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Combinatorics for the Third Grade Classroom

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A thesis  
presented to  
the faculty of the Department of Mathematics  
East Tennessee State University

In partial fulfillment  
of the requirements for the degree  
Master of Science in Mathematical Sciences

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by  
Rita Jane McFaddin  
August 2006

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Keywords: Fundamental Principle, Permutation, Combination, Pascal's Triangle,  
Elementary School

## ABSTRACT

Combinatorics for the Third Grade Classroom

by  
Rita Jane McFaddin

After becoming interested in the beauty of numbers and the intricate patterns of their behavior, the author concluded that it would be a good idea to make the subject available for students earlier in their educational experience. In this thesis, the author developed four units in combinatorics, namely Fundamental Principles, Permutations, Combinations, and Pascal's Triangle, which are appropriate for third grade level.

## DEDICATION

This thesis is dedicated to the many people who have supported me throughout this educational endeavor. To my husband, Ernest McFaddin, I could not have finished without your love, encouragement, and willingness to sacrifice. I love you! To my children, Kayla, Shaina, and Briana, for helping out at home and not complaining about all the time I have spent on studying and work. To my friends who have supported and encouraged me along the way. Most importantly I would like to thank my Heavenly Father who has given me the strength and courage to complete this journey.

## ACKNOWLEDGEMENTS

I would like to thank Dr. Bob Gardner for his help along the way. I would like to thank my friends and family for their help, encouragement, and support.

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## CHAPTER 1

### INTRODUCTION

“Counting is the way that numbers first entered our world. Four or five thousand years ago people first developed the concept of numbers, probably to order or quantify their possessions and to make transactions - my three pigs for your two cows. The numbers 1, 2, 3, 4, and so on can be used to count anything”[1].

During the author’s Spring 2004 semester at East Tennessee State University [ETSU], the author enrolled in Math 5010, Patterns and Problem Solving for Teachers, under Dr. Anant Godbole. It was during this class that the author first became interested in counting and combinatorics. The author’s interest was further sparked when she enrolled in Math 5026, Foundations and Structure of Math II, under George Poole in the Spring of 2005. It was then that she discovered how important the concept of counting, combinations, and permutations, really were. The author began to think how these concepts might be taught to young children, so that they can have an earlier start to discovering this area of mathematics. The information contained in this document is a resource for teachers in presenting combinatorics to young children.

#### Combinatorics in Elementary School

The author first researched to see at what level combinatorics was being taught. She discovered that combinations to some extent were already being taught at the third grade level. Students in Virginia schools are expected to list possible results of a given situation. Virginia Standard of Learning 3.23 states: The student will investigate and describe the concept of probability as chance and list possible results of a given situation [12]. After talking to several teachers, the author found that combinatorics was taught in

the elementary school by matching one to one the possible outcomes. The author also found that two to three days throughout the year was all the time that was spent on it.

## CHAPTER 2

### FUNDAMENTAL PRINCIPLES

One of the first questions that may arise from the given topic is “What is combinatorics?” “Combinatorics” is the name for the study of the methods of counting. We usually think of counting as a simple process. However, it can become complicated if whatever we wish to count cannot be readily visualized. In combinatorics, we are more concerned with counting ways of carrying out certain procedures rather than actually counting physical objects[8]. There are a few basic principles upon which much of combinatorics is based. They are: the Multiplication Principle of Counting, the Addition Principle of Counting, and the Subtraction Principle of Counting.

The Multiplication Principle of Counting states: Suppose a procedure can be divided into a first step and a second step, and that each distinct way of performing step one followed by step two results in a different outcome of the procedure. Suppose also that (1) there are  $m$  ways of performing step one, and then (2) there are  $n$  ways of performing step two, then the number of possible outcomes of the procedure is the product  $m \times n$  [8].

The Addition Principle of Counting is used when you have a choice of two methods of performing a procedure; then the number of ways of performing the procedure is found by adding the number of ways using the first method and the number of ways using the second method. The principle thus states: Suppose Task 1 can be done in  $m$  ways and Task 2 can be done in  $n$  ways. If there is no way of doing both tasks at once, then the number of ways of doing Task 1 or Task 2 is the sum  $m + n$  [8].

The Subtraction Principle of Counting is used if there is double counting

involved. The principle states: If there are  $k$  ways of doing both tasks at once, then the number of ways of doing Task 1 or Task 2 is  $m + n - k$  [8].

## CHAPTER 3

### COMBINATIONS UNIT

As we begin to teach combinatorics to young children, we need to keep several ideas in mind. Students need to understand that showing possible combinations is a way of analyzing data. Students will use a tree diagram to show possible combinations. A tree diagram is a drawing that connects two or more groups of objects. Students will apply the multiplication principle to find the number of combinations. The number of objects in one group and the number of objects in the other group are the factors. Finally, students will understand that in combinations, the order of objects is not important.

#### In-Class Examples

Example 1: Suppose Briana wakes up in the morning and finds that she has three pairs of shorts and four shirts, see Figure 1. How many ways can she dress for school? Her shorts are tan, white, and blue. Her shirts are green, yellow, orange, and red. How many possible combinations of shorts and shirts are there?



Figure 1: Shorts and Shirts

First, Briana must choose a pair of shorts. There are three ways of doing this. Then, she must choose a shirt. There are four ways of doing this. We can use a tree diagram to visualize the two step procedure, see Figure 2.

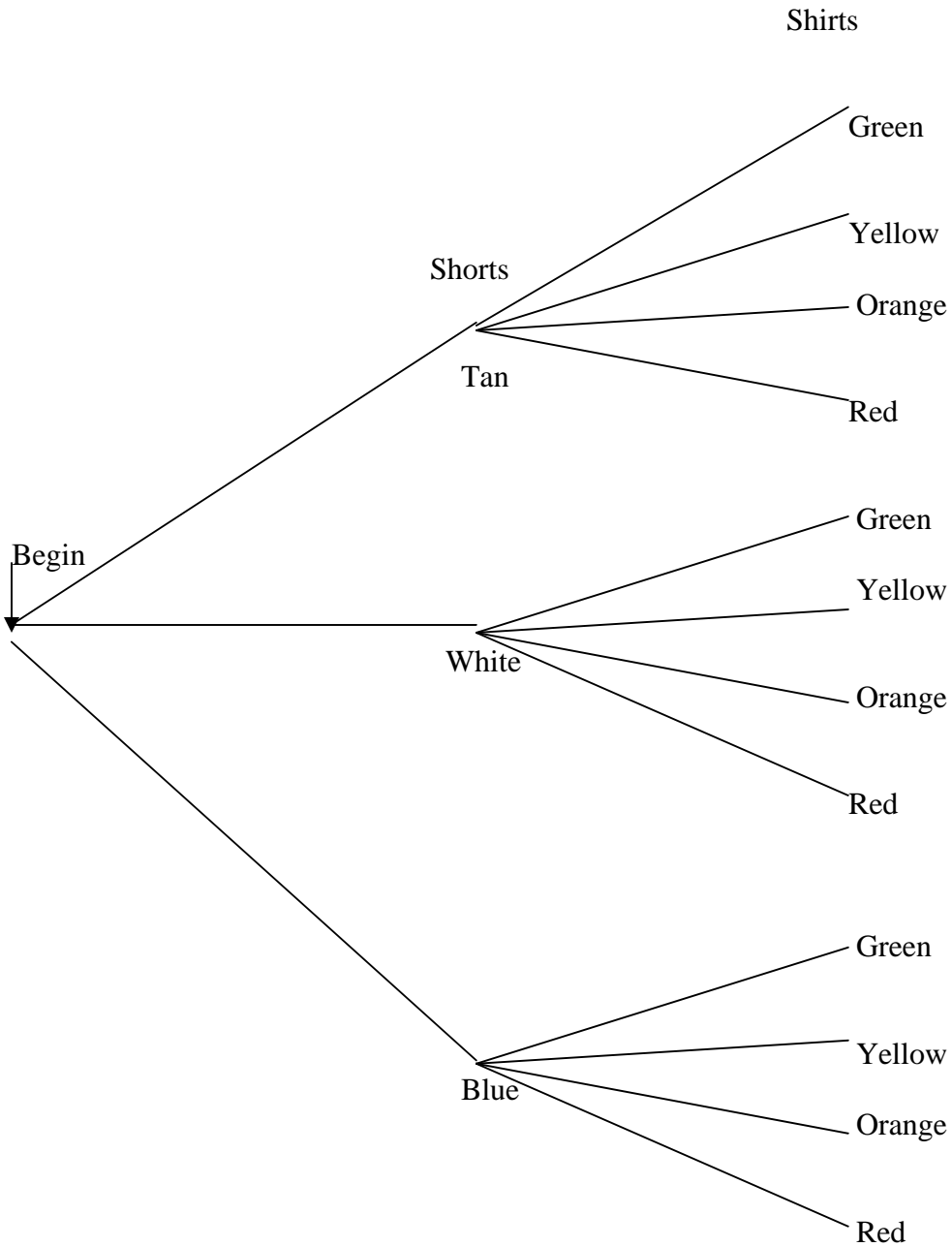


Figure 2: Tree Diagram of Shorts and Shirts



We see that there are three ways of completing STEP 1, represented by the first level branches in the tree diagram. Then for each branch in the first level, there are four second level branches, corresponding to the four ways of completing STEP 2. The points on the extreme right are leaves on the tree; these represent the ways of performing step one followed by step two. The number of leaves is the product of the number of first level options times the number of second level options:  $3 \times 4 = 12$ .

Example 2: Suppose you could have a ham, turkey, or roast beef sandwich for lunch. You could also have chocolate milk or strawberry milk. How many possible combinations of a sandwich and a drink are there?

What is Step 1? Step 1 is to list the sandwiches: HAM, TURKEY, and ROAST BEEF.

What is Step 2? For each kind of sandwich, draw branches for the kinds of drink you could have: chocolate milk and strawberry milk, see Figure 3.

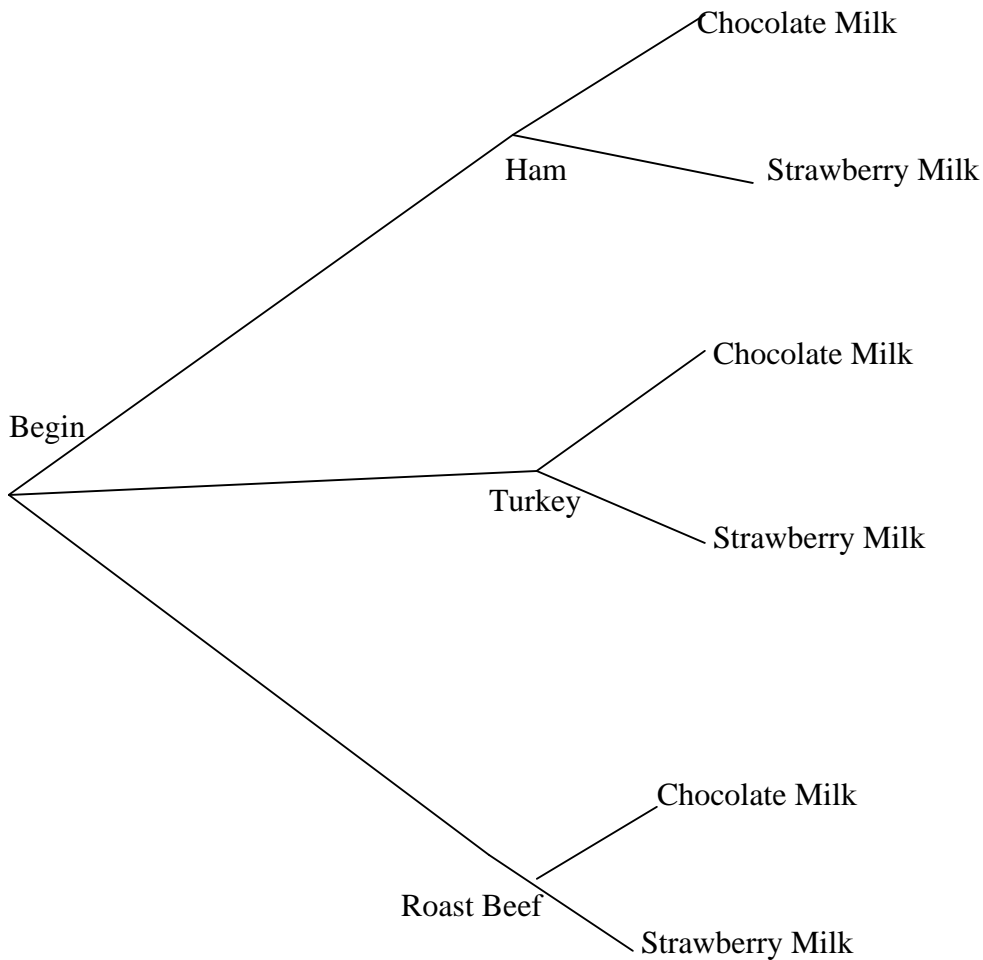


Figure 3: Tree Diagram Sandwich and Milk

So, there are six possible combinations for a sandwich and milk:  $2 \times 3 = 6$ .

Example 3: Shaina is going to a local bicycle shop to purchase a bicycle. She may choose a five speed, ten speed, or fifteen speed. She may choose a purple, red, or blue bike. How many possible combinations are there?

What is Step 1? Step 1 is to list the bicycle speed choices: FIVE, TEN, and FIFTEEN.

What is Step 2? Step 2 is to draw branches for the color choices: PURPLE, RED, and BLUE, see Figure 4.

