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The Role of the History of Mathematics in Middle School.

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The Role of The History of Mathematics
in Middle School

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by
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ABSTRACT

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It is the author's belief that middle school mathematics is greatly taught in isolation with little or no mention of its origin. Teaching mathematics from a historical perspective will lead to greater understanding, student inspiration, motivation, excitement, varying levels of learning, and appreciation for this subject. This thesis will develop four units that will incorporate original source documents and selected historical topics surrounding computation, numbers, and early calculating devices. Many of the units will center on the Rhind Papyrus and The Treviso Arithmetic. These units will be appropriate to middle school, with an emphasis on 6^{th} grade.

DEDICATION

This thesis is dedicated to my loving devoted family. To my husband, Richard thank you for giving tirelessly of yourself. Your devotion, strength, and love are a blessing to all who know you! I have always felt I could conquer the world with you by my side. To our children Levi, Noah, and Chelsey Rose your love, hugs, and kisses give me courage and inspire me to try harder each day! To my parents James D. and Mary Sue Baker, grandparents, brother J. Brad Baker, sister-in-law Vanessa Baker, and nephews Will and Ross. Thank you for all the prayers, help, and encouragement during my entire life. It is not easy to live with an energetic mathematician!

Most importantly I give thanks to the author of my faith, and creator of my soul, my Heavenly Father, who guides my steps, and leads me to the light!

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I give great appreciation to the over 1000 students who have endured my teaching both good and bad.

I also would like to thank my supportive division supervisors, Amy Merrihue and Belinda Mullins, for their continued support of my crazy classroom ideas. It has been a blessing and a privilege to work with you both during the last 20 years.

I also extend a big thank you to Karen Dee Michalowicz for sharing her knowledge and materials on historical mathematics. Thank you for teaching me how to make and use a Soroban Abacus.

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CHAPTER 1

INTRODUCTION

Role of Historical Mathematics in Middle Level Education

For thousands of years civilizations have worked to discover the hidden meaning of numbers. Early concepts of numbers can be found in artifacts 30,000 years old. However, it has been in the past 6,000 years that humans have been calculating and unraveling the mysteries of mathematics.

Egyptians in 3000 B.C. were using fractions and ratios. In 2000 B.C. the Babylonians used place value for the digits in numbers. In Greece "arithmetike" was developed to aid in counting. They used this arithmetic for the four operations of addition, subtraction, multiplication, and division. The Mayan system of numbers developed independently in the Americas before 500 B.C. (Boyer, 1991)

The need for "Civilized Society" to find answers to everyday problems as well as fascinate themselves with the infinite has lead to the use and exploration of mathematics. It is a common study that relates all cultures, all languages, and all civilizations. Studying mathematics through its rich history is a great way to bring students into the vast realm of mathematical potential. It is the author's belief that mathematics is generally taught in isolated subject areas with little or no mention of its origin. It is wrong to expect students to get excited about something that makes them feel helplessly at the mercy of their teacher. Teaching students to memorize a rule and repeat that rule hundreds of times to prove their concept competency is not mathematics.

The author fully understands that a loaded curriculum doesn't enable teachers to add "Historical Mathematics" as a new unit of study. Nor is it believed that this would add very much flavor to a middle school student's day! However, it is wholeheartedly believed that we send the wrong message when we simply run another worksheet. Instead of adding a new strand of learning, the plan is to take the parts of existing strands already taught in middle school and infuse historical mathematics where appropriate.

It is believed that students have learned to hate and even fear mathematics because of the delivery that it receives in the classroom. Mathematics should never be seen as an isolated discipline. Teaching students through discovery and historical perspectives will empower and excite learning. It will encourage students to see beyond their own little desk, beyond their teacher, beyond their classroom into an area of adventure and mission to learn, conquer and understand the who and why. Teaching mathematics through a historical vantage will breathe a breath of life into an "ancient study".

The opportunity of teaching from a historical perspective will give great steps toward more advanced problem solving and logical reasoning. These are two important and essential life skills that are gained from mathematics. However, these skills have been lost in our zest to score on the Standards of Learning Tests. Historical mathematics also leads to new ways of looking at old problems. This is of benefit to struggling learners who need new and different paths to investigate problems.

Incorporating history into mathematics can be done in two ways. First, the topic can be covered through the use of original source documents or through selected topics that fit into the curriculum. Teaching through original source documents can be a challenge and requires a greater understanding of the culture and happenings of the people who wrote the mathematics. Source teaching is very wonderful for development of theme units and easily lends itself to other disciplines such as language arts, science, and social studies. This is the case with the source documents used in this thesis: *The Rhind Papyrus* and *The Treviso Arithmetic*. The module on early calculating devices is an example of teaching selected historical topics that fit into current curriculum. Selected sources require less understanding of the civilization and are a great place to begin to use history in mathematics. Upon completion it is hoped that other teachers will be interested in using some or all of these materials in their classes, as well as searching out other materials that can be used in their classrooms. The goal of this document is to take existing concepts taught in sixth grade: number sense, computation, and calculating devices, and approach each with materials found in the *Rhind Papyrus*, *The Treviso Arithmetic*, Abacus, and John Napier's Rods.

It has been to the author's benefit to have taught middle school mathematics, science, and social studies during the past 20 years. During this time I had the opportunity to coordinate units that integrated mathematics with studies of the development of the Eastern Hemisphere. It does take great preparation to bring history into the math, but the benefits supersede the work load. While the space of this thesis will limit us to a meager amount of historical insight, perhaps it will encourage the reader to look for other historical units to add to those mentioned above.

People see and even teach mathematics in isolation. Yet mathematics is a part of society's need for solving its problems. A historical approach may help teachers of other curricula to see math not in isolation but a heartbeat toward advancement of every society.

To be a mathematician is to be a part of something larger. It doesn't mean to be a scholar who understands all higher mathematics. Mathematicians desire to make sense of the world through exploration and experimentation. . They are fascinated by seeking out old and new theories, believing that learning gives explanation and purpose. Children are the greatest and most eager seekers of knowledge. Mathematics should inspire our students to be lifelong learners! Mathematics when taught in an exciting way inspires richness of learning. It is a heartbeat to a dead curriculum.

Perhaps it would be a great time to share Smith's philosophy of mathematics education as given in an excerpt from his 1921 Religio Mathematici to the Mathematical Association of America:

> *"Mathematics increases the faith of a man who has faith; it shows him his finite nature with respect to the Infinite; it puts him in touch with immortality in the form of mathematical laws that are eternal; and it shows him the futility of setting up his childish arrogance of disbelief in that which he cannot see…We should teach the science venerable not merely for its technique; not solely for this little group of laws or that; not only for a body of unrelated propositions or for some examination set by the schools; but we should teach it primarily for the beauty of the discipline, for the "music of the spheres," and for the faith it gives in truth, in eternal law, in the Infinite, and in the reality of the imaginary; and for the feeling of humility that results from our comparison of the laws within our reach and those which obtain in the transfinite domain. With such a spirit to guide us, what teachers we would be!"* (1921).

NCTM

It is the recommendation of the National Council Teachers of Mathematics that history of mathematics be used as a valuable tool for both the teachers and students. Math can be learned and must be taught as an integral part of the development of great societies. It is inspiring and useful. In any society development is related to the evolution of mathematical advances. Math has advanced historically to help us make meaning in ways that language cannot. Mathematics doesn't operate in isolation but as a part of discovery and it gives meaning to the world.

Mathematics can and should be taught as great discovery. It is a topic that is not complete. It is still being discovered today. Students are inspired if they feel that they are an active part of discovery. Learning from the masters, students begin to realize that they are a part of something inspiring and greater than just rote learning.

Teaching Historical Mathematics will take considerable dedication and preparation by the teacher. The benefits to the student will be worth the sacrifice.

According to NCTM the history of mathematics alone will not function as a teaching tool unless the users (1) see significant purposes to be achieved in its introduction, (2) plan thoughtfully for its use to achieve these purposes. The aim is not to present the history of math itself but to use it for concrete purposes in our classrooms. NCTM also states that the age and background of the student and the ingenuity of the teacher must determine the way to use historical mathematics. They suggest brief historical anecdotes to illustrate as well as class discussions or working on a problem from a great mathematician. Other historical stories and topics are helpful for reports and independent study.

History shows that modern math is a combination of years of learning and discovery. Insight into the development of ideas can serve to improve both the curriculum makers' choices and the teacher's power to communicate insights and stimulate interests. (NCTM, 1998)

"Students should have numerous and varied experiences related to the cultural, historical, and scientific evolution of mathematics so that they can appreciate the role of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities. It is the intent of this goal-learning to value mathematics -- to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it is developed and the impact that interaction has on our culture and our lives." (Swetz, 1994).

Frank Swetz

Swetz is a noted author and expert on the history of mathematics. He is also the person who took the notes of David E. Smith and translated the entire Treviso Arithmetic. As a matter of fact, I see his name popping up on everything from the Treviso to NCTM documents. I believe that his opinion should be noted in this document.

"The history of mathematics supplies human roots to the subject. It associates mathematics with people and their needs. It humanizes the subject and, in doing so, removes some of its mystique. Mathematics isn't something magic and forbiddingly alien: rather, it's a body of knowledge developed by people over a 10,000 year period. These people, just like us and our students, made mistakes and were often puzzled, but they persisted and worked out solutions for their problems. Mathematics is and always was people-centered. Its teaching should recognize and build on this fact by incorporating the history of mathematics as a fundamental part of its learning." (Swetz, 1994)

Swetz states that teachers don't have time to teach another topic. He recommends that the best way to approach this topic is by carefully planning lessons that will complement mathematics learning. It is not something extra but something that enhances the curriculum that already exists. Swetz suggests, "That each teacher is different and needs to develop strategies that work for them." (Swetz, 1994) He does list five strategies from his book *Learning Activities from the History of Mathematics.*

1. A consideration of the people of mathematics -- the lives and work of selected mathematical personages.

2. Obtaining information about the origins and meaning of mathematical terms, symbols, and words.

3. Assigning classical or historical problems and noting their origins or significance.

4. Carrying out activities based on historical problems or discoveries.

5. Using historically based films or videotapes in classroom instruction. (Swetz, 1994)

Swetz suggests allowing students to participate actively in historical discovery. He suggests experiments in the measurement of pi, using Eratosthenes technique to obtain the circumference of the earth, Greek ruler and compass construction, and algebraic equations. The benefit of any historical math is that there is a great opportunity to reinforce skills and provide opportunity for enrichment.

