x 21	9. 23 x 10
10. 33 x 12	11. 21 x 44	12. 24 x 13
13. 43 x 13	14. 31 x 44	15. 34 x 20

10.3 Base Five Multiplication Answers

1. 2 x 4 13	2. 4 x 3 22	3. 3 x 3 14
4. 4 x 4 31	5. 12 x 2 44	6. 31 x 4 224
7. 22 x 11 1432	8. 40 x 21 11330	9. 23 x 10 1410
10. 33 x 12 3041	11. 21 x 44 12012	12. 24 x 13 2222
13. 43 x 13 4214	14. 31 x 44 20424	15. 34 x 20 10210

10.4 Base Two Multiplication

1. $\begin{array}{r} 101 \\ \underline{\text{x} \quad 11} \end{array}$	2. $\begin{array}{r} 11 \\ \underline{\text{x}\ 11} \end{array}$	3. $\begin{array}{r} 110 \\ \underline{\text{x} \quad 11} \end{array}$
4. $\begin{array}{r} 111 \\ \underline{\text{x}\ 100} \end{array}$	5. $\begin{array}{r} 1011 \\ \underline{\text{x} \quad 110} \end{array}$	6. $\begin{array}{r} 1100 \\ \underline{\text{x} \quad 111} \end{array}$
7. $\begin{array}{r} 1001 \\ \underline{\text{x}\ 1000} \end{array}$	8. $\begin{array}{r} 1110 \\ \underline{\text{x} \quad 111} \end{array}$	9. $\begin{array}{r} 1011 \\ \underline{\text{x} \qquad 10} \end{array}$

10.5 Base Two Multiplication Answers

1.	2.	3.
101 x 11 1111	11 x 11 1001	110 x 11 10010

4.	5.	6.
111 x 100 11100	1011 x 110 1000010	1100 x 111 101100

7.	8.	9.
1001 x 1000 1001000	1110 x 111 1100010	1011 x 10 10110

11 CREATING YOUR OWN BASE SYSTEM

11.1 Introduction

Shhh! Don't tell anyone. This unit is just for you. That's right, we're going to make our own secret code. We are going to do this with the things that you have learned throughout these lessons.

The different base systems have really been useful around the world for creating and deciphering codes. This is actually studied in college and by the governments across the globe. The practice of cryptography uses mathematics and computers to encrypt messages to make it harder to steal information. To do this properly, you need what is called a cipher which is a set code that at least two different groups have so they can share information secretly and securely. If you have ever played a video game and used cheat codes you have used cryptography and ciphers.

The first step of creating a secret code is to figure out what you are going to use as a cipher. This is where our newly found knowledge of the base systems come in handy. Let's use the base 5 system for an example and a code that you can use until you get your own figured out.

Knowing the base system we are going to use for a cipher is a good start, but we need to figure out how to use it now. We can start with a=1, b=2, c=3, d=4, e=10 and so on, or we could start with z=1, y=2 and so on.

Of course, if you would like to make it really difficult, we could start with a higher number or in the middle of the alphabet or something of that nature. We, however, are going to keep it fairly simple and use the base five system with a=1, b=2… y=100, z=101 (the complete cipher is at the end of this unit). This will now be our cipher for encoding and decoding messages.

Now that we have a complete cipher, the only thing left is to use it to make our own secret messages. Here try this one out!

23,1,21,14,24,12   34,10,3,33,10,40   3,30,4,10,34   14,34   1

22,30,40   30,11   11,41,24!!!

Did you figure it out? Great! For more messages, try the worksheets at the end of this unit. Also, make your own and use it when you write your next note to your friend.

11.2 Base Five Cipher

A= 1	N= 24
B= 2	O= 30
C= 3	P= 31
D= 4	Q= 32
E= 10	R= 33
F= 11	S= 34
G= 12	T= 40
H= 13	U= 41
I = 14	V= 42
J= 20	W=43
K= 21	X= 44
L= 22	Y= 100
M= 23	Z= 101

11.3 Code Breaking

1. 40,13,14,34 3,30,4,10 14,34 4,10,3,14,31,13,10,33,10,4 43,14,40,13 1 2,1,34,10 11,14,42,10 3,14,31,13,10,33.

2. 14 22,14,21,10 23,1,40,13.

3. 34,10,3,33,10,40 3,30,4,10,34 1,33,10 4,14,11,14,3,41,22,40 40,30 33,10,1,4.

4. 10,42,10,24 13,1,33,4,10,33 14,11 100,30,41 4,30,24,40 21,24,30,43 40,13,10 3,14, 31,13,10,33.

5. 40,13,14,34 41,24,14,40 13,1,34 2,10,10,24 1 22,30,40 30,11 11,41,24.

11.4 My Secret Cipher

A=	N=
B=	O=
C=	P=
D=	Q=
E=	R=
F=	S=
G=	T=
H=	U=
I =	V=
J=	W=
K=	X=
L=	Y=
M=	Z=

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