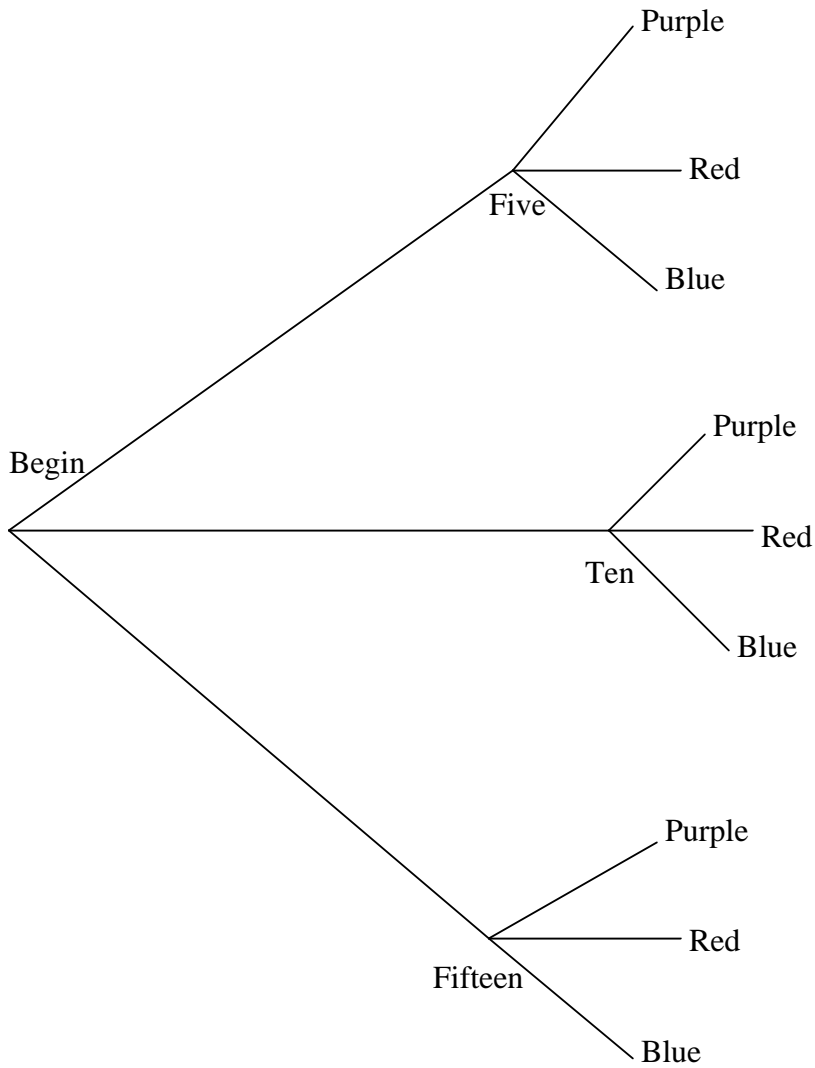


Figure 4: Tree Diagram Bicycle and Color

So, there are 9 possible combinations:  $3 \times 3 = 9$ .

Homework Exercises: Combinations

Directions: Make a tree diagram to show all the combinations. Tell how many combinations are possible.

1. The local ice cream parlor offers three milk shake flavors: vanilla, strawberry, and chocolate. They come in three sizes: small, medium, and large. How many

- possible combinations are there?
2. The Kountry Kitchen restaurant offers the following side order choices: tater tots, french fries, and onion rings. They come in two sizes: small and large. How many possible combinations are there?
  3. The deli at the mall offers the following food choices: hamburger, hotdog, and pizza. They offer the following drink choices: water, milk, soda, and juice. How many possible combinations are there?
  4. Kourtney goes to the Italian Eatery to order a pizza. She may choose from the following pizza toppings: cheese, pepperoni, and hamburger. She must choose a small, medium, or large size. How many possible combinations are there?
  5. While playing a game you must toss a coin and roll a die. When you toss a coin you can get heads or tails. When you roll a die you can get 1, 2, 3, 4, 5, or 6. How many possible combinations are there?

Answer Key: Combinations

1. Figure 5 gives the tree diagram for this exercise.

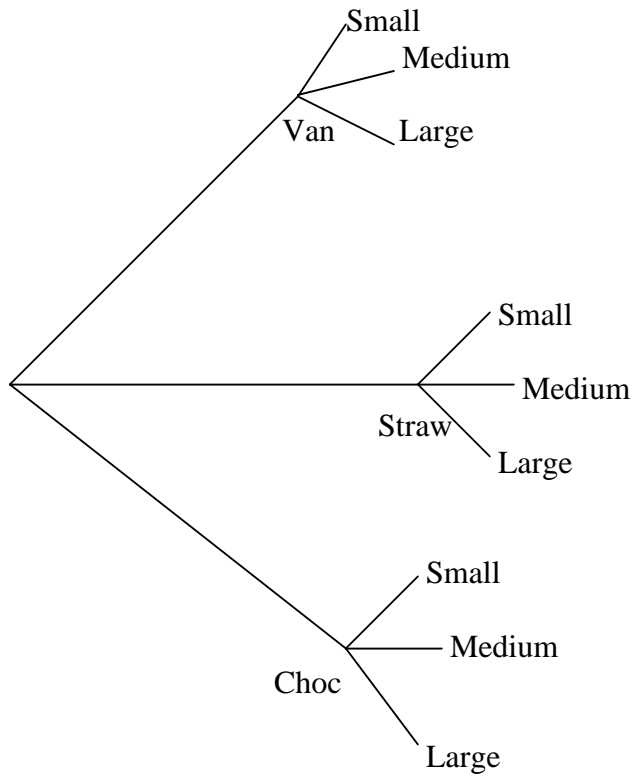


Figure 5: Tree Diagram Milk Shakes

There are three flavors of ice cream and three sizes to choose from:  $3 \times 3 = 9$ .

2. Figure 6 gives the tree diagram for this exercise.

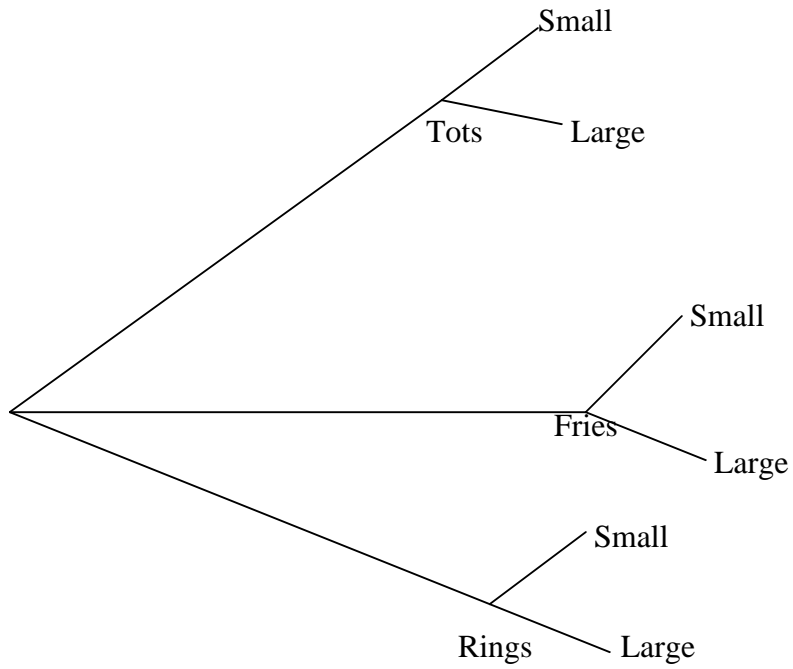


Figure 6: Tree Diagram Side Orders

There are three side order choices and two size choices:  $3 \times 2 = 6$ .

3. Figure 7 gives the tree diagram for this exercise.

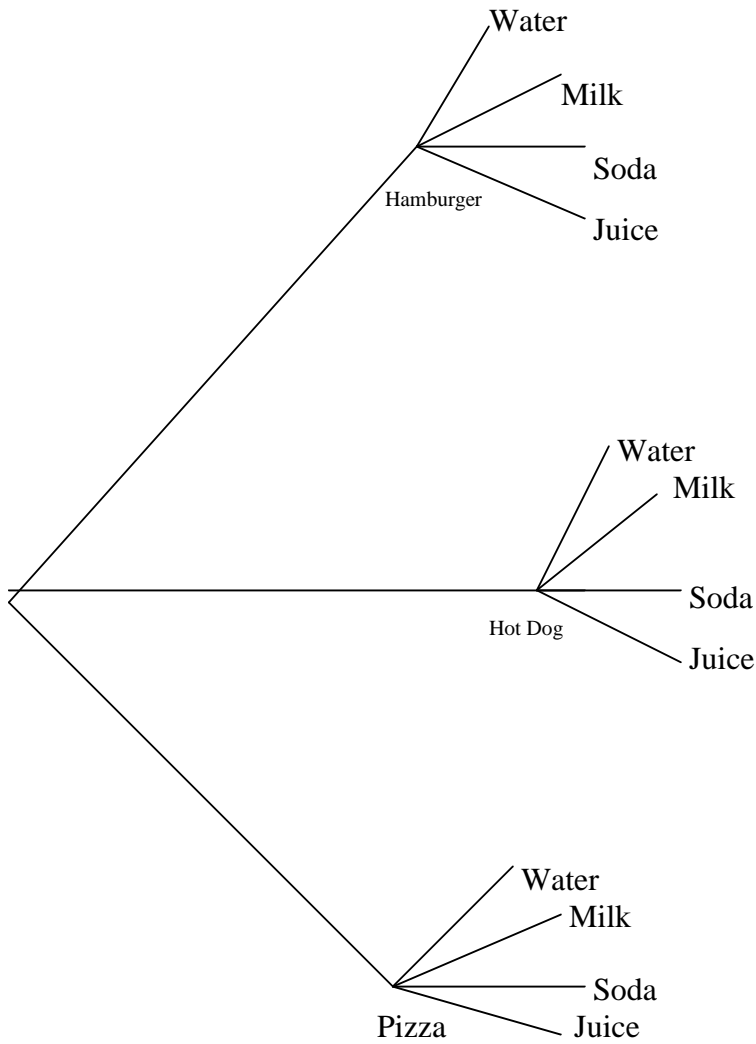


Figure 7: Tree Diagram Food Choices

There are three food choices and four drink choices:  $3 \times 4 = 12$ .

4. Figure 8 gives the tree diagram for this exercise

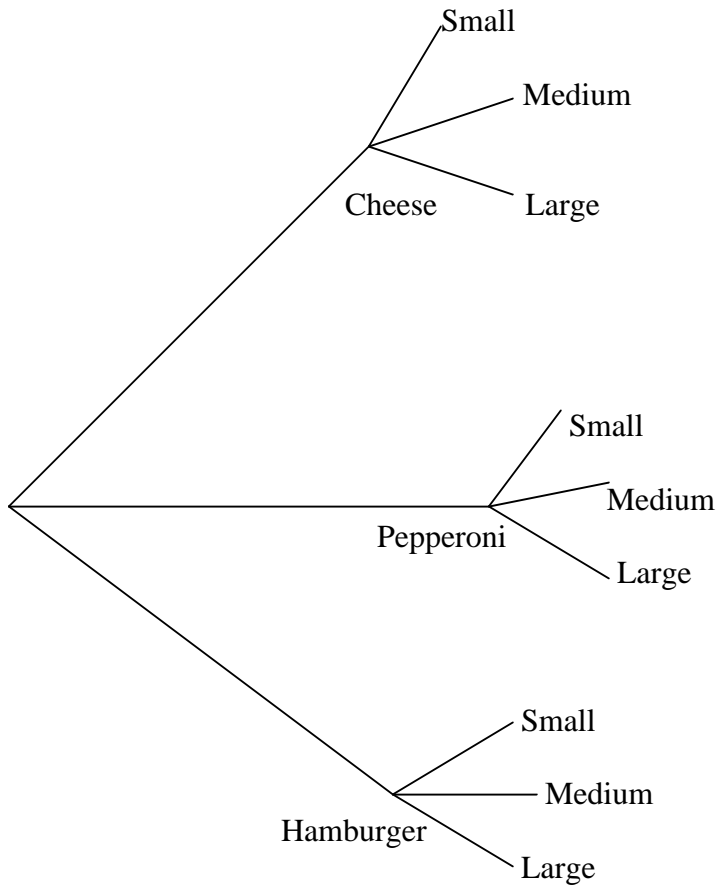


Figure 8: Tree Diagram Pizza Choices

There are three pizza topping choices and three size choices:  $3 \times 3 = 9$ .



5. Figure 9 gives the tree diagram for this exercise.

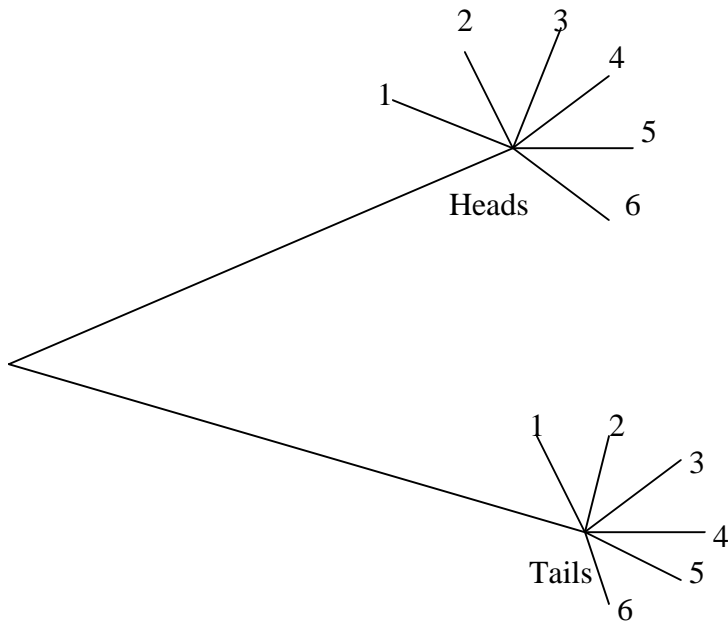


Figure 9: Tree Diagram Coin and Cube

There are two coin choices and six cube choices:  $2 \times 6 = 12$ .

### Three or More Steps

We have worked several examples of two step combinations. However, combinations can be done in three or more steps by using the same procedure as before – tree diagrams and multiplying.

Example 1: Flip a coin – a penny. There are two possible outcomes, heads and tails, see

Figure 10.