Wilbert & Luetta Reimer
Historical Connections AIMS Education Foundation

The value of using history in teaching mathematics continues to gain recognition in the United States and throughout the world. Providing a personal and cultural context for mathematics helps students sense the larger meaning and scope of their studies. When they learn how persons have discovered and developed mathematics, they begin to understand that posing a problem is often as valuable as the solution. (Reimer, 1995)

According to Reimer teachers should share biographical information and anecdotes as an introduction to new concepts in their mathematics classrooms. Historical mathematics is a huge motivational tool that inspires learning. They suggest that topics should support the concepts being taught or be presented for enrichment purposes.

Reimer and Reimer (1995) suggest integrating mathematics history in the following ways:

1. Read aloud mathematical stories to the class.
2. Students write about mathematical history topics.
3. Students perform plays, skits, or make videos about historical topics.
4. Hands-on experiences.
5. Through the arts.
6. Through the visual arts.

Sanderson Smith

Sanderson Smith is the author of *Agnesi to Zeno; 100 Vignettes from the History of Math*. Sanderson Smith reveals that mathematics taught in isolation without any mention of its origin and great history has led in many ways to dislike and fear of the subject. Mathematics is a people-centered subject and should be taught that way. Smith calls on dedicated mathematics teachers to bring history back into their classrooms. Smith emphasizes that students will be inspired learners of mathematics when topics are presented in historical perspective and classroom discussions are encouraged. Also, they should be exposed to discussions on how mathematics impacts their lives and how it has been used in many cultures over its evolution. (Smith, 1996)

The ICMI Study
John Fauvel and Jan van Maanen
Karen Michalowicz

The ICMI Study is unique in its construction. A group of mathematics educators gathered together in Luminy, France, April 20-25, 1998, to:

(1) Survey and assess the present state of the whole field.

(2) Provide a resource for teachers and researchers, and for those involved with curriculum development.

(3) Indicate lines of future research activity.

(4) Give guidance and information to policy-makers about issues relating to the use of history in pedagogy.

This function was carried out through the 11 chapters of the book. Their argument was based on the fact that mathematics has been around for thousands of years, and that using this vast history for the study of mathematics makes a difference. Chapter 6 is to be highlighted in this thesis, but the author points to the merit of each chapter and encourages the reader to read the book.

Chapter 6 was chosen for two strong reasons, first the author agrees with its statements, and secondly (perhaps more importantly) Karen Michalowicz has inspired the author since 1994 to use history of mathematics in classroom exercises. She is a brilliant mathematician and educator. Her opinions are important and valuable. In addition to three weeks of summer in 1994 and 1995, Karen has worked with the mathematics educators in Washington County Virginia during in-service training and on a consultant basis.

Chapter 6 Abstract (171-199): "The needs of students of diverse educational backgrounds for mathematical learning are increasingly being appreciated. Using historical resources teachers are better able to support the learning of students in such diverse situations as those returning to education, in under-resourced schools and communities, those with educational challenges, and mathematically gifted students." (Michalowicz, 2000)

While in recent years many universities and colleges have begun to include history of mathematics for prospective secondary-school teachers, it is still of minimal interest in the mathematics community. Those who teach mathematics at the university level are not the teachers of the diverse students of our school systems. When a mathematics teacher does use history of mathematics it is usually because of an amateur interest in historical mathematics and not because they were trained in this area.

The history of mathematics in the classroom should enlighten students to the vast study of mathematics. Primary mathematics teachers are encouraged to relate their units with social studies, geography, and history. While there is no concrete research to verify the importance of history in mathematics, there are countless reports given by primary teachers who have found success in the practice!

How to incorporate this topic into the already loaded curriculum? Teachers need proper training to be able to understand the history of mathematics and how it connects to arithmetic in the classroom. Teachers must also have access to materials to create their own classroom materials.

Teacher educators need to be committed to providing opportunities for teachers to learn about mathematical history and how to incorporate it into the classroom. Attitudes about mathematics are formed early in children's minds, and it is, therefore, important that even early education have some forms of historical mathematics.

There is much justification for the use of this area and the role it plays in allowing for diversity of learning style and abilities. It also broadens socio-cultural perspectives of the learner. It increases self confidence. It is a course that can be integrated into different levels of learners as well as learners of vast age differences. It fits just as easily in the primary as the secondary level if presented in an effective way.

History of Mathematics provides such opportunities to teach materials in different ways and varied levels. Everyone in the classroom is working on the same problem but at varied levels. Advanced students also benefit because they will find greater understanding in their future college math courses with increased understanding of how the mathematics was discovered and used. Because opinions form early, it is imperative that history of mathematics begins early and is a continued thread throughout a student's education.

Chi-Kailit, Man-Keung Siu, Ngai-Ying Wong

The Use of History in the Teaching of Mathematics: Theory, Practice, and Evaluation of Effectiveness: Education Journal Summer 2001. Chi-Kailit, Man-Keung Siu, Ngai-Ying Wong

This article was provided to the author by Man-Keung Siu, Department of Mathematics, The University of Hong Kong. It addresses the thoughts of several leading mathematicians on the topic of blending history with mathematics. While the teaching of mathematics is widely recognized as a useful tool, this article looked toward evidence of the effectiveness of historical mathematics to improve learning of mathematics. (Man-Keung, 2001)

The article was details an experiment of three weeks. The classroom in the experiment was compared to another classroom not in the experiment. A teaching capsule was designed for this pilot research. Pre-assessment and post-assessment of students' attitude towards mathematics, enjoyment of learning in mathematics, learning motivation, and mathematics self-concept were conducted at the start and at the end of the course.

In the experimental group the questionnaires were supplemented by a list of open-ended questions on how well they received the new way of teaching and how well they received the teaching material. Interviews with the teacher and six students from the experimental group on their perception of these three weeks of teaching/learning experience were also conducted.

Before the experiment, researchers interviewed students and evaluated the current teaching situation. It was found that classes were large (around 40 students) and the curriculum was mostly test driven. The classes were lecture, drill, and memorization, not much different from an American classroom. By interviewing 11 secondary school students, researchers learned that students found the lecture, practice, exam curriculum "boring". The tight schedule leaves no time to think. Students stated they preferred teachers who used activities, and experiments, that were creative, lively, patient, and caring.

The experimental module was the Pythagorean Theorem for secondary students. The existing curriculum material was analyzed and an attempt at changing the components in order to use history of mathematics was made. The module was revised according to the feedback obtained in the pretesting. Worksheets, problems taken from ancient classics, activities, manipulatives, and proofs of the Pythagorean Theorem were incorporated into the module.

The experiment lasted 3 weeks. The same teacher taught the experimental and the control group. He had a mathematics degree and 2 years of teaching experience. The groups received lessons of 40-minutes duration. The control group also learned the Pythagorean Theorem with traditional lecture, notes, and exam. Students' attitudes towards math, enjoyment of learning, motivation, and mathematics self-concept were assessed at the beginning and end of the experiment. Test scores were also taken, so the experiment had qualitative as well as quantitative data.

The results indicated that enjoyment, motivation, and self-concept rose in the experimental group. Self-concept was the greatest increase in the experimental group. Changes in the control group showed a decrease in enjoyment, self-concept, and motivation. Here again the largest change in the control group was the drop in self-concept. History increased enjoyment and self-concept. Students who reported in the interviews that they didn't like the use of history pointed that they liked the traditional form of teaching because they didn't like to read the extra materials. They preferred to memorize a rule and stick with it. Other students reported that the materials were more interesting and lively. They were less "bored". The teacher also stated that he liked the new teaching. It did, however, require more time, effort, and preparation.

The teacher in the experimental group showed high regard for the use of history. He found it to be feasible and manageable. There is no doubt that it did require more preparation on the part of the educator. Teachers should realize that in the beginning they can't expect to add more than a few units a year to their classrooms.

Test scores, however, did not show the increase that the researchers had hoped to prove. Sadly, educators realize that test scores drive curriculum. The qualities of the questions asked on a multiple choice test are questionable! Until we ask different questions about mathematics, the history of mathematics in relation to our classroom instruction will continue to lie only on the shoulders of teachers who have an interest in the topic. This answers the question as to why mathematics is seen only in isolation! It also explains why the topic is covered in limited amounts in college preparation classes, and then only at the discretion of college professors.

This experiment shows a clear gap between what is taught and what is assessed. When conventional testing relies heavily on paper and pencil exercises, drilling and practices remain the most "effective" way to push up test scores. Sadly!! Academic performance is definitely not the sole measure of the effectiveness of the use of history in the teaching of mathematics. Standard achievement is overemphasized at the expense of equally important topics like the history of mathematics in middle school. (Man-Keung, 2001)

It is the author's opinion that the responsibility for providing well rounded education depends solely on the well-informed, motivated teacher who may be the only person in a middle-school child's life who is capable of making the assessment of how to design a curriculum that will make our children well rounded, life-long, curious learners. It is also a high risk job! If scores fall teachers know who will be blamed. To add materials to such a state mandated curriculum is not for the weak of heart and requires preparation, extra work, and without question support from administration!

<u>Mediterranean Journal for Research in Mathematics Education,</u>
<u>"The Role of the History of Mathematics in Mathematics Education"</u>
<u>Volume 3 No. 1-2, 2004 Editor in Chief: Athanasios Gagatsis.</u>

During the 10th International Congress on Mathematical Education (ICME 10) held in Copenhagen on July 4-11, 2004, several activities were devoted to the relationship between the history of mathematics and the learning and teaching of mathematics. The study group topic was titled *<u>The Role of the History of Mathematics in Mathematics Education</u>*. The sessions were attended by about 70 people and represented more that 20 countries. The effort was to make clearer the proper use of this topic in the classroom. There were 13 presentations and follow-up discussions. This section will highlight the conclusions of these materials; however, full texts and materials can be found on the web or by contacting the editor-in-chief.