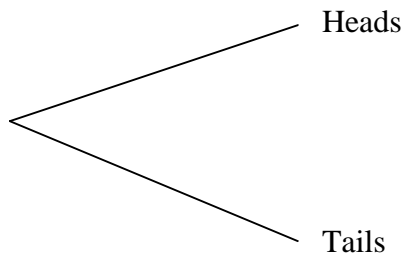


Figure 10: Penny

Now add a coin-a nickel. Flip a penny and a nickel. There are four possible outcomes, see Figure 11.

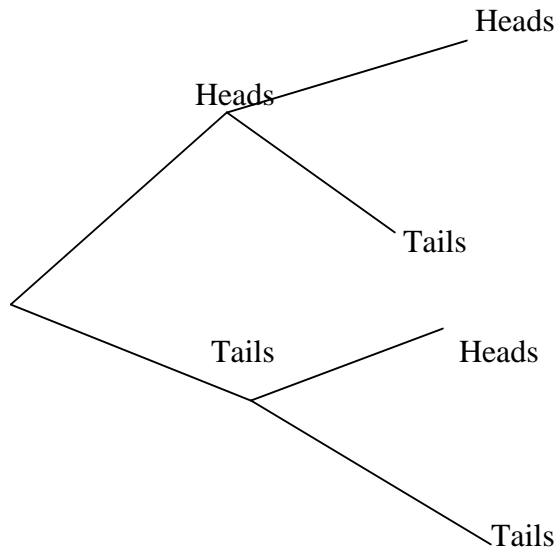


Figure 11: Nickel

Now add a dime. At this point a tree diagram becomes rather complicated. In fact, it becomes impractical. Therefore it is best to apply the multiplication principle. There are two penny outcomes, two nickel outcomes, and 2 dime outcomes:  $2 \times 2 \times 2 = 8$ .

Now add a quarter. There are two penny outcomes, two nickel outcomes, two dime outcomes, and 2 quarter outcomes:  $2 \times 2 \times 2 \times 2 = 16$ .

Example 2: Let's refer back to Briana's clothing choices – three pair of shorts and four shirts. Let's add a third choice – shoes. She may choose sandals, flip flops, or tennis shoes. How many possible combinations are there?

There are three short outcomes, four shirt outcomes, and three shoe outcomes:

$$3 \times 4 \times 3 = 36.$$

Example 3: Ernie has twelve coins. In how many different ways can he choose the

twelve coins?

There are two outcomes for coin number 1, two outcomes for coin number 2, two outcomes for coin number 3, two outcomes for coin number 4, two outcomes for coin number 5, two outcomes for coin number 6, two outcomes for coin number 7, two outcomes for coin number 8, two outcomes for coin number 9, two outcomes for coin number 10, two outcomes for coin number 11, and 2 outcomes for coin number 12:

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 4,032 .$$

NOTE: Third grade children will need to use a calculator to multiply numbers this large.

Example 4: Suppose a room has five lamps. Each lamp can be ON or OFF. How many different combinations of lighting the room are there?

There are two lamp 1 outcomes, two lamp 2 outcomes, two lamp 3 outcomes, two lamp 4 outcomes, and two lamp 5 outcomes:  $2 \times 2 \times 2 \times 2 \times 2 = 32 .$

### Homework Exercises: Three or More Steps

Directions: Use the multiplication principle to solve each of the following.

1. Katie is getting dressed for school. She may choose from four pairs of pants, two shirts, and six pairs of shoes. How many different ways could she get dressed?
2. Shaina goes to McDonald's for breakfast. She may choose from three kinds of biscuits: ham, sausage, or bacon. She may choose a side order: hash browns or fruit. She may choose a drink: milk, juice, or coffee. How many combinations of biscuit, side order, and drink are there?
3. During P.E., Monday through Friday, Kayla may choose between four sports: basketball, soccer, softball, and tennis. She must also choose between warm-up

exercises: running or jumping rope. How many combinations of days, sports, and warm-ups does she have?

Answer Key: Three or More Steps

1. There are four pair of pants, two shirts, and six pair of shoes:  $4 \times 2 \times 6 = 48$ .
2. There are three types of biscuits, two side order choices, and three drink choices:  
 $3 \times 2 \times 3 = 18$ .
3. There are five days, four sport choices, and two warm-up exercises:  $5 \times 4 \times 2 = 40$ .

Further Applications of the Multiplication Principle

Example 1: In the state of Virginia, how many different license plates can be made if each plate is to display three letters followed by three numbers?

Step 1: Question: How many letters in the English alphabet?

Answer: 26

So, there are 26 choices for the first letter, 26 choices for the second letter, and 26 choices for the third letter:  $26 \times 26 \times 26$ .

Step 2: Question: How many digits are used for numbers?

Answer: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9

There are 10 digits. Those ten digits are used to make all numbers. So, there are 10 choices for the first number, 10 choices for the second number, and 10 choices for the third number:  $10 \times 10 \times 10$ .

Step 3: Put the two steps together:  $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$  possible outcomes.

Note: Third grade students will need to use a calculator to multiply these numbers.

Example 2: A dance class has 9 boys and 11 girls. How many ways can the teacher choose a couple consisting of 1 boy and 1 girl to demonstrate the chicken dance?

Step 1: The teacher must choose a boy. There are 9 ways of choosing a boy.

Step 2: The teacher must choose a girl. There are 11 ways of choosing a girl.

Step 3: Apply the multiplication principle:  $9 \times 11 = 99$ .

Next suppose that the teacher may dress the girl in a dress, jeans, or a chicken suit. The teacher may dress the boy in jeans or a chicken suit. How many ways can she choose to dress the couple?

Step 1: Choose a boy. There are 9 ways to choose a boy.

Step 2: Choose a girl. There are 11 ways to choose a girl.

Step 3: Dress the boy. There are 2 ways to dress the boy.

Step 4: Dress the girl. There are 3 ways to dress the girl.

So, the number of possibilities is:  $9 \times 11 \times 2 \times 3 = 594$ .

Example 3: How many different two digit even numbers are there?

Step 1: Question: How many ways do we have of choosing the first digit?

Answer: There are 9 ways of choosing the first digit because there are 10 digits and you can't use a zero as the first digit.

Step 2: Question: How many ways do we have of choosing the second digit?

Answer: There are 5 ways of choosing the second digit. It must be a 0, 2, 4, 6, or 8 if the number is even.

Step 3: Apply the multiplication principle:  $9 \times 5 = 45$ .

### Homework Exercises: Multiplication Principle

Directions: Use the multiplication principle to solve the following problems.

1. A family consists of a mother, father, 2 girls, and 3 boys. How many different ways can the family choose one girl to wash the dishes and one boy to dry the dishes?
2. A family consists of a mother, father, 2 girls, and 3 boys. How many different ways can a boy, a girl, and a parent go shopping?
3. Katie is taking a science test. How many different ways can she answer all the questions on the test if the test has 10 true-false questions?
4. How many possible 5-digit zip codes are there?
5. How many 4-digit numbers are there that are odd?

#### Answer Key: Multiplication Principle

1. Step 1: Choose a girl. There are 2 ways to do this.  
Step 2: Choose a boy. There are 3 ways to do this.  
Step 3: Apply the multiplication principle:  $2 \times 3 = 6$  possibilities.
2. Step 1: Choose a boy. There are 3 ways to do this.  
Step 2: Choose a girl. There are 2 ways to do this.  
Step 3: Choose a parent. There are 2 ways to do this.  
Step 4: Apply the multiplication principle:  $3 \times 2 \times 2 = 12$  possibilities.
3. Step 1: Choose true as an answer. There are 10 ways to do this.  
Step 2: Choose false as an answer. There are 10 ways to do this.  
Step 3: Apply the multiplication principle:  $10 \times 10 = 100$  possibilities.
4. Step 1: Choose the number of digits for the first position. There are 9 choices for

this. The first digit can't be 0.

Step 2: Choose the number of digits for the second position. There are 10 choices for this.

Step 3: Choose the number of digits for the third position. There are 10 choices for this.

Step 4: Choose the number of digits for the fourth position. There are 10 choices for this.

Step 5: Choose the number of digits for the fifth position. There are 10 choices for this.

Step 6: Apply the multiplication principle:  $9 \times 10 \times 10 \times 10 \times 10 = 90,000$  possibilities.

5. Step 1: Choose the number of ways of choosing the first digit. There are 9 ways of doing this because the first digit can't be a zero.

Step 2: Choose the number of ways of choosing the second digit. There are 10 ways of doing this.

Step 3: Choose the number of ways of choosing the third digit. There are 10 ways of doing this.

Step 4: Choose the number of ways of choosing the fourth digit. This digit must be odd. The odd digits are 1, 3, 5, 7, and 9. There are 5 ways of doing this.

Step 5: Apply the multiplication principle:  $9 \times 10 \times 10 \times 5 = 4,500$  possibilities.

### Addition Principle of Counting

As we have seen, the multiplication principle applies to procedures consisting of a number of steps, or tasks, each of them to be carried out in order. Our next example

illustrates a second fundamental principle of counting; this principle applies to procedures where there are a number of tasks, but only one of them is to be carried out.

Example 1: A P.E. class consists of: 9 girls from Mrs. McFaddin's class, 8 boys from Mrs. McFaddin's class, 10 girls from Mrs. Richardson's class, and 11 boys from Mrs. Richardson's class. How many ways are there of choosing a girl from Mrs. McFaddin's class or a boy from Mrs. Richardson's class?

This problem is different than the other problems we have been solving. Now instead of performing both of two tasks we are to perform only one or the other of them. There are 9 ways of selecting a girl from Mrs. McFaddin's class, and 11 ways of selecting a boy from Mrs. Richardson's class. We need to carry out only one of these tasks. To find the number of ways of selecting one student, we put the 9 girls together with the 11 boys, to obtain  $9 + 11 = 20$  possibilities for choosing one student according to these rules.

Example 2: John will draw one card from a standard deck of playing cards. How many ways can he draw a king or a queen?

Step 1: Question: How many kings are there in a deck of cards?

Answer: There are 4 kings in a deck of cards.

Step 2: Question: How many queens are there in a deck of cards?

Answer: There are 4 queens in a deck of cards.

Step 3: If there are 4 kings and 4 queens in a deck of cards then John can't choose a king and a queen on the same draw. So, by applying the addition principle, there are  $4 + 4 = 8$  ways of choosing a king or a queen.



Example 3: In a pack of 12 colored pencils there are 2 red, 2 green, 1 yellow, 2 blue, 1 black, 1 brown, 1 gray, 1 purple, and 1 orange pencil. In one draw, how many ways can a student choose either a red or a green pencil?

Step 1: Question: How many red pencils are there?

Answer: There are 2 red pencils.

Step 2: Question: How many green pencils are there?

Answer: There are 2 green pencils.

Step 3: Apply the addition principle of counting. The student can't choose a red and a green pencil in one choice. So, there are  $2 + 2 = 4$  ways of choosing a red or a green pencil.

#### Homework Exercises: Addition Principle

Directions: Use the addition principle of counting to answer the following problems.

1. Kayla goes to the local pet store to choose a new pet. They have the following pets to choose from: 9 dogs, 12 cats, 24 hamsters, and 6 iguanas. How many ways are there of choosing a cat or a hamster?
2. Alex wins a prize for playing a game of Hot Seat in class. He may choose from the following prizes: 4 yo-yos, 6 slap bracelets, or 10 balls. How many ways are there of choosing a slap bracelet or a ball?
3. Chelsey goes to the store to purchase a candy bar. She may choose from 15 Hershey bars, 22 Reese cups, or 5 Snicker bars. How many ways are there of choosing a Reese cup or a Hershey bar?
4. Alyssa is making a mosaic. She may use 20 red squares, 20 green squares, 20 blue squares, 20 white squares, or 20 yellow squares. How many ways are there

of choosing a red square or a blue square?

5. There are 5 flights from Bristol to Orlando with a stop in Atlanta, one direct flight from Bristol to Orlando, and 7 flights from Bristol to Orlando with a stop in Charlotte. In how many ways can the McFaddin family travel from Bristol to Orlando for their vacation?

Answer Key: Addition Principle

1. There are 12 cats and 24 hamsters at the pet store. There are  $12 + 24 = 36$  choices.
2. There are 6 slap bracelets and 10 balls in the prize box. There are  $6 + 10 = 16$  choices.
3. There are 22 Reese cups and 15 Hershey bars at the store. There are  $22 + 15 = 37$  choices.
4. There are 20 red squares and 20 blue squares available for making a mosaic. There are  $20 + 20 = 40$  choices.
5. There are 5 flights from Bristol to Orlando with a stop in Atlanta, 1 nonstop flight from Bristol to Orlando, and 7 flights from Bristol to Orlando with a stop in Charlotte. There are  $5 + 1 + 7 = 13$  choices.

Subtraction Principle of Counting

Now, that we have had some practice with the addition principle of counting, we can carry it one step further and apply the subtraction principle.

We have already learned that when we have a choice of two methods of performing a procedure, then the number of ways of performing the procedure is found by adding the number of ways using the first method and the number of ways of using the second method. However, sometimes when you do this, double counting occurs. You

must subtract in this situation [8].

Example 1: Christian will draw one card from a standard deck of playing cards. How many ways can he choose a queen or a red card?

Question: How many ways are there of choosing a queen?

Answer: There are four ways.

Question: How many ways are there of choosing a red card?

Answer: There are twenty-six ways. (There are fifty-two cards in a deck. Half of them are red and half of them are black.) Demonstrate this to the students using a deck of cards.

Question: Is it possible to draw a card that is a queen and a red card on the same draw?

In other words, is it possible to draw a red queen?

Answer: Yes. There are two red queens in a deck of cards.

So, there are four ways of choosing a queen plus twenty-six ways of choosing a red card:  $4 + 26 = 30$ .

The problem is that we have counted the red queens twice. We counted them once in the queen count and again in the red count. Therefore, we must subtract two, because we counted the two red queens twice. So, there are thirty minus two ways of choosing a queen or a red card:  $30 - 2 = 28$  ways.

Example 2: A family consists of a mother, a father, two girl children, and three boy children. How many ways can the family choose a male or a child to take out the trash?

Question: How many ways are there of choosing a male?

Answer: There's one father and three boy children. So, there are  $1 + 3 = 4$  ways of choosing a male.

Question: How many ways are there of choosing a child to take out the trash?

Answer: There are two girl children and three boy children. So, there are  $2 + 3 = 5$  ways of choosing a child to take out the trash. So, there are  $4 + 5 = 9$  ways of choosing a male or a child to take out the trash. The problem is that we counted the boy children twice. Therefore we must subtract the three boy children:  $9 - 3 = 6$ . So, there are six ways of choosing a male or a child to take out the trash.

Example 3: A family consists of a mother, a father, four girl children, and six boy children. How many ways can the family choose a female or a child to wash the car?

Question: How many ways are there of choosing a female?

Answer: There's one mother and four girl children. So, there are  $1 + 4 = 5$  ways of choosing a female.

Question: How many ways are there of choosing a child?

Answer: There are four girl children and six boy children. So, there are  $4 + 6 = 10$  ways of choosing a child.

So, there are  $5 + 10 = 15$  ways of choosing a female or a child to wash the car.

The problem is that we counted the four girl children twice. Therefore, we must subtract the four girl children:  $15 - 4 = 11$ . So, there are 11 ways of choosing a female or a child to wash the car.

### Homework Exercises: Subtraction Principle

Directions: Use the subtraction principle to solve each of the following problems.

1. You are to draw one card from a deck of 52 cards. How many ways can you choose a king or a black card?
2. You are to draw one card from a deck of 52 cards. How many ways can you choose a two, a five, or a black card?
3. In a high school math class, there are six girls in the tenth grade, eight boys in the tenth grade, five girls in the ninth grade, and four boys in the ninth grade. In how many ways can the teacher choose a girl or a tenth grader to answer a question?
4. In the above example, in how many ways can the teacher choose a ninth grader or a boy to call the roll?

### Answer Key: Subtraction Principle

1. Question: How many ways are there of choosing a king?

Answer: There are four ways.

Question: How many ways are there of choosing a black card?

Answer: There are 26 ways.

Question: Is it possible to draw a black king?

Answer: Yes, there are 2 black kings in a deck of 52 cards.

So, there are  $26 + 4 = 30$  ways of choosing a king or a black card. The problem is that you counted the black kings twice. So, there are  $30 - 2 = 28$  ways of choosing a king or a black card.

2. Question: How many ways are there of choosing a two?

Answer: There are 4 ways of choosing a two.

Question: How many ways are there of choosing a five?

Answer: There are 4 ways of choosing a five.

Question: How many ways of choosing a black card?

Answer: There are 26 ways of choosing a black card.

So, there are  $4 + 4 + 26 = 34$  ways of choosing a 2, 5, or a black card. However, we counted the two black twos and the two black fives twice. So, we need to subtract four:  $34 - 4 = 30$ . Therefore, there are 30 ways of choosing a 2, 5, or a black card.

3. Question: How many girls are in the class?

Answer: There are 6 girls in the tenth grade and there are 5 girls in the ninth grade. So, there are  $6 + 5 = 11$  girls in the class.

Question: How many tenth graders are in the class?

Answer: There are 6 tenth grade girls and 8 tenth grade boys. So, there are  $6 + 8 = 14$  tenth graders in the class. So, there are  $11 + 14 = 25$  ways of choosing a girl or a tenth grade student. The problem is that you counted the tenth grade girls twice. So, there are  $25 - 6 = 19$  ways the teacher can choose a girl or a tenth grader to answer a question.

4. Question: How many ninth graders are in the class?

Answer: There are 5 girls and 4 boys in the ninth grade. So,  $5 + 4 = 9$  ninth grade students.

Question: How many boys are in the class?

Answer: There are 8 tenth grade boys and 4 ninth grade boys in the class. So, there are  $8 + 4 = 12$  boys in the class. So, there are 9 ninth graders plus 12 boys

or 21 ways of choosing a ninth grader or a boy to call the roll. The problem is that we counted the four ninth grade boys twice. So, we must subtract them:

$21 - 4 = 17$ . So, there are 17 ways of choosing a ninth grader or a boy to call the roll.

## CHAPTER 4

### PERMUTATIONS UNIT

In this unit we will examine a concept in counting called permutations.

The number of ways that you can change the order of a set of things is called the number of permutations of that set of things. For example, how many different ways can you arrange the letters in the word WHO? By using the letters W, H, and O you can make the following letter arrangements: WHO, WOH, HWO, HOW, OHW, and OWH. Each different letter arrangement is called a permutation of the word WHO. In finding permutations it is important to remember that order matters[7].

#### Permutation Activity 1

In order to understand the concept of permutations, young children need to have some hands on activity. This activity will allow them to manipulate cards to see how permutations work[6].

Supplies: cards with various letters of the alphabet (no letters repeat): M A T H.

Directions:

1. Students work in pairs. Distribute a set of cards to each pair of students. A set of cards should consist of four cards. The task is to find all four-letter arrangements of the cards. The arrangements do not have to spell actual words. Students should record or write down each arrangement.
2. Encourage students to develop a strategy for finding their answers.
3. Discuss the findings with the class.

#### Permutation Activity 2

We have previously worked with finding all possible four letter combinations of



words. In this activity we are going to work with a different set of cards. We are going to find all possible combinations of five letter words[6].

Supplies: cards with various letters of the alphabet: S H I N E.

Directions:

1. Students work in pairs. Distribute a set of cards to each pair. A set of cards should consist of five cards. The task is to find all five- letter arrangements of the cards. The arrangements do not have to spell actual words. Students should record or write down each arrangement.
2. Encourage students to develop a strategy for finding their answers.
3. Discuss the findings with the class.
4. Students could also be asked to make a tree diagram showing their findings.

#### Follow-Up Activity

As a follow-up activity, give each pair of students a set of four cards where one of the letters repeat: B A L L.

As before, the task is to find all the four-letter arrangements of the cards. The arrangements do not have to spell actual words. Ask students to record each arrangement. Ask students to make observations about their answers. (The students should observe that the number of possible arrangements of the letters of BALL will be the same as the letters of MATH. The difference is that some of the arrangements will be the same[6].

#### In Class Exercises

Example 1: How many ways can you arrange the letters in the word STOP?

Words that start with “S”: STOP STPO SOTP SOPT SPTO SPOT

Words that start with “T”: TSOP TSPO TOSP TOPS TPSO TPOS

Words that start with “O”: OSTP OSPT OTSP OTPS OPST OPTS

Words that start with “P”: PSTO PSOT PTSO PTOS POST POTS

There are 24 ways to order the letters in STOP. You might ask if there is a rule to follow. The answer is yes. The rule is as follows:

1. There are 4 ways to pick the first letter. It can be “S”, “T”, “O”, or “P”.
2. After you pick the first letter there are 3 ways to pick the second letter.
3. After you pick the first two letters, there are 2 ways to pick the third letter.
4. After picking the first three letters, there is only one letter left to pick.

So, the number of ways to order the letters in “STOP” is  $4 \times 3 \times 2 \times 1 = 24$  ways.

Example 2: How many ways can you arrange the letters in the word BOY?

Words that start with “B”: BOY BYO

Words that start with “O”: OBY OYB

Words that start with “Y”: YOB YBO

There are 6 ways to order the letters in BOY. If you follow the rule, you will get  $3 \times 2 \times 1 = 6$ .

Example 3: How many ways can you arrange the letters in HONAKER?

1. There are 7 ways to pick the first letter.
2. After you pick the first letter, there are 6 ways to pick the second letter.
3. After you pick the second letter, there are 5 ways to pick the third letter.
4. After you pick the third letter, there are 4 ways to pick the fourth letter.
5. After you pick the fourth letter, there are 3 ways to pick the fifth letter.

6. After you pick the fifth letter, there are 2 ways left to pick the sixth letter.
7. After you pick the sixth letter, there's one way to pick the seventh letter.

So, there are  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$  ways to arrange the letters in HONAKER .

NOTE: Third grade students will need to use a calculator to multiply numbers that are this large.

Example 4: How many ways can you arrange the letters in BASKET?

Now that you understand the process that is involved, we can move straight to the rule.

So, there are  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$  ways to arrange the letters in the word BASKET.

#### Homework Exercises: Permutations

1. How many different ways can you arrange the letters in the word SUN?
2. How many different ways can you arrange the letters in the word GIRL?
3. How many different ways can you arrange the letters in the word TIGER?
4. How many different ways can you arrange the letters in the word STUDENT?
5. How many different ways can you arrange the letters in your name?

#### Answer Key: Permutations

1. There are  $3 \times 2 \times 1 = 6$  ways to arrange the letters in the word SUN.
2. There are  $4 \times 3 \times 2 \times 1 = 24$  ways to arrange the letters in the word GIRL.
3. There are  $5 \times 4 \times 3 \times 2 \times 1 = 120$  ways to arrange the letters in the word TIGER.
4. There are  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$  ways to arrange the letters in the word STUDENT.

5. Answers will vary according to the number of letters in each name.

Word Length	Permutations	Result
1	1	1
2	$2 \times 1$	2
3	$3 \times 2 \times 1$	6
4	$4 \times 3 \times 2 \times 1$	24
5	$5 \times 4 \times 3 \times 2 \times 1$	120
6	$6 \times 5 \times 4 \times 3 \times 2 \times 1$	720
7	$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$	5,040

#### More Permutations

Now, let's suppose that you only want to choose a few letters out of a word. For example, you only want to choose 2 letters out of the word "TABLE". Here are all the ways to pick them:

TA TB TL TE AT AB AL AE BT BA  
BL BE LT LA LB LE ET EA EB EL

There are 20 pairs. You may want to know if there is an easier method than writing them all down. There is a rule [7]. Here it is:

1. There are 5 ways to choose the first letter.
2. After you choose the first letter, there are 4 ways to choose the second letter.

So, the number of two letter permutations of the five letter word "TABLE"

is:  $5 \times 4 = 20$ .

General Rule:

If you have a word with “N” letters in it, then:

- to select 2 letters, the number of permutations is  $n \times (n - 1)$

Example: TABLE,  $5 \times (5 - 1) = 5 \times 4 = 20$

- to select 3 letters, the number of permutations is  $n \times (n - 1) \times (n - 2)$

Example: TABLE,  $5 \times (5 - 1)(5 - 2) = 5 \times 4 \times 3 = 60$

- to select 4 letters, the number of permutations is  
 $n \times (n - 1) \times (n - 2) \times (n - 3)$

Example: TABLE,  $5 \times (5 - 1)(5 - 2)(5 - 3) = 5 \times 4 \times 3 \times 2 = 120$

Example 1: Suppose we want to find the number of ways to arrange the three letters of the word DOG in different two-letter groups where DO is different from OD and there are no repeated letters. Because order matters, we’re finding the number of permutations of size two that can be taken from a set of size three. We could list them: DO, DG, OD, GD, OG, and GO. Or we could apply the rule:  $3 \times (3 - 1) = 3 \times 2 = 6$ .

Example 2: Now, let’s suppose that we have ten letters and want to make groupings of four letters. It’s harder to list all those permutations. So, it is best to apply the rule:

$$10 \times (10 - 1) \times (10 - 2) \times (10 - 3) = 10 \times 9 \times 8 \times 7 = 5,040.$$

Example 3: If the five letters,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$  are put into a hat, in how many different ways could you draw out three letters?

Here you must think, I have a set of five and I want to choose three. So, you apply the rule:  $5 \times (5 - 1) \times (5 - 2) = 5 \times 4 \times 3 = 60$ . There are 60 different ways to choose a set of

three out of the five letters.

Example 4: You have 21 students in your room. You want to put your students in groups of three. You want the students in order from left to right. In how many different ways could you do this?

Here you must think, I have 21 students, and I want to choose groups of three. So, you apply the rule:  $21 \times (21 - 1) \times (21 - 2) = 21 \times 20 \times 19 = 7,980$  ways.

### Homework Exercises: More Permutations

Directions: Use the permutations rule to solve the following problems.

1. You go to the local ice cream shop. They have 20 different flavors of ice cream. You want to make a cone that has two scoops of ice cream on it, a top layer and a bottom layer. In how many different ways could you choose two scoops from the 20 flavors?
2. Suppose you want to find the number of ways to arrange the five letters in the word PIZZA in different three-letter groups. In how many different ways can we do this?
3. You go back to the local ice cream shop. Today they only have fourteen flavors. You want to make a cone that has three scoops of ice cream on it, a top layer, a middle layer, and a bottom layer. In how many different ways could you choose three scoops from the 14 flavors?
4. Suppose we want to find the number of ways to arrange the six letters in the word TIGERS in different three-letter groups. In how many different ways can we do this?

### Answer Key: More Permutations

1. There are 20 flavors of ice cream. You are to choose two scoops. In order to find out how many different ways you can choose two scoops, apply the permutations rule:  $20 \times (20 - 1) = 20 \times 19 = 380$ . So, there are 380 ways to choose two scoops of ice cream from the 20 flavors.
2. There are five letters in the word PIZZA. You want to arrange the letters in groups of three. In order to find out how many different ways you can choose 3 from 5, apply the permutations rule:  $5 \times (5 - 1) \times (5 - 2) = 5 \times 4 \times 3 = 60$ . So, there are 60 ways to choose three letters from five.
3. There are 14 flavors of ice cream. You are to choose three scoops. In order to find out how many different ways you can choose three scoops, apply the permutations rule:  $14 \times (14 - 1) \times (14 - 2) = 14 \times 13 \times 12 = 2,184$ . So, there are 2,184 ways to choose three scoops from 14 flavors.
4. There are six letters in the word TIGERS. You want to arrange the letters in groups of three. In order to find out how many different ways you can choose 3 from 6, apply the permutations rule:  $6 \times (6 - 1) \times (6 - 2) = 6 \times 5 \times 4 = 120$ . So, there are 120 ways to choose three letters from six.