Introducing a historical dimension to mathematics education involves three different areas: mathematics, history, and didactics. It is reported that history is a vehicle to reflect on the nature of mathematics as a socio-cultural process, and history is a possible way to conceive and understand mathematical topics. It is a strong way to enrich teacher education at all levels, both by introducing courses in the history of mathematics and its relation to other disciplines and letting teachers become acquainted with historically inspired material that can be or has been used in the classroom. In this way teachers can begin to introduce history as a dimension in teaching mathematics at all levels.

Materials should aim to motivate and guide the teacher, improve the teaching approach, or understand better students' difficulties and their ways of learning mathematics.

The discussions of this group make it clear that the actual implementation of the history of mathematics can enhance the learning and teaching of mathematics. The most important point of the meeting may be that the history of mathematics is best used in integrating the subject into classrooms rather than trying to make it something that is a course by itself. This is not to say that it alone is not a valuable subject in isolation, but it may have stronger impacts when integrated into a topic already taught. As educators we already know the benefits of integration in all areas of education to build relationships between subject matter and provide continuity for our students.

As a final remark the group points out that despite its importance, history of mathematics is not to be regarded as a panacea to all issues in mathematics educations, just as mathematics is not the only subject worth learning. It is the harmony of mathematics with culture and civilization that makes the subject worth studying. It provides a more important role in the education of the whole student.

VDOE

This section will take the computation strand from the sixth grade Virginia Standards of Learning and engage the student in alternative activities from various historical topics. The suggested materials will provide opportunities to advance the strand and develop concepts in problem solving, logical reasoning, and also computation.

Computation/Number Sense (VDOE, 2005)

Define: According to the State of Virginia computation is defined by three standards:

6.6a: solve problems that involve addition, subtraction, multiplication, and/or division with fractions and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less, and express their answers in simplest form. (VDOE, 2005)

6.6b: find the quotient, given a dividend expressed as a decimal through thousandths and divisor expressed as a decimal to thousandths with exactly one non-zero digit. (VDOE, 2005)

6.7: The student will use estimation strategies to solve multi-step practical problems involving whole numbers, decimals, and fractions (rational numbers). (VDOE, 2005)

6.8: The student will solve multi-step consumer-application problems involving fractions and decimals and present data and conclusions in paragraphs, tables, or graphs. (VDOE, 2005)

Students in the middle grades use problem solving, mathematical communication, mathematical reasoning, connections, and representations to integrate understanding within this strand and across all the strands. (VDOE, 2005)

Students develop conceptual and algorithmic understanding of operations with integers and rational numbers through concrete activities and discussions that bring meaning to why procedures work and make sense. (VDOE, 2005)

Students develop and refine estimations strategies and develop an understanding of when to use algorithms and when to use calculators. Students learn when exact answers are appropriate and when, as in many life experiences, estimates are equally appropriate. (VDOE, 2005)

Students learn to make sense of the mathematical tools they use by making valid judgments of the reasonable of answers. (VDOE, 2005)

Students reinforce skills with operations with whole numbers, fractions and decimals through problem-solving and application activities. (VDOE, 2005)

CHAPTER 2

RHIND PAPYRUS

(Original Source Document)

Background information: Much of what we know about Egyptian mathematics is based on two important papyri. *The Rhind Papyrus* is named for Mr. A.H. Rhind, who donated it to the British Museum in London, England in 1863. A few fragments of the papyrus can be found in the Brooklyn Museum in New York City. *The Moscow Papyrus*, is in the Museum of Fine Arts in Moscow. Both of these texts as well as other less important mathematical papyri were written between 2060 B.C. and 1580 B.C.

The Rhind Papyrus was purchased by Alexander Henry Rhind, a Scottish antiquarian, in Thebes, Egypt in 1858. It may have originally been taken illegally from an excavation site near Ramesseum. The papyrus was discovered in a tomb in Thebes. It is sometimes called the Ahmes Papyrus. Ahmes was not the author. He was a scribe who copied the material from an earlier papyrus.

The original *Rhind Papyrus* was probably written about 2000 B.C. It is our chief source of information on ancient Egyptian mathematics. It is 33 centimeters tall and 5 meters long. The papyrus contains 87 problems and solutions on the four operations, equations, progressions, volumes of granaries, linear equations, and other mathematical problems.

The Moscow Papyrus is about 15 feet long and 3 inches wide. It was purchased by V.S. Golenishchev in 1947 and later sold to the Moscow Museum of Fine Arts. It contains 25 practical problems and was written around 1700 B.C. (Wikipedia, 2005)

Both papyri were written in a cursive script called hieratic. In modern works it is common to transliterate this script into hieroglyphics that were used to record information on monuments and tombs. The Egyptian Hieroglyphic writing enabled them to write whole numbers up to 1,000,000. In this notation there was a special sign for every power of 10.

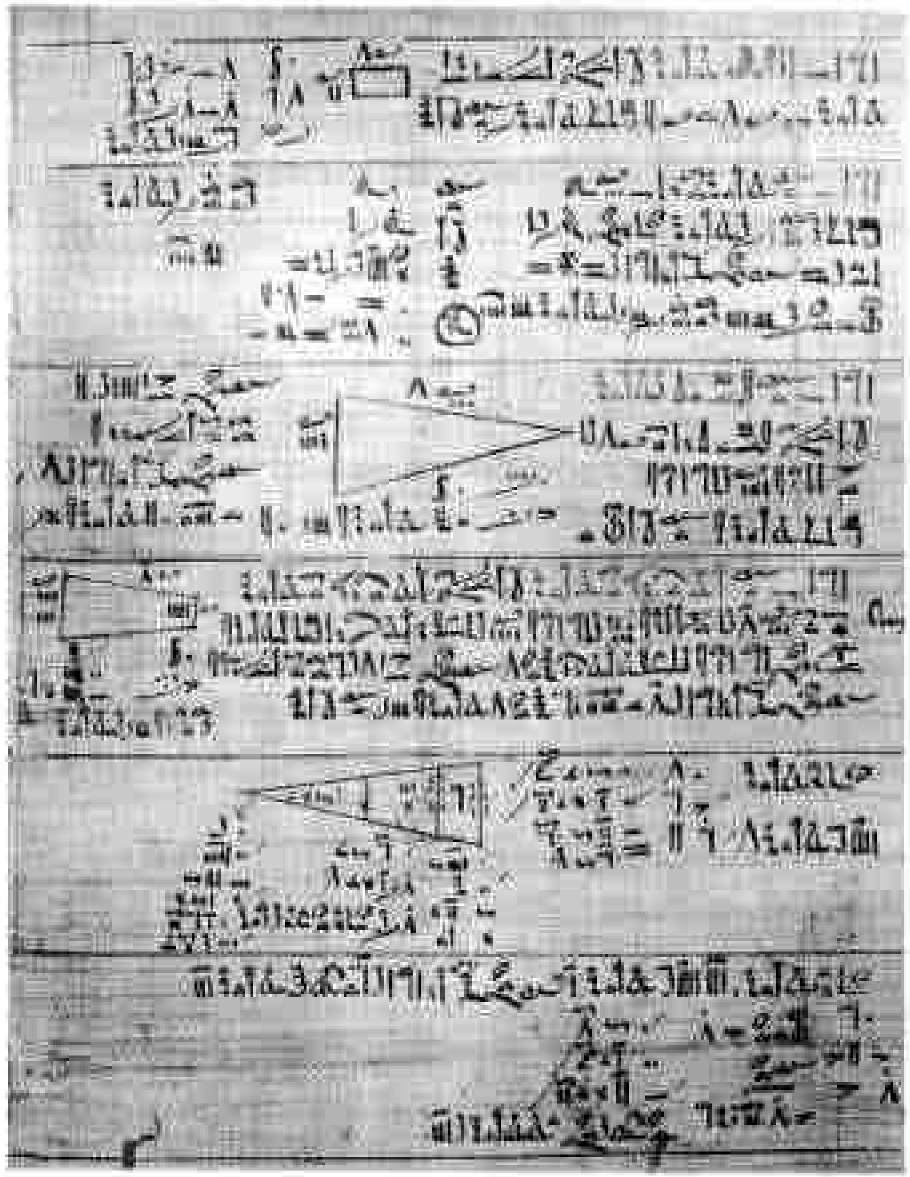
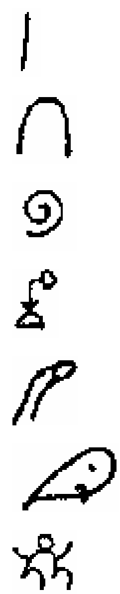


Figure 1. Rhind Papyrus

(Saint Andrews University, Scotland, 1997).

Table 1

Hieroglyphic Numbers

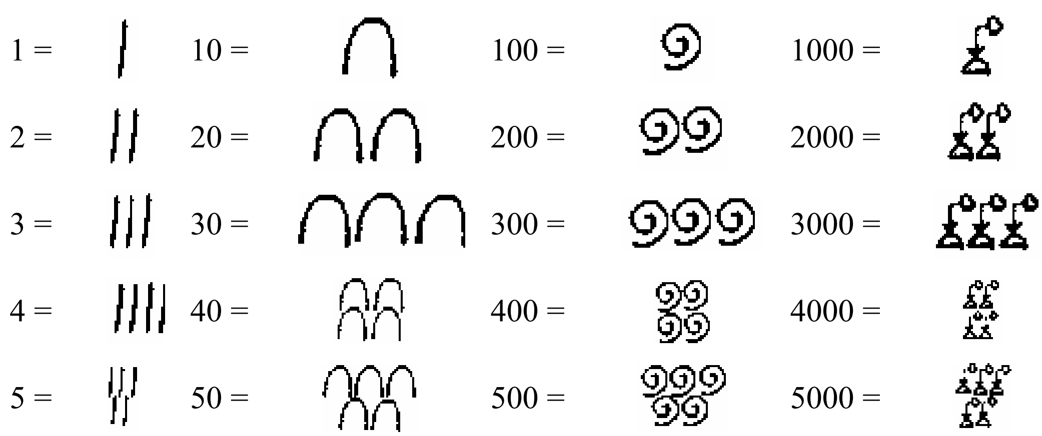


1 =	staff
10 =	heel bone
100 =	coil of rope
1000 =	lotus flower
10,000 =	pointing finger
100,000 =	tadpole
1,000,000 =	kneeling genie

(Holt, 2000)

This hieroglyphic numeration was a written version of a concrete counting system using material objects. To represent a number, the sign for each decimal order was repeated as many times as necessary. To make it easier to read the repeated signs were placed in groups of two, three, or four and arranged vertically.(Holt, 2000)

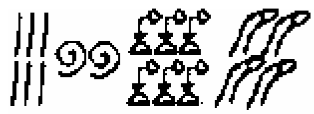
Example 1.  (Holt, 2000)



(Holt, 2005)

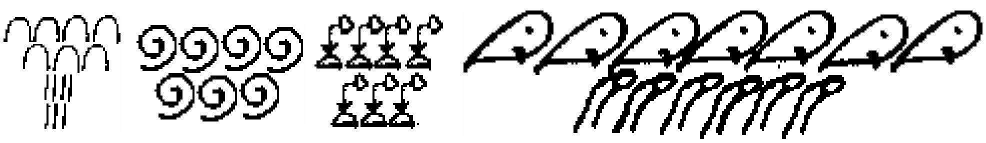
In writing the numbers, the largest decimal order would be written first. The numbers were usually written from right to left. However, there are many texts written with the largest numbers to the left. The point for students is to use the Egyptian numbers and develop greater understanding of the Egyptian society.