### Factorial Representation of Permutations

Today we are looking at the symbol “!”. The symbol, “!”, in permutations means to multiply. For example,  $5!$  means  $5 \times 4 \times 3 \times 2 \times 1 = 120$ .  $3!$  means  $3 \times 2 \times 1 = 6$ .  $6!$  means  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ .

Example 1: Five different books are on a shelf. In how many different ways can you arrange them?

Answer: There are  $5!$  ways to arrange them:  $5 \times 4 \times 3 \times 2 \times 1 = 120$ .

Example 2: There are 6 letters in the word TIGERS. In how many different ways can you arrange them?

Answer: There are  $6!$  ways to arrange them:  $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ .

Example 3: There are  $6!$  permutations of the 6 letters of the word SQUARE. In how many of them is R the second letter?

Answer:    R            

Let R be the second letter. Then there are 5 ways to fill the first spot, 4 ways to fill the third spot, 3 ways to fill the fourth spot, 2 ways to fill the fifth spot, and 1 way to fill the last spot. There are  $5!$  such permutations:  $5 \times 4 \times 3 \times 2 \times 1 = 120$  ways.

Example 4: There are  $5!$  permutations of the word VIDEO. In how many of them is D the second letter?

Answer:    D         

Let D be the second letter. Then there are 4 ways to fill the first spot, 3 ways to fill the third spot, 2 ways to fill the fourth spot, and 1 way to fill the last spot. There are  $4!$  Such permutations:  $4 \times 3 \times 2 \times 1 = 24$  ways.

Example 5: There are  $3!$  permutations of the word CAT. In how many of them is C the last letter?



Answer: \_\_\_ \_ C

Let C be the last letter. Then there are 2 ways to fill the first spot and 1 way to fill the second spot. There are  $2!$  such permutations:  $2 \times 1 = 2$  ways.

Example 6: There are  $7!$  permutations of the word STUDENT. In how many of them is E the fifth letter?

Answer: \_\_\_ \_ \_ \_ E \_\_\_

Let E be the fifth letter. Then there are 6 ways to fill the first spot, 5 ways to fill the second spot, 4 ways to fill the third spot, 3 ways to fill the fourth spot, 2 ways to fill the sixth spot, and 1 way to fill the last spot. There are  $6!$  such permutations:

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ ways [5].}$$

### Homework Exercises: Factorials

Directions: Use factorials to solve the following problems.

1. There are  $4!$  permutations of the 4 letters of the word BLUE. In how many of them is the letter U the second letter?
2. There are  $6!$  permutations of the 6 letters of the word ORANGE. In how many of them is the letter O the last letter?
3. There are  $8!$  permutations of the 8 letters of the word FLAVORED. In how many of them is the letter V the third letter?
4. There are  $10!$  permutations of the 10 letters of the word MICROWAVES. In how many of them is the letter W the fourth letter?

Answer Key: Factorials

1. \_\_\_ U \_\_\_

Let U be the second letter. Then there are 3 ways to fill the first spot, 2 ways to fill the third spot, and 1 way to fill the last spot. There are  $3!$  such permutations:  
 $3 \times 2 \times 1 = 6$ .

2. \_\_\_ \_\_\_ \_\_\_ \_\_\_ O

Let O be the last letter. Then there are 5 ways to fill the first spot, 4 ways to fill the second spot, 3 ways to fill the third spot, 2 ways to fill the fourth spot, and 1 way to fill the fifth spot. There are  $5!$  such permutations:  $5 \times 4 \times 3 \times 2 \times 1 = 120$ .

3. \_\_\_ \_\_\_ V \_\_\_ \_\_\_ \_\_\_

Let V be the third letter. Then there are 7 ways to fill the first spot, 6 ways to fill the second spot, 5 ways to fill the fourth spot, 4 ways to fill the fifth spot, 3 ways to fill the sixth spot, 2 ways to fill the seventh spot, and 1 way to fill the last spot. There are  $7!$  such permutations:  $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5,040$ .

4. \_\_\_ \_\_\_ \_\_\_ W \_\_\_ \_\_\_ \_\_\_

Let W be the fourth letter. Then there are 9 ways to fill the first spot, 8 ways to fill the second spot, 7 ways to fill the third spot, 6 ways to fill the fifth spot, 5 ways to fill the sixth spot, 4 ways to fill the seventh spot, 3 ways to fill the eighth spot, 2 ways to fill the ninth spot, and 1 way to fill the last spot. There are  $9!$  such permutations:  $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$ .

## CHAPTER 5

### MIXING THINGS UP

In this lesson students will discover, through interactive play on the computer, that a combination is a set of objects in which order is not important. Students will need to have access to a computer and the internet. Take the students to the computer lab. Have them log on to [www.hellam.net/math2000/combi.html](http://www.hellam.net/math2000/combi.html). On this site they will have access to three puzzles. The puzzles are called “Football Strips”, “Ice Cream Cones”, and “Cars”. Have them to click on “Football Strips”. Give children time to manipulate the puzzle and come up with an answer to the question of “How many different football strips can you draw?” The children should be able to solve the puzzle by clicking on the different colors. When they answer the question correctly, the ball will bounce and they will be sent to the home page where they can choose the next puzzle “Ice Cream Cones”. They will then follow the same procedure to solve this puzzle. The question this time is “How many ice cream cones can you draw?” When the question has been answered correctly the students are sent back to start where they can choose the final puzzle “Cars”. The question for this puzzle is “How many cars can you draw?” The children may then manipulate the choices to solve the puzzle. Once all the students have solved the puzzles then the class will return to the classroom. The students will then brainstorm some other things that you could experiment with such as faces, animals, pizzas, houses, spaceships, aliens, etc. They will then choose one and make up a similar puzzle of their own using paper and crayons[4].

#### Puzzles

Remind students that a combination is a set of objects in which order is not

important. Review the puzzles; Football Strips, Ice Cream Cones, and Cars. Allow students to share the puzzles that they designed. Discuss the combinations of objects as they are presented.

### Permutations and Combinations

In this lesson we will be combining the idea of permutations and combinations. Remember from our prior lessons that the order of the objects is important in permutations. In other words  $abc$  is different from  $bca$ . In combinations, however, we are only concerned that  $a, b,$  and  $c$  have been chosen. In other words  $abc$  and  $bca$  are the same combination [5]. Here are all the combinations of  $abcd$  taken three at a time:  $abc, abd, acd,$  and  $bcd$ . There are four such combinations. We call this:

*The number of combinations of four things taken three at a time.*

Now, how are the number of combinations related to the number of permutations?

Let's look at the number of permutations for each combination:

$abc$	$abd$	$acd$	$bcd$
$acb$	$adb$	$adc$	$bdc$
$bac$	$bad$	$cad$	$cbd$
$bca$	$bda$	$cda$	$cdb$
$cab$	$dab$	$dac$	$dbc$
$cba$	$dba$	$dca$	$dcb$

Each column is the  $3!$  permutations of that combination. But they are all one combination because order does not matter. Hence, there are  $3!$  times as many permutations as combinations. Therefore, in order to find the number of combinations of 4 options, choosing 3, take the number of permutations and divide by the number of permutations that each combination generates.

Number of permutations of 4 options, choosing 3:  $4 \times 3 \times 2 = 24$ .

Number of permutations that each combination generates:  $1 \times 2 \times 3 = 6$ .

Now divide:  $24 \div 6 = 4$ .

Example 1: How many combinations are there of 5 cookies taken 4 at a time?

Number of permutations of 5 cookies, choose 4:  $5 \times 4 \times 3 \times 2 = 120$ .

Number of permutations that each combination generates:  $1 \times 2 \times 3 \times 4 = 24$ .

Now divide:  $120 \div 24 = 5$ .

Example 2: How many combinations are there of 8 students chosen 6 at a time?

Number of permutations of 8 students, choose 6:  $8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20,160$ .

Number of permutations that each combination generates:  $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ .

Now divide:  $20,160 \div 720 = 28$ .

Example 3: How many combinations are there of 8 dogs chosen 2 at a time?

Number of permutations of 8 dogs, choose 2:  $8 \times 7 = 56$ .

Number of permutations that each combination generates:  $1 \times 2 = 2$ .

Now divide:  $56 \div 2 = 28$ .

Example 4: There are 12 boys in Mrs. McFaddin's class. In how many ways can Mrs. McFaddin choose 2 boys from the 12?

Number of permutations of 12 boys, choose 2:  $12 \times 11 = 132$ .

Number of permutations that each combination generates:  $1 \times 2 = 2$ .

Now divide:  $132 \div 2 = 66$ .

Example 5: There are 14 girls in Mrs. McFaddin's class. In how many ways can Mrs. McFaddin choose 3 girls from 14?

Number of permutations of 14 girls, choose 3:  $14 \times 13 \times 12 = 2,184$ .

Number of permutations that each combination generates:  $1 \times 2 \times 3 = 6$ .

Now divide:  $2,184 \div 6 = 364$ .

### Homework Exercises: Permutations and Combinations

Directions: Use the number of permutations and the number of each permutation that each combination generates to solve the following problems.

1. There are 6 toys from which to choose and you are to choose 2 of them. In how many ways can you choose 2 toys from the 6 toys?
2. There are 12 pizza toppings from which to choose and you are to choose 3 of them. In how many ways can you choose 3 toppings from the 12?
2. There are 4 kittens from which to choose and your mom says that you may choose 2 to take home. In how many different ways can you choose 2 kittens from the 4?
3. There are 8 toppings for your hamburger. In how many different ways can you choose 4 toppings for your hamburger?
5. There are 48 games to choose from at the store. Your dad says that you may have 3 games to take home. In how many different ways can you choose the 3 games?

### Answer Key: Permutations and Combinations

1. There are 6 toys from which to choose and you are to choose 2 of them. In how many ways can you do this?

Number of permutations of 6 toys, choose 2:  $6 \times 5 = 30$ .

Number of combinations that each permutation generates:  $1 \times 2 = 2$ .

Now divide:  $30 \div 2 = 15$ .

2. There are 12 pizza toppings from which to choose and you are to choose 3 of them.

In how many ways can you do this?

Number of permutations of 12 toppings, choose 3:  $12 \times 11 \times 10 = 1,320$ .

Number of combinations that each permutation generates:  $1 \times 2 \times 3 = 6$ .

Now divide:  $1,320 \div 6 = 220$ .

3. There are 4 kittens from which to choose and you are to choose 2 of them. In

how many ways can you do this?

Number of permutations of 4 kittens, choose 2:  $4 \times 3 = 12$ .

Number of combinations that each permutation generates:  $1 \times 2 = 2$ .

Now divide:  $12 \div 2 = 6$ .

4. There are 8 toppings from which to choose and you are to choose 4 of them.

In how many ways can you do this?

Number of permutations of 8 toppings, choose 4:  $8 \times 7 \times 6 \times 5 = 1,680$ .

Number of combinations that each permutation generates:  $1 \times 2 \times 3 \times 4 = 24$ .

Now divide:  $1,680 \div 24 = 70$ .

5. There are 48 games from which to choose and you may choose 3 of them. In how

many ways can you do this?

Number of permutations:  $48 \times 47 \times 46 = 103,776$ .

Number of combinations that each permutation generates:  $1 \times 2 \times 3 = 6$ .

Now divide:  $103,776 / 6 = 17,296$ .

### More Combinations

Example 1: Let's suppose that we have 10 posters to choose from. We want to choose 3

of them to hang on the wall in our classroom. In how many different ways could we do this?

We can think of this in terms of 10 posters, choose 3. The order does not matter.

Step 1:  $10 \times 9 \times 8 = 720$

Step 2: Each of these can be arranged in  $3!$  ways:  $1 \times 2 \times 3 = 6$

Step 3:  $720 \div 6 = 120$  different ways to choose the posters.

Example 2: Let's suppose that we have 25 books to choose from. We want to choose 2 of them to check out. In how many different ways can you do this?

Think in terms of 25 books, choose 2.

Step 1:  $25 \times 24 = 600$

Step 2:  $1 \times 2 = 2$

Step 3:  $600 \div 2 = 300$  different ways to choose the books.

Example 3: You are going to summer camp. You have 5 swimming suits. Your mom says that you may only take 3 of them with you. In how many different ways can you do this?

Think in terms of 5 suits, choose 3.

Step 1:  $5 \times 4 \times 3 = 60$

Step 2:  $1 \times 2 \times 3 = 6$

Step 3:  $60 \div 6 = 10$  different ways to choose the swimming suits.



Example 4: You are at a salad bar. There are 35 items on the bar. You want to put 5 items on your salad. In how many different ways can you select the 5 items?

Think in terms of 35 items, choose 5.

$$\text{Step 1: } 35 \times 34 \times 33 \times 32 \times 31 = 38,955,840$$

$$\text{Step 2: } 1 \times 2 \times 3 \times 4 \times 5 = 120$$

$$\text{Step 3: } 38,955,840 \div 120 = 324,632 \text{ different ways.}$$

Example 5: You are going to the beach. You go on the internet and find that there are 14 different routes to take. You want to choose 2 different routes. In how many different ways could you do this?

Think in terms of 14 routes, choose 2.

$$\text{Step 1: } 14 \times 13 = 182$$

$$\text{Step 2: } 1 \times 2 = 2$$

$$\text{Step 3: } 182 \div 2 = 91 \text{ different ways.}$$

Example 6: You go to the local shoe store. They have 32 pairs of shoes in your size. In how many different ways could you choose 4 pair to take home?

Think in terms of 32 choose 4.

$$\text{Step 1: } 32 \times 31 \times 30 \times 29 = 863,040$$

$$\text{Step 2: } 1 \times 2 \times 3 \times 4 = 24$$

$$\text{Step 3: } 863,040 \div 24 = 35,960 \text{ different ways.}$$

### Homework Exercises: More Combinations

Directions: Use combinations and permutations to solve each of the following

problems.

1. You want to choose a soda from the machine. There are ten different types of sodas in the machine. You want to choose three sodas. In how many different ways can you do this?
2. At the local store there are 15 different kinds of salad dressings. Your mom told you to choose 3 different kinds. In how many ways can you do this?
3. You are designing a flower arrangement. You have 22 different kinds of flowers to choose from. You need to choose 4 kinds to include in your arrangement. In how many different ways can you do this?
4. There are 7 different balls to choose from. You need to choose 5 balls. In how many different ways can you do this?
5. You have 25 channels to choose from. You may choose 5 from the 25 channels to have on your T.V. In how many different ways can you do this?

Answer Key: More Combinations

1. Think 10 sodas, choose 3. In how many ways can you do this?

Step 1:  $10 \times 9 \times 8 = 720$

Step 2:  $1 \times 2 \times 3 = 6$

Step 3:  $720 \div 6 = 120$  different ways

2. Think 15 salad dressings, choose 3. In how many ways can you do this?

Step 1:  $15 \times 14 \times 13 = 2,730$

Step 2:  $1 \times 2 \times 3 = 6$

Step 3:  $2,730 \div 6 = 455$  different ways

3. Think 22 flowers, choose 4. In how many ways can you do this?

Step 1:  $22 \times 21 \times 20 \times 19 = 175,560$

Step 2:  $1 \times 2 \times 3 \times 4 = 24$

Step 3:  $175,560 \div 24 = 7,315$  different ways

4. Think 7 balls, choose 5. In how many ways can you do this?

Step 1:  $7 \times 6 \times 5 \times 4 \times 3 = 2,520$

Step 2:  $1 \times 2 \times 3 \times 4 \times 5 = 120$

Step 3:  $2,520 \div 120 = 21$  different ways

5. Think 25 channels, choose 5. In how many ways can you do this?

Step 1:  $25 \times 24 \times 23 \times 22 \times 21 = 6,375,600$

Step 2:  $1 \times 2 \times 3 \times 4 \times 5 = 120$

Step 3:  $6,375,600 \div 120 = 53,130$  different ways

### Combination and Permutation Problems

Example 1: There are 6 girls and 6 boys on Kourtney's t-ball team. In how many different ways can the coach select a team of 9 players? The team consists of 4 girls and 5 boys.

The first step in solving this problem is to identify how many ways there are of selecting a boy. There are 6 boys and you need to choose 5. So,  $6 \times 5 \times 4 \times 3 \times 2 = 720$  divided by  $1 \times 2 \times 3 \times 4 \times 5 = 120$  is equal to 6.

The second step in solving this problem is to identify how many ways there are of selecting a girl. There are 6 girls and you need to choose 4. So,  $6 \times 5 \times 4 \times 3 = 360$  divided by  $1 \times 2 \times 3 \times 4 = 24$  is equal to 15.

The third step is then to multiply the number of ways of choosing a boy by the number of

ways of choosing a girl:  $5 \times 15 = 90$  ways.

Example 2: There are 10 boys and 12 girls in Mrs. McFaddin's third grade class. She wants to select a group of 3 students from the class to work on a math project. She wants the group to consist of 2 girls and 1 boy. In how many ways can she do this?

Step 1: How many ways can she choose the 2 girls?

$12 \times 11 = 132$  divided by  $1 \times 2 = 2$  which is equal to 66 ways

Step 2: How many ways can she choose the boy?

$10 \div 1 = 10$  ways.

Step 3: Multiply the number of ways of choosing a girl by the number of ways of choosing a boy:  $66 \times 10 = 660$  ways.

Example 3: At a local ice cream shop there are 3 different types of cones and 52 different flavors of ice cream. In how many ways can you choose 1 cone with 2 scoops of ice cream?

Step 1: How many ways can you choose a cone?

$3 \div 1 = 3$  ways

Step 2: How many ways can you choose 2 scoops of ice cream?

$52 \times 51 = 2,652$  divided by  $1 \times 2 = 2$  equals 1,326

Step 3: Multiply the number of ways of choosing a cone by the number of ways of choosing 2 scoops of ice cream:  $3 \times 1,326 = 3,978$  ways.

Example 4: You are packing your suitcase to go on vacation. You have 15 different

shirts and 9 different pairs of shorts. You need to choose 5 shirts and 5 pairs of shorts. In how many ways can you choose the 5 outfits?

Step 1: How many ways can you choose the shirts?

15 shirts, choose 5

$$15 \times 14 \times 13 \times 12 \times 11 = 360,360 \text{ divided by } 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ equals } 3,003$$

Step 2: How many ways can you choose the shorts?

9 shorts, choose 5

$$9 \times 8 \times 7 \times 6 \times 5 = 15,120 \text{ divided by } 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ equals } 126$$

Step 3: Multiply the number of ways of choosing a shirt by the number of ways of choosing shorts:  $3,003 \times 126 = 378,378$  ways.

#### Homework Exercises: Combination and Permutation Problems

Directions: Use your knowledge of combinations and permutations to solve each of the following problems.

1. At the local pizza place there are 8 choice for meat toppings and 10 choices for vegetable toppings. You want to choose 3 meats and 5 vegetables for your pizza. How many ways can you choose the 8 toppings?
2. Mr. Matthews has 24 boys and 26 girls in his P.E. class. He wants to choose 6 students for a volleyball team. The team will consist of 3 boys and 3 girls. In how many ways can he choose his team?
3. Mr. Hubbard has 12 Senior boys and 14 Junior boys going out for the football team. He wants to choose 8 Seniors and 3 Juniors for his team. In how many different ways can he do this?
4. I visit the local candy shop. There are 18 different types of candy bars and 10

different types of lollipops. I want to choose a bag of candy consisting of 5 bars of candy and 5 lollipops. In how many different ways can I do this?

Answer Key: Combination and Permutation Problems

1. Step 1: In how many ways can you choose the meats?

8 meats, choose 3

$$8 \times 7 \times 6 = 336 \text{ divided by } 1 \times 2 \times 3 = 6 \text{ equals } 56 \text{ ways}$$

- Step 2: In how many ways can you choose the vegetables?

10 vegetables, choose 5

$$10 \times 9 \times 8 \times 7 \times 6 = 30,240 \text{ divided by } 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ equals } 252$$

- Step 3: Multiply the number of ways to choose the meats by the number of ways

to choose the vegetables:  $56 \times 252 = 14,112$

2. Step 1: In how many ways can you choose the boys?

24 boys, choose 3

$$24 \times 23 \times 22 = 12,144 \text{ divided by } 1 \times 2 \times 3 = 6 \text{ equals } 2,024$$

- Step 2: In how many ways can you choose the girls?

26 girls, choose 3

$$26 \times 25 \times 24 = 15,600 \text{ divided by } 1 \times 2 \times 3 = 6 \text{ equals } 2,600$$

- Step 3: Multiply the number of ways to choose the boys by the number of ways

to choose the girls:  $2,024 \times 2,600 = 5,262,400$

3. Step 1: In how many ways can you choose the 12 seniors?

12 Seniors, choose 8

$$12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 19,958,400 \text{ divided by}$$

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40,320$$

Step 2: In how many ways can you choose the 11 juniors?

11 Juniors, choose 3

$$11 \times 10 \times 8 = 990 \text{ divided by } 1 \times 2 \times 3 = 6 \text{ equals } 165$$

Step 3: Multiply the number of ways to choose a senior by the number of ways to choose a junior:  $495 \times 165 = 81,675$  ways.

4. Step 1: In how many ways can you choose the candy bars?

18 candy bars, choose 5

$$18 \times 17 \times 16 \times 15 \times 14 = 1,028,160 \text{ divided by } 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ equals}$$

8,568

Step 2: In how many ways can you choose the lollipops?

10 lollipops, choose 5

$$10 \times 9 \times 8 \times 7 \times 6 = 30,240 \text{ divided by } 1 \times 2 \times 3 \times 4 \times 5 = 120 \text{ equals } 252$$

Step 3: Multiply the number of ways to choose a candy bar by the number of ways to choose a lollipop:  $8,568 \times 252 = 2,158,136$

## CHAPTER 6

### PASCAL'S TRIANGLE

Pascal's Triangle is an arithmetical triangle that is used for some neat and clever things in mathematics. The triangle was named after Blaise Pascal who lived from 1623 – 1662 [3]. He was a French mathematician who made many contributions to the area of mathematics. However, his greatest contribution was the work he did in developing the properties and applications of this triangle which was named after him [2].

#### How to Construct the Triangle

You start out with the top two rows: 1, and 1, 1, see Figure 12. Then to construct each entry in the next row, you look at the two entries above it (the entry to the left and right of it). At the beginning and the end of each row, when there's only one number above it, put a 1. If you were to add the numbers above it to the left and to the right; you are adding 1 and 0, so you get 1 [11]. You can continue doing this, adding as many lines as you would like. Now, using Worksheet #1, we will practice constructing Pascal's Triangle[10].

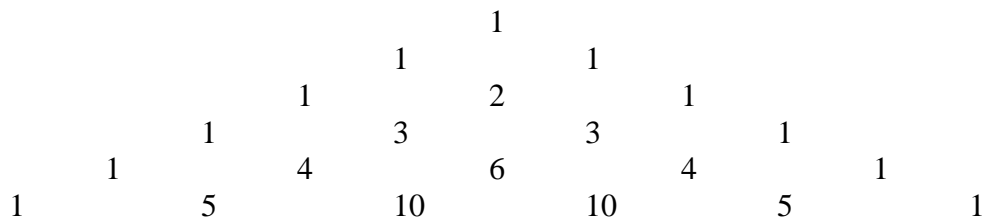
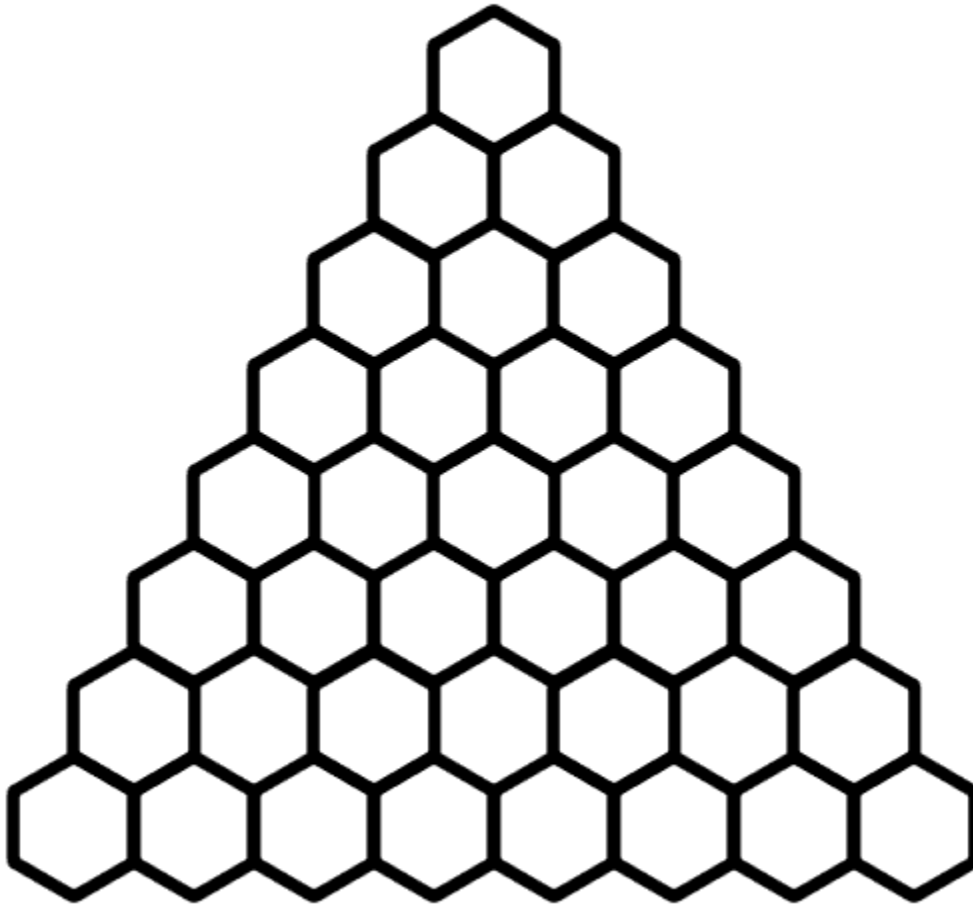


Figure 12: Pascal's Triangle



# Discovering Patterns

Name \_\_\_\_\_

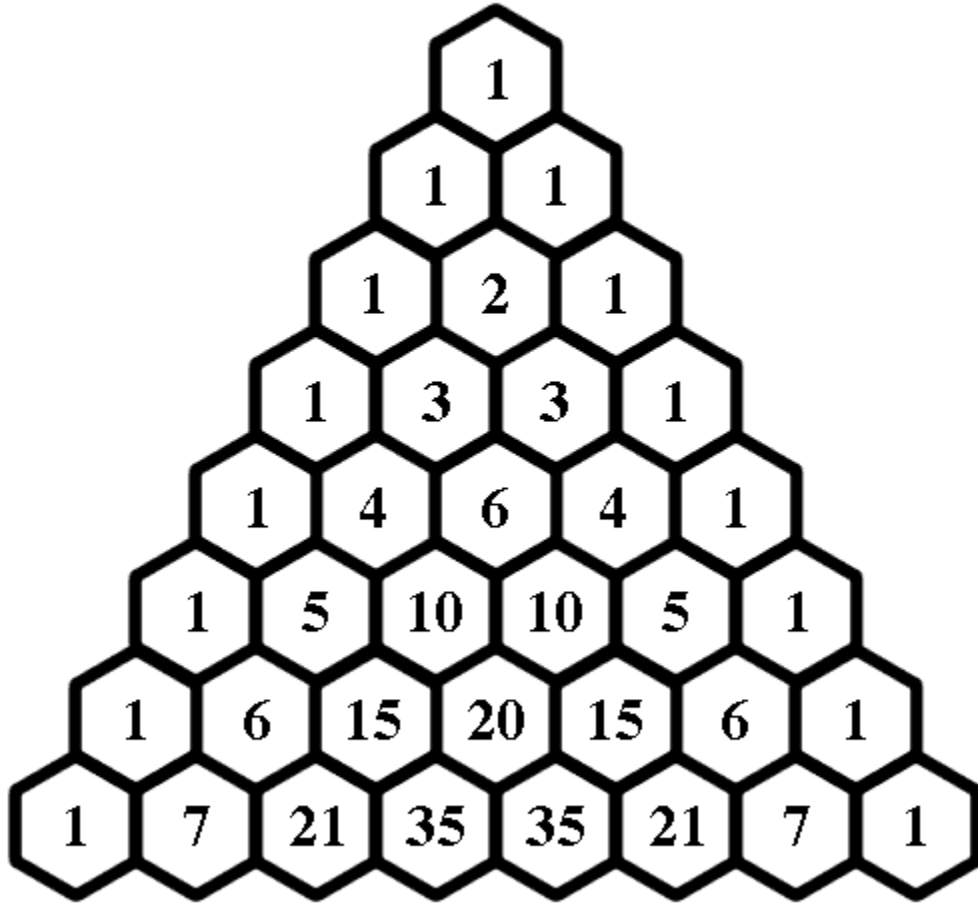


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Figure 13: Blank Triangle

# Discovering Patterns

Name \_\_\_\_\_



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Figure 14: Completed Triangle

### Patterns on Pascal's Triangle

Now that we have had practice constructing the triangle, we are going to look for patterns in the triangle. Study the numbers in the triangle.