Example 2

46,206 = 

Below are some examples from tomb inscriptions.

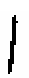
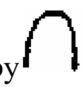
A B C D



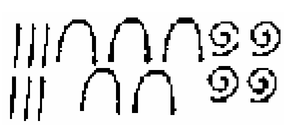
77 700 7000 760,000

(Holt, 2000)

Addition and Subtraction

The techniques used by the Egyptians are essentially the same as those used by modern mathematicians. The Egyptians added by combining symbols. They would combine all the units () together, then all of the tens () together, then all of the hundreds () etc. If the scribe had more than 10 units (), they would replace those 10 units by . They would continue to do this until the number of units left was less than 10. This process was continued for the tens, replacing ten tens with , etc. (Holt, 2000)

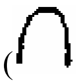
For example, if the scribe wanted to add 456 and 265, the problem would look like this

 (= 456)

 (= 265)

(Holt, 2000)

The scribe would then combine all like symbols to get something like the following

They would then replace the 11 units () with a unit () and a 10 (). They would then have 1 unit and 12 tens. The 12 tens would be replaced by 2 tens and 1 one-hundred. When they were finished they would have 721, which they would write as

.

Subtraction was done much the same way as we do it today except that when one has to borrow, it is done with writing 10 symbols instead of a single one. (Holt, 2005)

Hieroglyphic Class Exercises

Write the following numbers as the ancient Egyptians would have, be careful to follow their rules for writing numbers. Remember that we write larger numbers to the left in our place value system. The Egyptians did the opposite.

1. 525

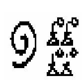
2. 6,859

3. 45,471

4. 1,345,678

Write the following hieroglyphics in base ten.

5. 

6. 

Solve these problems.

7. A man rents  donkeys to move his goods. The Egyptian Logistics Company charges  copper deben per donkey per day. The man leases the donkeys for

 days. How much does he own the "ELC"?

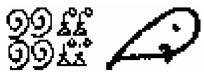
8. If each donkey eats  bundles of straw a day and the cost per bundle is  copper deben how much will they eat in  days

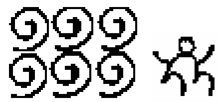
Enrichment:

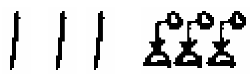
Refer to http://www.eyelid.co.uk/maths1.htm and solve the Mathematics problem for 10-11 year olds.

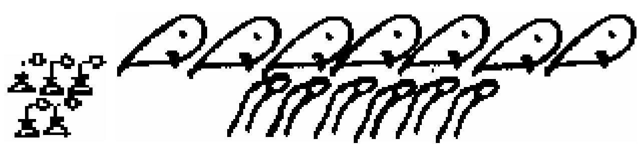
<u>Hieroglyphic Independent Exercises</u>

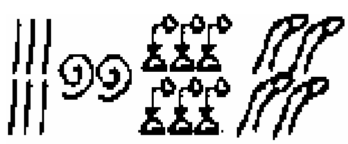
Write the following hieroglyphics in base ten.

1. 

2. 

3. 

4. 

5. 

Write the following numbers in hieroglyphics.

6. 4,090

7. 99,009

8. 20, 701

Answer the following problems.

9. Isis sells her hand painted scarab beetles for ∩ copper deben each. If she sells the beetles for a week, and each day the total number of beetles sold doubles. How many beetles did she sell, and how many deben did she earn, if the first day she sold || beetles? (Hint: make a chart and look for a pattern.)

10. Zaporia trades hiro cards with his friend Xylphonia. They trade every day. How many minutes did they trade cards this week if each traded 1 minute the first day, 4 minutes the second day, 9 minutes the third day, 16 minutes the fourth day, and so on until day seven?

Make a table and look for a pattern.

Enrichment*:* Use the classroom internet to access the following. If possible print out your answers.

Refer to http://www.eyelid.co.uk/maths1.htm and solve the Mathematics problem for 10-11 year olds.

Egyptian Multiplication

The Egyptian's method of multiplication is fairly clever but can take longer than the modern-day method. This is how they would have multiplied 5 by 29.

*1   29

2   58

*4   116

1 + 4 = 5   29 + 116 = 145

Example 1

When multiplying they would have begun with the number they were multiplying; 29 and double it for each line. (29, 58, 116...) Now pick out the numbers in the first column that add up to the first number (5). They used the distributive property of multiplication over addition.

29(5) = 29(1 + 4) = 29 + 116 = 145

Example 2

54 multiplied by 7

*1   54

*2   108

*4   216

1+2+4   54+108+216

So 54(7) = 54(1+2+4) = 54+108+216 = 378

It is the author's opinion that this method will encourage a student to think about the true concept of multiplying , which is just fancy addition. Often it has been a classroom observation that students dive into a canned algorithm of "multiply, carry one…"

This method will lead students to look at multiplication in smaller segments of addition. It will allow students to manipulate the numbers in different arrays, which allows for multiple correct class responses.

It has obvious benefits for the learning disabled who have difficulty memorizing multiplication facts. The memorization of multiplication facts for some students can be an insurmountable stumbling block, which leads to fear and even contempt for mathematics.

Egyptian multiplication also gives an opportunity for the advanced student in looking at the many combinations of number patterns and an opportunity to look further into Egypt and hieroglyphics as well as the algorithms used by other cultures.

Multiplication Class Exercises

Solve these multiplication problems using the Egyptian Style of Multiplication.

1. $18 \times 4 =$

2. $75 \times 9 =$

3. $245 \times 5 =$

4. $25 \times 12 =$

5. $145 \times 25 =$

After you solve the problems using Egyptian multiplication, write your answers to the questions in hieroglyphics. Show your work!

Refer to http://www.eyelid.co.uk/calc.htm to check your work on this Egyptian calculator.

Multiplication Independent Exercises

Solve these multiplication problems using the Egyptian Style of Multiplication. Show your multiplication steps in hieroglyphics.

1. $234 \times 4 =$

2. $56 \times 7 =$

3. $128 \times 12 =$

4. $134 \times 32 =$

5. $45 \times 24 =$

After you solve the problems using Egyptian multiplication, write your answers to the questions in hieroglyphics.

Refer to http://www.eyelid.co.uk/calc.htm to check your work on this Egyptian calculator.

Egyptian Division

Egyptian division was similar to their multiplication.

Example 1

$98 \div 7$

Egyptians thought of this problem as 7 times some number equals 98. Again the problem was worked in columns.

1	7
2	*14
4	*28
8	*56

$2 + 4 + 8 = 14 \quad 14 + 28 + 56 = 98$

First begin with the largest number in the right column that is less than the dividend, and at each step add the largest number on the right that keeps the sum less than the dividend.

In this example the numbers in the right-hand column are marked out so their sum is 98, then the corresponding numbers in the left-hand column are summed to get the quotient.
The answer is 14. $98 = 14 + 28 + 56 = 7(2 + 4 + 8) = 7\times 14$

Example 2

$90 \div 6$

Again think of the problem as 6 times some number equals 98. Write the problem in columns.

1	*6
2	*12
3	*24
4	*48

Add the numbers on the right-hand column that add to 90, and then add the corresponding numbers in the left-hand column to get the quotient. The answer is 15.

$90 = 6 + 12 + 24 + 48 = 6(1 + 2 + 4 + 8) = 6 \times 15$

Example 3

This works well for division without remainders. Let's try a division that will not work out evenly.

$59 \div 8$

1	*8
2	*16
4	*32
8	64

No matter how the numbers are added they will not equal 59. So 59 is not evenly divisible by 8. However we can get very close!

$59 = 8 + 16 + 32 + (3 \text{ remainder}) = 8(1 + 2 + 4) + 3$

$59 = 8 \times 7 + 3$

It is the author's opinion that this type of division will lend itself more easily to estimation skill enhancement. A weakness of the traditional division algorithm is the elimination of the process in which students need to think in terms of estimation of the entire answer. Egyptian division does break down the problem into much smaller parts, yet it forces the student to go back and look at the divisor and dividend again. Before a student begins solving these or any division problem he/she must be led to look at the entire problem and always to ask, "How many times do you think it will divide or what do you think is the best estimate." In this age of automation, estimation is an invaluable skill. This type of division also allows weaker students an opportunity to divide without the complicated algorithm that is prone to careless mistakes during the multi-step process. While many students still struggle with multiplication, most can double numbers without complication.

<u>Division Class Exercises</u>

Solve the following division problems using Egyptian division. Write your answers in hieroglyphics. Estimate your answers before you begin.

1. 111 ÷ 3 =

2. 177 ÷ 3 =

3. 320 ÷ 64 =

4. 590 ÷ 118 =

5. 1,500 ÷ 125 =

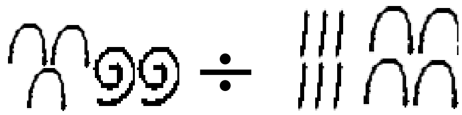
6. 