Question: What patterns do you see in the arrangement of the numbers?

Possible Answers: Natural Numbers (1, 2, 3, 4)

Triangular Numbers (1, 3, 6, 10)

Row of Ones

Question: Can you predict the next row of numbers?

Possible Answer: Yes. Continue the pattern.

Question: Add the numbers in each row. Is there a pattern in the sums of these numbers?

Answer: Yes, the sum of the rows double.

Question: Do any numbers repeat?

Answer: Yes. The ones start and end each row.

Question: Can you find a pattern in the diagonal numbers?

Possible Answers: Yes. The first diagonal row is ones. The second diagonal row is counting by ones. The third diagonal row is add 2, add 3, add 4, add 5, etc.

Now that you are aware of some of the patterns in the triangle you are going to do an activity in which you will color the odd and even numbers. Then you will look for more patterns.

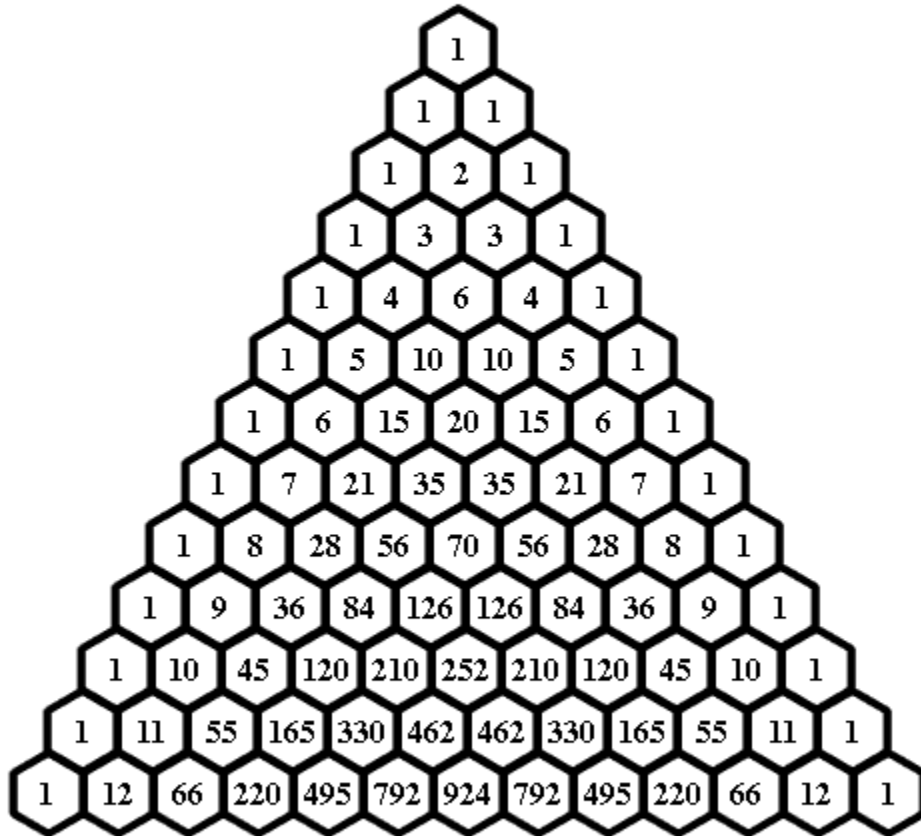
Directions for Activity: Distribute a copy of Worksheet #2 to each student. Color the odd numbers green and the even numbers red. Look for a pattern.

### The Mathematics in the Patterns

1. Students practice the concept of even and odd numbers by coloring odd-numbered spaces green. They should see that the triangle is outlined in green spaces because the number 1 is odd.
2. The Commutative Property of Addition. This property, together with the structure of Pascal's Triangle, explains the symmetry that can be observed in the colors of each row of numbers. It is not necessary for students to know or understand this property to be able to appreciate the symmetry in the coloring: if the triangle is folded through its center, spaces of the same color will fall on top of each other.
3. The sum of two odd numbers is an even number. When two green numbers are close to each other, the number below it will be red, see Figure 16.
4. The sum of two even numbers is an even number. When two red numbers are next to each other, the number below it will be red, see Figure 16.
5. The sum of an odd number and an even number is an odd number. When a green number is next to a red number, the number below it will be green [10].

# Discovering Patterns

Name \_\_\_\_\_



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Figure 15: Twelve Rows of Pascal's Triangle

Answer Key for Worksheet #2

(First Ten Rows)

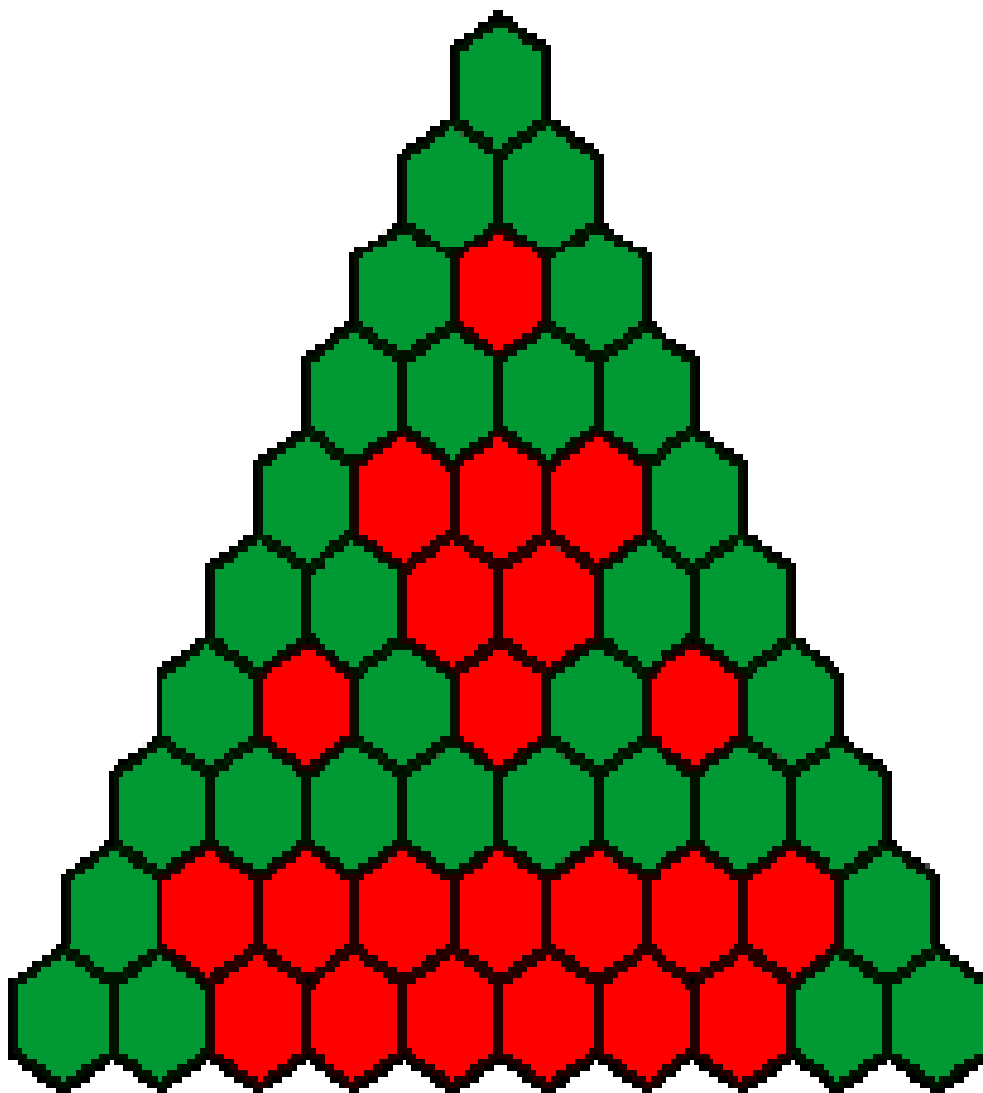


Figure 16: Red and Green Patterns

### Reading Pascal's Triangle

When reading Pascal's Triangle, people usually give a row number and a place in that row, beginning with row zero and place zero [10]. For instance, the number 20 appears in row 6, place 3.

Question: Where would you find the number 28?

Answer: Row 8, place 2.

Question: Where would you find the number 35?

Answer: Row 7, place 3 and 4.

Question: Where would you find the number 56?

Answer: Row 8, place 3 and 5.

Question: Where would you find the number 2?

Answer: Row 2, place 1.

Question: Where would you find the number 210?

Answer: Row 10, place 4 and 6.

### Homework Exercises: Pascal's Triangle

Directions: Use Worksheet #2, see Figure 15, to answer the following questions.

1. Where would you find the number 6?
2. Where would you find the number 252?
3. Where would you find the number 70?
4. Where would you find the number 495?
5. Where would you find the number 55?

### Answer Key: Pascal's Triangle

1. Row 4, place 2
2. Row 10, place 5
3. Row 8, place 4
4. Row 12, place 4 and 8
5. Row 11, place 1 and 10

### Pascal's Triangle and Combinatorics

In this lesson we are going to learn how to use Pascal's Triangle to find combinations [11].

Example 1: Let's say you have five books on a shelf, and you want to know how many ways you can pick two of them to read. It doesn't matter which book you read first. So, this problem amounts to the question "how many different ways can you pick two objects from a set of five objects?" You need to think 5 choose 2. The answer is the number in the second place in the fifth row, see Figure 17.

The figure shows Pascal's Triangle with rows numbered 0 to 5. The values in each row are: Row 0: 1; Row 1: 1, 1; Row 2: 1, 2, 1; Row 3: 1, 3, 3, 1; Row 4: 1, 4, 6, 4, 1; Row 5: 1, 5, 10, 10, 5, 1. The value 6 is highlighted in the second position of the fifth row.

				1				
			1	2	1			
		1	3	3	1			
	1	4	6	4	1			
1	5	10	10	5	1			
1	6	15	20	15	6	1		
1	7	21	35	35	21	7	1	

Figure 17: Five Choose Two

Example 2: Let's say that you have six coats in your closet, and you want to know how many ways you can pick three of them to wear. It doesn't matter which coat you wear first. So, this problem amounts to "how many different ways can you pick three objects from a set of six?" You need to think 6 choose 3. The answer is the number in the third



place in the sixth row, see Figure 18.

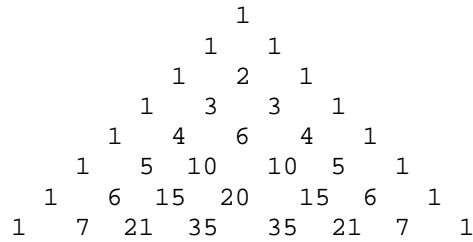


Figure 18: Six Choose Three

Example 3: You go to the local pizza parlor. Mariano tells you that he has 9 different pizza toppings. Today’s special is a large pizza with three toppings. In how many different ways can you choose the three toppings. In other words “how many ways can you pick three objects from a set of nine?” You need to think in terms of 9 choose 3.

The answer is the number in the third place of the ninth row, see Figure 19.

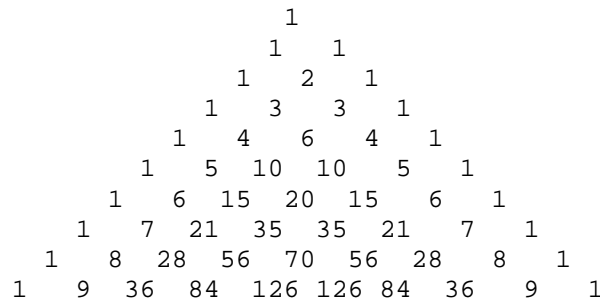


Figure 19: Nine Choose Three

Example 4: In the problem above, in how many different ways can you choose a pizza with four toppings? In other words “how many ways can you pick four objects from a set of nine?” You need to think in terms of 9 choose 4. The answer is the number in the fourth place of the ninth row, see Figure 20.

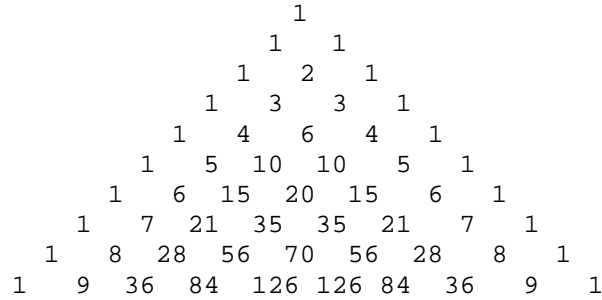


Figure 20: Nine Choose Four

Homework Exercises: Pascal and Combinatorics

Directions: Use Worksheet #2 to answer the following questions.