7. 245 ÷ 4 =

Refer to http://www.eyelid.co.uk/calc.htm to check your work on this Egyptian calculator.

<u>Division Independent Exercises</u>

Solve the division problems using Egyptian division. Write your answers in hieroglyphics. Estimate your answers before you begin.

1.  =

2. 44,184 ÷ 789 =

3. 53,676 ÷ 852 =

4. 19,971 ÷ 951 =

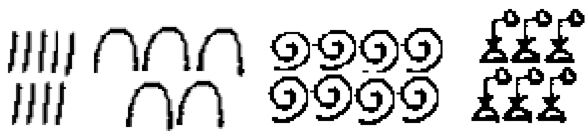
5. 256 ÷ 6 =

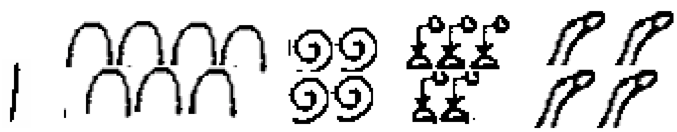
Refer to http://www.eyelid.co.uk/calc.htm to check your work on this Egyptian calculator.

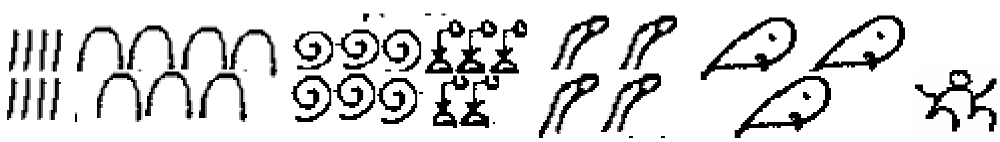
Answer Key

Hieroglyphic Class Exercises

1. 

2. 

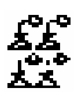
3. 

4. 

5. 445

6. 4,100

7. 4,000.00 deben or



8. 2,000.00 deben or

Answer Key

Hieroglyphics Independent Exercises

1. 104,400

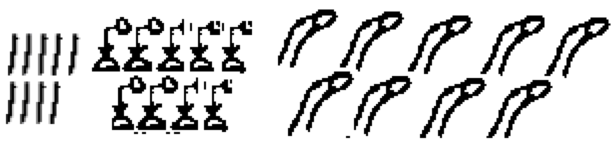
2. 1,000,600

3. 3,003

4. 765,000

5. 46,206

6. 

7. 

8. 

9.

DAY	BEETLES	TOTAL
1	2	2
2	4	6
3	8	14
4	16	30
5	32	62
6	64	126
7	128	254

TOTAL SOLD 254 = 2,540.00 deben

10.

DAY	MINUTES	TOTAL
1	1	1
2	4	5
3	9	14
4	16	30
5	25	55
6	36	91
7	49	140

Total minutes: 140

Extension activities for advanced students may include writing the charts in hieroglyphics, looking for patterns, and finding formulas for the totals. Also this is a wonderful opportunity to discuss the concept of sums of perfect squares. Develop a formula after the discussion, if appropriate to the learner's level of ability.

Answer Key

Multiplication Class Exercises

(Some answers may vary)

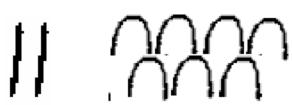
1. $18 \times 4 =$

1 18

2 36

*4 72

*18(4) = 18(2+2) = 36+36 =72



2. $75 \times 9 =$

*1 75

2 150

4 300

*8 600

*75(9) = 75(4+4+1) = 300+300+75 = 675

or *75(9) = 75(8+1) = 600+75= 675

3. 245×5 =

*1 245

2 490

*4 980

*245(5) = 245(4+1) = 980+245 = 1,225

or *245(5) = 245(2+2+1) = 490+490+245 = 1,225

4. 25×12 =

1 25

2 50

*4 100

* 8 200

*25(12) = 25(8+4) = 200+100= 300

or *25(12) = 25(4+4+4) = 100+ 100+ 100 = 300

5. 145×25 =

*1	145
2	290
4	580
*8	1,160
*16	2,320

*145(25) = 145(16 +8+1) = 2,320+1,160+145 = 3,625

Answer Key

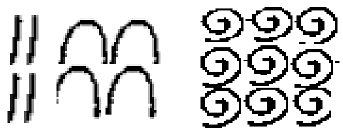
Multiplication Independent Exercises

(Answers may very)

1. 234×4 =

1	234
*2	468

*234(4) = 234(2+2) = 468+468 = 936



2. 56×7 =

*1 56

*2 112

*4 224

*56(7) = 56(4+2+1)= 224+112+56 = 392

3. 128×12 =

1 128

2 256

*4 512

*8 1024

*128(12) = 128(8+4) = 1,024+512= 1,536

4. 134×32

1	134
2	268
4	536
8	1,072
*16	2,144

*134(32) = 134(16+16) = 2,144+2,144 = 4,288

5. 45×24=

1	45
2	90
4	180
*8	360
*16	720

*45(24) = 45(24) = 45(16+8) = 720+360 = 1,080

Answer Key

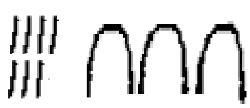
Division Classroom Exercises

(Answers methods may vary)

1. 111 ÷ 3 =

1	*3
2	6
4	*12
8	24
16	48
32	*96

*111 = 96+ 12+ 3 = 3(32 + 4 +1) so 3(37) = 111



2. 177 ÷ 3 =

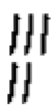
1	*3
2	*6
4	12
8	*24
16	*48
32	*96

*177 = 96 + 48 + 24 + 6 + 3 = 3(32 + 16 + 8 + 2 + 1) so 3(59) = 177

3. 320 ÷ 64 =

1	*64
2	128
4	*256

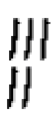
*320 = 256 + 64 = 64(4 + 1) so 64(5) = 320



4. 590 ÷ 118 =

1	*118
2	236
4	*472

*590 = 472 + 118 = 118(4+1) = 118(5) =590



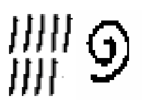
5. 1,500 ÷ 125 =

1	125
2	250

4 *500

8 *1,000

*1,500 = 1,000 + 500 = 125(8 + 4) = 125(12) = 1,500

6. 

7. 245 ÷ 4 =

1 *4

2 *8

4 *16

8 *32

16 *64

32 *128

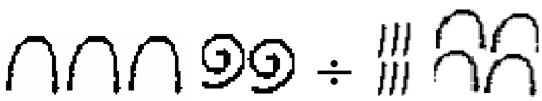
*245 = 128+ 64 + 32 + 16 + 8 = 4(32 + 16 + 8 + 4 + 2 + 1) + 1 = 4(61) + 1

So this problem has a remainder of 1. Refer to http://www.eyelid.co.uk/calc.htm to check your work on their Egyptian calculator.

Answer Key

Division Independent Exercises

(Answer methods may vary)

1. 

$230 \div 46 =$

1	*46
2	92
4	*184

*230 = 184 + 46 = 46(4+1) = 46(5) = 230

2. 44,184 $\div$ 789 =

1	789
2	1,578
4	3,156
8	*6,312
16	*12,624
32	*25,248

*44,184 = 25,248 + 12,624 + 6,312 = 789(32 + 16 + 8) = 789(56)

3. 53,676 ÷ 852 =

1 *852

2 *1,704

4 *3,408

8 *6,816

16 *13,632

32 *27,264

*53,676 = 27, 264 + 13, 632 + 3,408 + 6,816 + 1,704 + 852 = 852(32+16+8+4+2+1)

53,676 = 852×63

4. 19,971 ÷ 951 =

1 *951

2 1902

4 *3804

8 7608

16 *15,216

*19,971 ÷ 951 = 15,216 + 3,804 + 951 = 951(16 + 4 + 1) = 951(21)

5. 256 ÷ 6 =

1	6
2	*12
4	24
8	*48
16	96
32	*192

256 = (192 + 48 + 12) + remainder 4 = 6(32 + 8 + 2) +4 = 6(42) + 4

So this problem has a remainder of 4.

Suggested Classroom Web-sites

http://webinstituteforteachers.org/99/teams/egyptmath/mathproblems.htm (great site for solving problems independently)

http://www.42explore2.com/egypt.htm

http://www.eyelid.co.uk/numbers.htm

http://www.touregypt.net/featurestories/numbers.htm

http://www.eyelid.co.uk/calc.htm (Egyptian calculator)

http://www.mathcats.com/explore/oldegyptianfractions.html

http://www.arab.net/egypt/et_sphinx.htm

http://saxakali.com/COLOR_ASP/historymaf2.htm

http://mathworld.wolfram.com/EgyptianFraction.html

http://www.surfnetkids.com/pyramids.htm

CHAPTER 3

EGYPTIAN FRACTIONS

(A brief section on using these fractions in middle school)

One of the most interesting and even unusual parts of the *Rhind Papyrus* is the use of the Egyptian unit fraction. It is not at all clear why Egyptians chose to use this type of manipulation of numbers to solve fraction problems. It is fascinating to look at this advanced civilization and realize that they used this system for 2,000 years. It may have been because of their form of writing or perhaps a traditional form that remained unquestioned. Whatever the reason, it does provide problem solving opportunities for some students. This type of fraction will certainly enrich advanced students, so it should be presented in such a way as not to frustrate lower level students.

The Egyptians had an interesting way of using fractions. To represent a fraction they always wrote a fraction with a numerator of one. These are called unit fractions. Examples are $\frac{1}{2}, \frac{1}{5}, \frac{1}{25} \ldots$ Because they are parts of $\frac{1}{n}$ their notation could not be used to write the fractions as we know them $\frac{2}{3}, \frac{5}{6}, \frac{4}{9} \ldots$

Egyptians were able to write their fractions as a sum of unit fractions.

Example 1

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{2}$$

Egyptians would never have written the fraction $\frac{3}{4}$ as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. So, in our classroom examples we will not use these repeated addends as correct representations of the unit fraction.

Example 2

$\frac{6}{7} = \frac{1}{2} + \frac{1}{3} + \frac{1}{42}$ NOT $\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$ for any repeated use of any other unit fraction.

The *Rhind Papyrus* contains many unit fractions represented in what is referred to as 2/n tables. These sites contain conversions for unit fractions to help speed up the conversion of fractions to unit fractions.

Table 2

2/n Table Rhind Papyrus

2/3 = 1/2 + 1/6 ::: 2/5 = 1/3 + 1/15 ::: 2/7 = 1/4 + 1/28

2/9 = 1/6 + 1/18 ::: 2/11 = 1/6 + 1/66 ::: 2/13 = 1/8 + 1/52 + 1/104

2/15 = 1/10 + 1/30 ::: 2/17 = 1/12 + 1/51 + 1/68 ::: 2/19 = 1/12 + 1/76 + 1/114

2/21= 1/14 + 1/42 ::: 2/23 = 1/12 + 1/276 ::: 2/25 = 1/15 + 1/75

2/27 = 1/18 + 1/54 ::: 2/29 = 1/24 + 1/58 + 1/174 + 1/232

2/31 = 1/20 + 1/124 + 1/155 ::: 2/33 = 1/22 + 1/66

2/35 = 1/25 + 1/30 + 1/42 [this is neither 2/7*5 or 2/5*7] ::: 2/37 = 1/24 + 1/111 + 1/296

2/39 = 1/26 + 1/78 ::: 2/41 = 1/24 + 1/246 + 1/328

2/43 = 1/42 + 1/86 + 1/129 + 1/301 ::: 2/45 = 1/30 + 1/90

2/47 = 1/30 + 1/141 + 1/470 ::: 2/49 = 1/28 + 1/196 ::: 2/51 = 1/34 + 1/102

2/53 = 1/30 + 1/318 + 1/795 ::: 2/55 = 1/30 + 1/330 (use 2/11 instead of 2/5)

2/57 = 1/38 + 1/114 ::: 2/59 = 1/36 + 1/236 + 1/531

2/61 = 1/40 + 1/244 + 1/488 + 1/610 ::: 2/63 = 1/42 + 1/126

2/65 = 1/39 + 1/195 ::: 2/67 = 1/40 + 1/335 + 1/536 ::: 2/69 = 1/46 + 1/138

2/71 = 1/40 + 1/568 + 1/710 ::: 2/73 = 1/60 + 1/219 + 1/292 + 1/365

2/75 = 1/50 + 1/150 ::: 2/77 = 1/44 + 1/308

2/79 = 1/60 + 1/237 + 1/316 + 1/790 ::: 2/81 = 1/54 + 1/162

2/83 = 1/60 + 1/332 + 1/415 + 1/498 ::: 2/85 = 1/51 + 1/255 ::: 2/87 = 1/58 + 1/174

2/89 = 1/60 + 1/356 + 1/534 + 1/890 ::: 2/91 = 1/70 + 1/130 (do not use 2/7)

2/93 = 1/62 + 1/186 ::: 2/95 = 1/60 + 1/380 + 1/570 (do not use 2/5)

2/97 = 1/56 + 1/679 + 1/776 ::: 2/99 = 1/66 + 1/198

2/101 = 1/101 + 1/202 + 1/303 + 1/606 (Richardson, 2000)

Other such tables can be found at http://noisefactory.co.uk/maths/history/hist006.html and http://www.math.buffalo.edu/mad/Ancient-Africa/best-egyptian-fraction.html

In these 2/n tables all values of up to 101 are used, the fractions with even denominators are omitted; perhaps it indicates that the Egyptians thought these to be obvious.

The first entry in the *Rhind Papyrus* table is $\frac{2}{3}$ to which is assigned the expression $\frac{1}{2}+\frac{1}{6}$. Each entry for the multiples of three in the denominator is grouped as family, other denominators are also grouped as a family.

The next table entry is $\frac{2}{5}$ to which they give a value $\frac{1}{3}+\frac{1}{15}$. The table continues in this manner. It is interesting to mention that multiples are "sieved out", indicating that Egyptians had their own version of the Sieve of Eratosthenes. Mathematicians also feel that the Egyptians had a knowledge of prime and composite numbers, as shown by the ending of the table at 101. Most composites are sieved out. The number 55 should have been sieved out as a multiple of 5, but they chose to keep it as a multiple of 11. Also the composites 35, 91, and 95 were not treated as composites.

The *Rhind Papyrus* does clearly show that a thousand years before Pythagoras the Egyptians had some awareness of primes, composites, and number sieves.

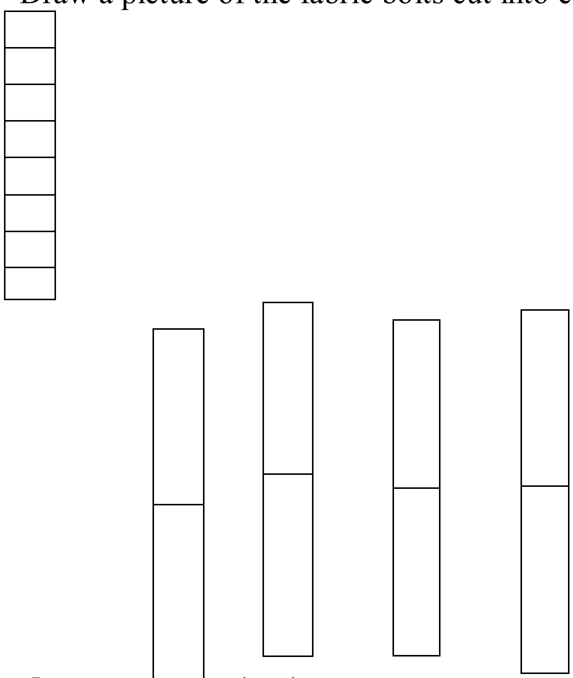
A practical use of this type of fraction in ancient Egypt may have been used to represent how much land was flooded by the Nile River. Instead of reporting that $\frac{2}{5}$ of their land was flooded, they would have said $\frac{1}{3}+\frac{1}{15}$ of their land was flooded.

Every ordinary fraction can be written as an Egyptian fraction. One practical use of this type of fraction is shown in the following example:

Example3:

Suppose that Cleopatra wishes to share his 5 bolts of purple fabric with 8 workers in the Snail Factory. Now think about the fraction that is involved, $\frac{5}{8}$.

Draw a picture of the fabric bolts cut into equal sections for the workers to share.



$\frac{5}{8} =$ $\frac{1}{2} + \frac{1}{8}$

The previous 5 pictures represent the 5 bolts of purple fabric. These bolts must be shared with eight Egyptians. The first bolt is cut into 8 pieces, and each person gets one. The remaining 4 bolts can be cut into 2 pieces and each person gets one of the 8 halves. Looking at this picture it is easy to see that each worker would receive one of the $\frac{1}{2}$ and one of the $\frac{1}{8}$.

Many web-sites contain interesting proofs that explain the process of using Egyptian fractions. The Theory of the Greedy Algorithm can be found at http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fractions/egyptian.html#calc1. Some advanced students may find it beneficial to consider this proof as an explanation for finding Egyptian fractions.

<u>Egyptian Fractions Class Exercises</u>

Solve the following using the method of Egyptian fractions found in example #3. Hint : Draw a picture.

1. Suppose Thebes wishes to divide 3 bolts of his beautiful purple fabric with 4 workers.

2. What if Thebes wishes to divide 2 bolts among 5 people?

3. What if Thebes wishes to divide 4 bolts among 10 people?

4. What about 15 bolts to share among 14 people?

5. Write the fraction $\frac{3}{7}$ as a sum of unit fractions.

Egyptian Fractions Independent Exercises

Write the following as a sum of unit fractions.

1. $\frac{3}{4}$

2. $\frac{4}{5}$

3. $\frac{4}{7}$

4. $\frac{5}{8}$

5. $\frac{3}{10}$

6. $\frac{7}{20}$

Use the following site to check your answers with the Egyptian fraction calculator. Also experiment with at least five fractions that you create yourself.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fractions/egyptian.html#calc1

Go to the Egyptian Cats web page and work at least 10 problems.

http://www.mathcats.com/explore/oldegyptianfractions.html

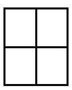
Egyptian Fraction Answer Key

Class Exercises

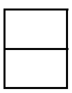
(Answers may vary)

Solve using the method of Egyptian fractions found in example #3.

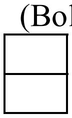
1. Suppose Thebes wishes to divide 3 bolts of his beautiful purple fabric with 4 workers. Draw a picture if you would like to use that problem solving strategy. Each little square is a bolt of fabric. Bolt one is cut into 4 pieces, so each worker gets one of four pieces. Bolts 2 and 3 are cut into two pieces, allowing each of the four workers to get one half of a bolt. Each worker gets $\frac{1}{2}$ and $\frac{1}{4}$ bolts, or $\frac{3}{4} = \frac{1}{4} + \frac{1}{2}$



(Bolt 1)



(Bolt 2)



(Bolt 3)

2. What if Thebes wishes to divide 2 bolts among 5 people?

$$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}$$

3. What if Thebes wishes to divide 4 bolts among 10 people.

$$\frac{4}{10} = \frac{2}{5} = \frac{1}{3} + \frac{1}{15}$$

4. What about 15 bolts to share among 14 people?

$$\frac{14}{15} = \frac{1}{2} + \frac{1}{3} + \frac{1}{10}$$

5. Write the fraction $\frac{3}{7}$ as a unit fraction.

$$\frac{3}{7} = \frac{1}{3} + \frac{1}{11} + \frac{1}{231}$$

<u>Egyptian Fraction Independent Exercises</u>

<u>Answer Key</u>

<u>(Answers may vary)</u>

1. $\frac{3}{4}$

$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$$

2. $\frac{4}{5}$

$$\frac{4}{5} = \frac{1}{2} + \frac{1}{4} + \frac{1}{20}$$

3. $\frac{4}{7}$

$$\frac{4}{7} = \frac{1}{2} + \frac{1}{14}$$

4. $\frac{5}{8}$

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

5. $\frac{3}{10}$

$$\frac{3}{10} = \frac{1}{4} + \frac{1}{20}$$

6. $\frac{7}{20}$

$$\frac{7}{20}=\frac{1}{3}+\frac{1}{60}$$

Use the following sites to check your answers with the Egyptian fraction calculator. Also experiment with at least five fractions that you create yourself.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fractions/egyptian.html#calc1

http://www.mathcats.com/explore/oldegyptianfractions.html

It is the author's opinion that teaching Egyptian fractions does have a place in middle level education. First, it does provide some practical ways of dividing whole number amounts into fractions, and fractions into unit parts. Many students have problems dividing into fractional groups. It also gives much practice in looking for common multiples and seeing the many ways the same fractions can be written.