1. Mrs. McFaddin has 12 girls in her room. In how many different ways can she choose a group of 3 girls from her class?
2. Mrs. McFaddin has 10 boys in her class. In how many different ways can she choose a group of 3 boys from her class?
3. The concession stand has 7 different types of snacks. In how many different ways can you choose 2 snacks from the concession stand?
4. The local ice cream shop has 11 flavors of ice cream. In how many different ways can I choose 3 scoops of ice cream?
5. The local soda shop has 9 different flavors of soda. In how many different ways can I choose 4 flavors of soda?

Answer Key: Pascal and Combinatorics

1. 12 choose 3  
220 different ways
2. 10 choose 3  
120 different ways
3. 7 choose 2



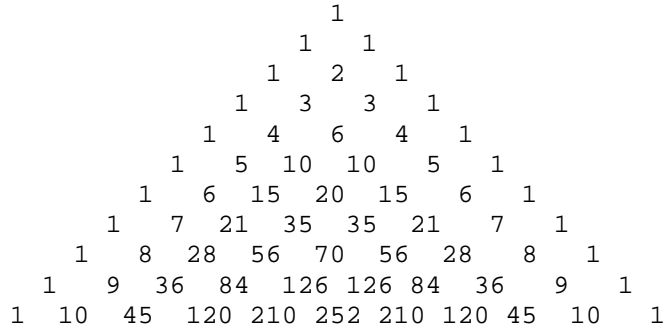


Figure 22: Ten Choose Two

Next, let's ask the question, "How many different three-topping pizzas can you order when choosing from ten toppings?" Think 10 choose 3. By using Pascal's Triangle, we find place 3 in row 10. Thus, the answer is 120, see Figure 23.

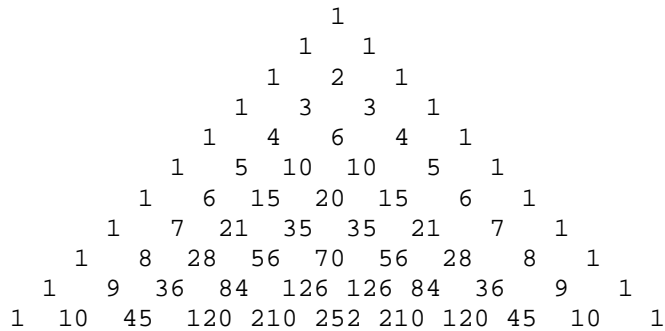


Figure 23: Ten Choose Three

Let's now ask the question, "How many different four-topping pizzas can you order when choosing from ten toppings?" Think 10 choose 4. By using Pascal's Triangle, we find place 4 in row 10. Thus the answer is 210, see Figure 24.

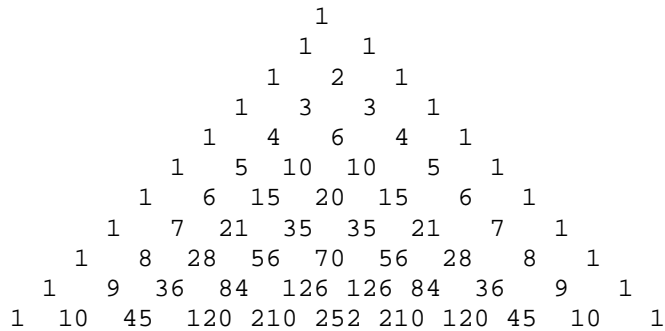


Figure 24: Ten Choose Four

You can now begin to see the pattern to what we are doing. We can now use Pascal's Triangle to answer the question of, "What's the total number of different pizza combinations that can be made given a choice of 10 toppings?" We look at row 10 and find the following:

- 1 pizza with no toppings
- 10 different pizzas with 1 topping
- 45 different pizzas with 2 toppings
- 120 different pizzas with 3 toppings
- 210 different pizzas with 4 toppings
- 252 different pizzas with 5 toppings
- 210 different pizzas with 6 toppings
- 120 different pizzas with 7 toppings
- 45 different pizzas with 8 toppings
- 10 different pizzas with 9 toppings
- 1 pizza with 10 toppings

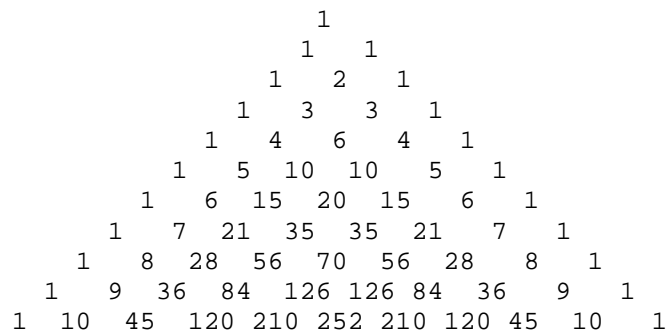


Figure 25: Sum of Combinations Row 10

To find the sum of the numbers in row 10, see Figure 25:

$$1 + 10 + 45 + 120 + 210 + 252 + 210 + 120 + 45 + 10 + 1 = 1,024$$

Example 1: What is the total number of jewelry combinations that can be made given a choice of 7 pieces of jewelry? To answer this question let's refer to row 7 of Pascal's Triangle, see Figure 26.

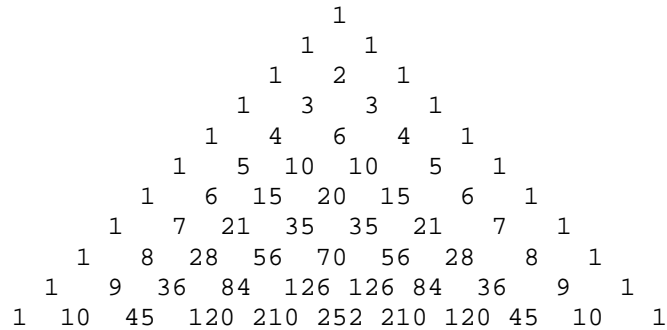


Figure 26: Sum of Combinations Row 7

Row 7 tells us the following:

- 1 jewelry combination with 0 pieces of jewelry
- 7 jewelry combinations with 1 piece of jewelry
- 21 jewelry combinations with 2 pieces of jewelry
- 35 jewelry combinations with 3 pieces of jewelry
- 35 jewelry combinations with 4 pieces of jewelry
- 21 jewelry combinations with 5 pieces of jewelry
- 7 jewelry combinations with 6 pieces of jewelry
- 1 jewelry combination with 7 pieces of jewelry

To find the sum of the combinations add the numbers in row 7:

$$1 + 7 + 21 + 35 + 21 + 7 + 1 = 128$$

Example 2: What is the total number of different sundae topping combinations that can be made given a choice of 8 toppings? To answer this question refer to row 8 of Pascal's Triangle, see Figure 27.

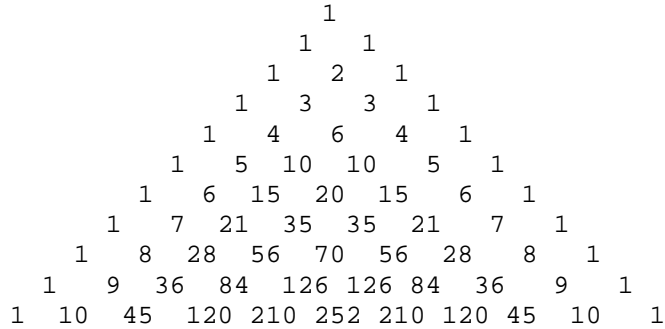


Figure 27: Sum of Combinations Row 8

Row 8 tells us the following:

- 1 sundaes with no toppings
- 8 sundaes with 1 topping
- 28 sundaes with 2 toppings
- 56 sundaes with 3 toppings
- 70 sundaes with 4 toppings
- 56 sundaes with 5 toppings
- 28 sundaes with 6 toppings
- 8 sundaes with 7 toppings
- 1 sundaes with 8 toppings

To find the sum of all the combinations add the numbers in row 8:

$$1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256$$

Homework Exercises: Sum of Combinations

Directions: Use Pascal's Triangle and the sum of combinations to solve the following problems.

1. What is the total number of different picture combinations that can be made given a choice of 4 pictures?
2. What is the total number of different pizza combinations that can be made given a

choice of 6 toppings?

3. What is the total number of different sundae combinations that can be made given a choice of 5 toppings?
4. What is the total number of different hamburger toppings that can be made given a choice of 9 toppings?
5. What is the total number of fingernail polish combinations that can be made given a choice of 12 colors?

Answer Key: Sum of Combinations

1. Refer to row 4 of Pascal's Triangle, see Figure 28.

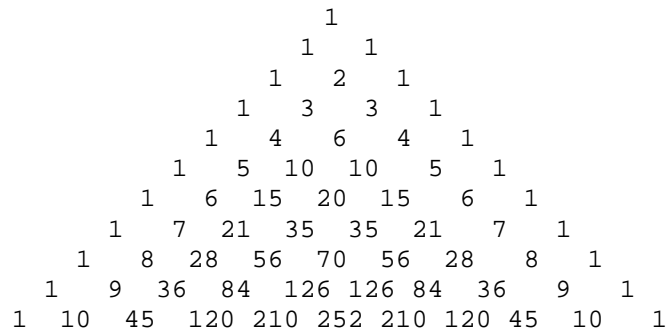


Figure 28: Sum of Combinations Row 4

$$1 + 4 + 6 + 4 + 1 = 17 \text{ total different combinations.}$$

2. Refer to row 6 of Pascal's Triangle, see Figure 29.

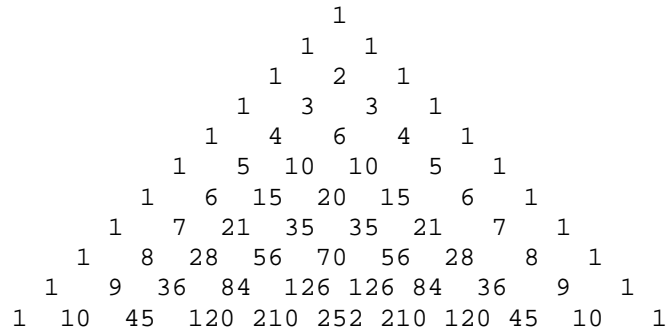


Figure 29: Sum of Combinations Row 6

$$1 + 6 + 15 + 20 + 15 + 6 + 1 = 64 \text{ total combinations.}$$



3. Refer to row 5 of Pascal's Triangle, see Figure 30.

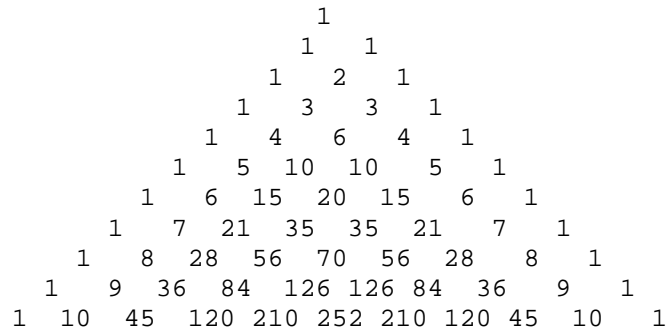


Figure 30: Sum of Combinations Row 5

$$1 + 5 + 10 + 10 + 5 + 1 = 32 \text{ total combinations.}$$

4. Refer to row 9 of Pascal's Triangle, see Figure 31.

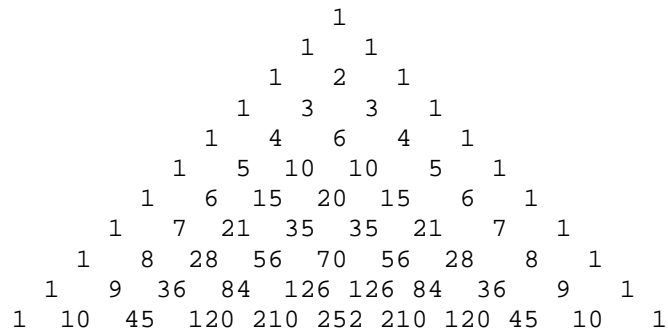


Figure 31: Sum of Combinations Row 9

$$1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1 = 402 \text{ total combinations.}$$

5. Refer to row 12 of Pascal's Triangle, see Figure32.

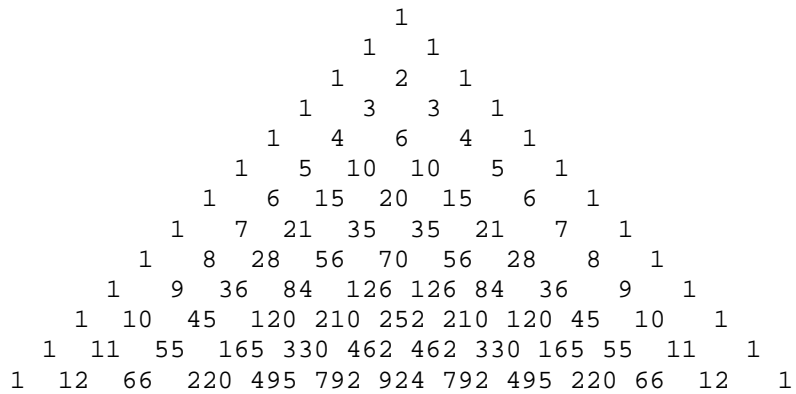


Figure 32: Sum of Combinations Row 12

$$1 + 12 + 66 + 220 + 495 + 792 + 924 + 792 + 495 + 220 + 66 + 12 + 1 = 4,096 \text{ combinations.}$$

## SUMMARY

Since the Combinatorics Units were designed to introduce third grade students to combinations and permutations, the author thought that it would be appropriate to teach the units to her third grade class. In the spring of 2006, after having administered the SOL test, the author decided to teach the Fundamental Principle, Permutations, and Combinations units.

The class really enjoyed the material. The author/teacher spent three days on each unit as this was all the time that she had. The students were very interested in the different ways to count. The author/teacher felt that the units were easy to follow which allowed everything to flow smoothly. The students wished that they had more time to spend on this type of math. The author hopes to teach the units to her summer school students. She also hopes to encourage other teachers in the area to try the units with their classes as an alternative to traditional math classes.

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