Students may find it easier to compare fractions using Egyptian fractions when working with <, =, and >.

While it may be confusing for struggling students at first, it provides limitless opportunities for advanced problem solving. It is certainly a subject that can be understood at its most elementary level yet provide for very different levels of understanding. Many different groups in a classroom could be working on these fractions at their appropriate levels and still find them understandable and challenging at the same time.

Suggested Classroom Web-sites

http://www.math.buffalo.edu/mad/Ancient-Africa/mad_ancient_egyptroll2-n.html

http://www.math.buffalo.edu/mad/Ancient-Africa/mad_ancient_egyptpapyrus.html

http://www.mathcats.com/explore/oldegyptianfractions.html

http://math.truman.edu/~thammond/history/RhindPapyrus.html

http://noisefactory.co.uk/maths/history/hist006.html

http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Egyptian_papyri.html

http://aleph0.clarku.edu/~djoyce/ma105/2byn.html

http://www.math.washington.edu/~greenber/Rhind.html

CHAPTER 4

TREVISO ARITHMETIC 1478

(Original Source Document)

The *Treviso Arithmetic* is an Italian mathematics textbook written by an anonymous teacher in Treviso, Italy in 1478. David Eugene Smith translated parts of the *Treviso Arithmetic* for educational purposes in 1907. Frank J. Swetz translated the *Treviso* using Smith's notes in 1987. The Treviso is the earliest known printed mathematics book in the West, and one of the first printed European textbooks dealing with a science. The *Treviso Arthmetic* is a practical book intended for self study and for use in Venetian trade. It is written in the Venetian dialect and communicated knowledge to a large population. It helped to end the monoply on mathematical knowledge and gave important information to the middle class. It was not written for a large audience, but intended to teach mathematics of everyday currency in Italy.

Original copies of the *Treviso Arithmetic* are extremely rare. There appears to have been only one edition of the work. Frank Swetz's complete translation can be found in *Capitalism & Arithmetic: The New Math of the 15th Century*. Swetz used a copy of the Treviso housed in the Manuscript Library at Columbia University. The volume found its way to this collection via a curious route. Maffeo Pinelli (1785), an Italian bibliophile, is the first known owner. After his death his library was purchased by a London book dealer and sold at auction on February 6, 1790. The book was obtained for three shillings by Mr. Wodhull. About 100 years later the Arithmetic appeared in the library of Brayton Ives, a New York lawyer. When Ives sold the collection of books at auction, George Plimpton, a New York publisher, acquired the Treviso and made it an acquisition to his extensive collection of early scientific texts. Plimpton donated his library to Columbia University in 1936. (Swetz, 1987)

The book was printed in quarto work which means each page is divided into fourths. There are 123 pages of text with 32 lines of print to a page. The pages are unnumbered, untrimmed and have wide margins. Some of the margins contain written notes. The size of the book is 14.5 cm by 20.6 cm.

Gutenberg's press in 1450 paved the way to the publication of books. The Treviso became one of the first mathematics books that were written for the expansion of human knowledge. It gave opportunity for the common person to learn the art of computation instead of only a privileged few. The Treviso Arithmetic provided an early example of the Hindu-Arabic numeral system and computational algorithms. (Swetz, 1987)

Capitalism and Arithmetic: The New Math of the 15th Century by Frank J. Swetz is divided into seven chapters. Chapter 1 is a perspective and background on the atmosphere and culture of the day as clearly and accurately described by Frank Swetz. Chapter 2 provides David Smith's free translation of the book. Chapters 3, 4, 5, and 6 show computation skills as related to Italy at the time of the Treviso. Chapter 7 focuses on the social, economic, and commercial aspects of the culture.

Chapter 1 describes Italy emerging into a renaissance. It was a time of opportunity for economic wealth and growth of knowledge and great spiritual awakening. The Treviso is a prime example of a book of opportunity. Merchants of Venice were bright businessmen. By the time of the book's publication Venice was a trading powerhouse in Europe. The need for common people to learn arithmetic and commercial mathematics was becoming commonplace. Addition and subtraction were taught in many countries of Europe, but to learn multiplication and division a student might be best taught in Italy by a reckoning master.

Reckoning schools were formed in Italy and then spread northward along trade routes. Knowledge, especially commercial mathematics, was not just a luxury for the rich any longer. Gutenberg's moveable type in 1450 enabled rapid spread of ideas throughout Europe.

The *Treviso Arithmetic* appeared in 1478. Its practical algorithms provided opportunity for self study of commercial math. It certainly led the way to development of mathematical knowledge in the middle ages. (Swetz, 1987)

This next section will focus on the use of the excess of nines in addition, multiplication, subtraction, and division. It is an example of how an original source document can be used in conventional classrooms. The picture that the book paints of pre-Renaissance Italy could serve as a wonderful beginning for thematic units across the curriculum!

Excess of Nines Addition

Example 1

$$\begin{array}{r} 59 \\ +38 \\ \hline 97 \end{array} \quad /7$$

To check this addition the Treviso Arithmetic would tell the reader to add the 5 and 9 in 59. This sum is 14. Subtract the excess of 9 and the answer is 5. In 38 add the 3 and 8. The sum is 11. Subtract the excess of 9 and the answer is 2. Add the 5 and 2 together. This leaves you with 7. Write this number beside the sum of the problem. Then take out the excess of 9 in the number 97. If the excess in the sum equals the excess of 9 in the addends the addition checks.

While this is a wonderful way to manipulate numbers, the student must be warned of the flaws in the system. The excess of 9 does give a quick test for accuracy, but it does not take into account number position. If we reordered the numbers in the sum in the wrong order we would still expect to get the same excess of 9, and therefore, have a feeling of false accuracy. In the first example an answer of 79 would render the same excess; however, it is not the correct answer.

Example 2:

$$\begin{array}{r} 1{,}467 \\ +\ 251 \\ \hline 1{,}718 \end{array} \quad /8$$

To check the addition find the excess of 1,467. In this case the excess of nine must be found a second time because the sum of the digits is 18. This gives an excess equal to 0. The excess of nines in 251 is 8. 8+0=8. The excess of 1,718 is also 8. So we have checked the problem.

Subtraction is the opposite of addition and could be checked in much the same way. Looking at the two previous examples, you could change these to subtraction by rewriting them. Then check in the same manner.

<u>Excess of Nines Addition Class Exercises</u>

Using the method of the excess of nines from the *Treviso Arthmetic* add the following numbers and then check your answers.

1. $$\begin{array}{r} 569 \\ +\ 189 \\ \hline \end{array}$$

2. $$\begin{array}{r} 5{,}721 \\ +\ 524 \\ \hline \end{array}$$

3. $$\begin{array}{r} 9{,}639 \\ +5{,}481 \\ \hline \end{array}$$

4. $$\begin{array}{r} 72{,}816 \\ +19{,}827 \\ \hline \end{array}$$

5. $$\begin{array}{r} 432{,}791 \\ +153{,}961 \\ \hline \end{array}$$

Make your own problem and check with the excess of nines method.

<u>Excess of Nines Addition Independent Exercises</u>

Using the method of the excess of nines from the *Treviso Arithmetic,* add the following numbers and then check your answers.

1. 245
   +198
   ———

2. 1,897
   + 2,436
   ———

3. 67,887
   + 40,599
   ———

4. 894,512
   + 86,739
   ————

Do you have a strategy for getting the 9s out of your problems? What is it?

<u>Enrichment</u>

1. Read about the history of commercial Italy during the printing of the *Treviso Arithmetic* in 1478. Compare the printing date to the invention of the Gutenberg Press in Germany.

2. In what country was the Treviso first published?

3. How was it different from other mathematics textbooks?

4. What symbol was often used for the number 1, because the printer didn't have enough digits?

5. Give an example of excess of 9 that checks even though the addition is incorrect.

Chessboard Multiplication

About 1478 in Treviso, Italy, the *Treviso Arithmetic* showed 4 ways of multiplying. These were called the Gelosia Method. The Gelosia Method was first used in India in the tenth century, and became popular in Europe in the fourteenth and fifteenth centuries.

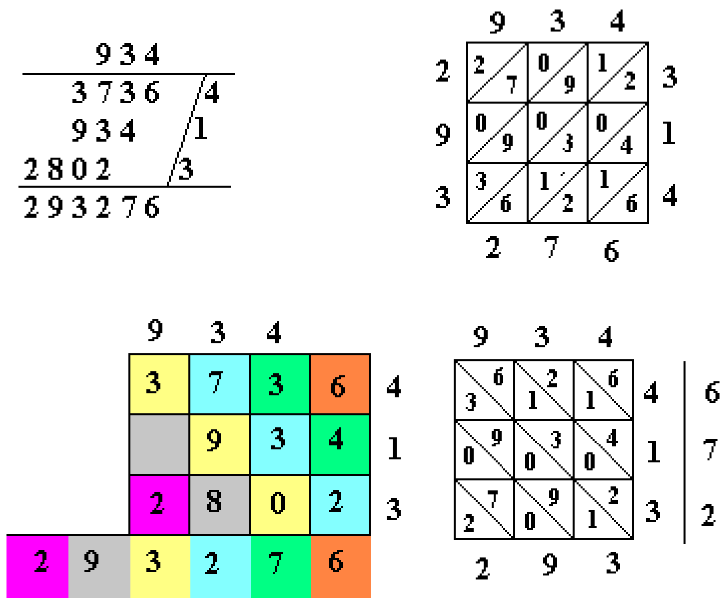


Figure 2

Chessboard Multiplication

These four pictures were contributed by Tina Gonzales, Wichita State University, Department of Mathematics and Statistics. (Gonzales,2000) Most of these methods were adapted in many different cultures. There are so many ways of multiplying, one hardly knows which method to use. Problem solving is finding ways to a solution. If one method is not suited for you, never hurts to try another.

Now let's take a few examples of checkerboard multiplication, also called the lattice method or box method.

Example # 1

842

$\times$ 431

	8	4	2	
3	$^{3}/_{2}$	$^{1}/_{6}$	$^{0}/_{8}$	4
6	$^{2}/_{4}$	$^{1}/_{2}$	$^{0}/_{6}$	3
2	$^{0}/_{8}$	$^{0}/_{4}$	$^{0}/_{2}$	1
	9	0	2	

The factors are listed along the top and sides of the table. The products of each set of factors are written inside of one cell of the table divided by a slash. Notice that $2 \times 4 = 8$ and it is written $^{0}/_{8}$. Notice that $4 \times 4 = 16$ and it is written $^{1}/_{6}$. Then add up the diagonals, listing the sums around the outer cells of the table. If it is necessary to regroup in the addition, then "carry" to the next column. This is, of course, another opportunity to insert a lesson on place value!

842

$\times$ 431

362,902

The author suggests graph paper for this activity because it will greatly aid in the neatness of the method. As in most forms of multiplication, keeping all number columns straight is a definite plus. Once the grille has been drawn, this type of multiplication is faster, requires less carrying, and is easily checkable.

Example # 2

697

$\times$ 358

	6	9	7	
2	1/8	2/7	2/1	3
4	3/0	4/5	3/5	5
9	4/8	7/2	5/6	8
	5	2	6	

On the graph paper enter the factors along the top and side of the paper. When multiplying 7×3, write the product in the form $2/1$. Continue in this manner. Add the columns together around the outside edges.

$$\begin{array}{r} 697 \\ \times\ 358 \\ \hline \end{array}$$

249,526

Chessboard Multiplication Class Exercises

Solve the following multiplication problems using the Chessboard Multiplication as described in the *Treviso Arithmetic*. Use your graph paper to keep things straight.

1. $\begin{array}{r} 647 \\ \times 548 \\ \hline \end{array}$

2. $\begin{array}{r} 891 \\ \times 563 \\ \hline \end{array}$

3. $\begin{array}{r} 1{,}597 \\ \times 326 \\ \hline \end{array}$

4. $\begin{array}{r} 2{,}648 \\ \times 1{,}589 \end{array}$

5. $\begin{array}{r} 2{,}065 \\ \times 703 \\ \hline \end{array}$

Chessboard Multiplication Independent Exercises

Solve the following multiplication problems using the Chessboard Multiplication as described in the Treviso Arithmetic. Use your graph paper to keep things straight.

1. $\begin{array}{r} 457 \\ \times 123 \\ \hline \end{array}$

2. $\begin{array}{r} 5{,}780 \\ \times 360 \\ \hline \end{array}$

3. $\begin{array}{r} 50{,}279 \\ \times 1{,}567 \\ \hline \end{array}$

4. $\begin{array}{r} 9{,}875 \\ \times 9{,}582 \\ \hline \end{array}$

5. $\begin{array}{r} 9{,}009 \\ \times 5{,}050 \\ \hline \end{array}$

Enrichment: Explain why the Chessboard Multiplication works.
Will it work with Napier's Rods? Research Napier's Chessboard Multiplication.

Excess of Nines Multiplication

In multiplication the process of excess of nines still works, but it is slightly different in its application. This time we will find the excess of each set of factors. We will then multiply that excess and cast out the excess of nines from the product of the excess. This product will be the excess found in the product of the entire problem.

Example 1:

$$\begin{array}{r} 9{,}279 \\ \times\ 8 \\ \hline \end{array}$$

The excess of 9,279 is 0. The excess of 8 is 8. Multiply $0 \times 8 = 0$. The excess of nine in the product will be 0.

$$\begin{array}{ll} 9{,}279 & /0 \\ \times 8 & /8 \\ \hline 74{,}223 & /0 \end{array}$$

Example 2:

$$\begin{array}{ll} 12{,}359 & /2 \\ \times\ 38 & /2 \\ 46{,}9642 & /4 \end{array}$$

The excess of nine in 12,359 is 2. The excess of 38 is 2. Multiplying $2 \times 2 = 4$. The excess of 469,642 must be 4.

Example 3:

$$\begin{array}{ll} 2{,}591 & /8 \\ \times\ 323 & /8 \\ 836{,}893 & /1 \end{array}$$

The excess of nine in 2,591 is 8. The excess of 323 is also 8. Multiplying $8 \times 8 = 64$.

The excess of nine in 6 + 4 is 1. The excess of 9 in 8 + 3 + 6 + 8 + 9 + 3 is also 1. According to the Treviso this checks the multiplication.

Multiplication Class Exercises

Check the answers we got from the chessboard/lattice method of multiplication by casting out nines.

1. 647

$\times$548

35,4556

2. 891

$\times$563

501,633

3. 1,597

$\times$ 326

530,662

4. 2,648

$\times$1,589

4,207,672

5. 2,065
× 703

1,451,693

Multiplication Independent Exercises

Check the following problems using the excess of nine for multiplication method.

1. 457
×123

56,211

2. 5,780
×360

2,080,800

3. 50,279
×1,567

78,787,193

4. 9,875
×9,582

94,622,520

5. 9,009
×5,050

45,485,450

Enrichment: What explanation can be given as to why this works? Have you found situations that appear to give correct answers, but then upon further examination you find that the Treviso Method did not work?

Excess of Nines Division

The Treviso also gives a check for the division algorithm. It is a little more complicated than the addition and multiplication, but it is well worth the effort.

Example #1

7,624 0 Remainder

÷ 2

3,812

5	0
2	1

Quotient Excess	Remainder
Divisor	Quotient Excess x Divisor + Remainder

Example 2

In this example the excess of the quotient 3,812 is found by adding the numbers 3 + 8 + 1 + 2. The total is 14, which has an excess of 5. Multiply this excess by the divisor 2. The answer is 10. Add the 10 to the remainder 0. The total is 10 again. The excess of 10 is 1. Then check to see if the dividend 7624 has the same excess of 1. So it checks with the Treviso Method.

54,324 Remainder 0

÷ 6

9,054

Again refer to the table:

Quotient Excess	Remainder
Divisor	Quotient Excess x Divisor + Remainder

The excess of the quotient 9,045 is 0. The divisor is 6. The remainder is 0. Multiply the quotient excess and the divisor add the remainder and you get 0. If the dividend has an excess of 0 then the algorithm is proven. The excess of 54,324 is 0!

0	0
6	0

Example 3

 3,569
÷ 54
―――

 66 Remainder 5

Quotient Excess	Remainder
Divisor	Quotient Excess x Divisor + Remainder

The excess of the quotient is 5. The divisor is 54. The remainder is 5. $(5\times54)+5$ gives an excess of 5. The excess of the dividend is also 5.

5	5
54	5

Note that you could have equally found the answer by taking out the excess of 9 in 54. Your table would have then looked like:

5	5
0	5

Division Class Exercises

Check the following answers using the excess of nines for division algorithm.

1. 5,649 ÷ 3 = 1,883 remainder 0

2. 9,876 ÷ 6 = 1,646 remainder 0

3. 19,086 ÷ 52 = 367 remainder 2

4. 38,148 ÷ 52 = 732 remainder 5

5. 8,979 ÷ 11 = 815 remainder 2

Division Independent Exercises

Check the following answers using the excess of nines for division algorithm.

1. 1,5216 ÷ 951 = 18 remainder 0

2. 627 ÷ 25 = 25 remainder 2

3. 127,257 ÷ 169 = 753 remainder 0

4. 58,105 ÷ 605 = 96 remainder 25

5. 8,119 ÷ 9 = 902 reminder 1

Enrichment: Make up other problems that will test the excess check. Research David E. Smith. What part does he play in the translation of the Treviso Arithmetic? Can you give examples that reveal an incorrect answer that still checks?

Answer Key

Excess of Nines Addition Class Exercises

Using the method of the excess of nines from the Treviso Arithmetic add the following numbers and then check your answers. Hint: to check addition you add the excess of the addends.

1. 569 /2
+ 189 /0

758 /2

2. 5,721 /6
+ 524 /2

6,245 /8

3. 9,639 /0
+5,481 /0

15,120 /0

4. 72,816 /6
+19,827 /0

92,643 /6

5. 432,791 /8
+153,961 /7

586,752 /6

Make your own problem and check with the excess of nines method. Share answers.

Answer Key

Excess of Nines Addition Independent Exercises

Using the method of the excess of nines from the Treviso Arithmetic add the following numbers and then check your answers. Hint: to check addition you add the excess of the addends.

1. 245 /2
+198 /0

443 /2

2. 1,897 /7
+ 2,436 /6

4,333 /4

3. 67,887 /0
+ 40,599 /0

108,486 /0

4. 894,512 /2
+ 86,739 /6

981,251 /8

Answer Key

Chessboard Class Exercises

Solve the following multiplication problems using the Chessboard Multiplication as described in the Treviso Arithmetic. Use graph paper to keep things straight.

1. 647

$\times$548

	6	4	7	
3	3/0	2/0	3/5	5
5	2/4	1/6	2/8	4
4	4/8	3/2	5/6	8
	5	5	6	

354,556

2. 891

$\times$563

	8	9	1	
5	4/0	4/5	0/5	5
0	4/8	5/4	0/6	6
1	2/4	2/7	0/3	3
	6	3	3	

501,663

3. 1,597

$\times$326

	1	5	9	7	
0	0/3	1/5	2/7	2/1	3
5	0/2	1/0	1/8	1/4	2
2	0/6	3/0	5/4	4/2	6
	0	6	2	2	

520,622

4. 2,648

$\times$1,589

	2	6	4	8	
0	0/2	0/6	0/4	0/8	1
4	1/0	3/0	2/0	4/0	5
2	1/6	4/8	3/2	6/4	8
0	1/8	5/4	3/6	7/2	9
	7	6	7	2	

4,207,672

5. 2,065

$\times$ 703

	2	0	6	5	
1	1/4	0/0	4/2	3/5	7
4	0/0	0/0	0/0	0/0	0
5	0/6	0/0	1/8	1/5	3
	1	6	9	5	

1,451,695

Answer Key

Chessboard Independent Exercises

Solve the following multiplication problems using the Chessboard Multiplication as described in the *Treviso Arithmetic*. Use your graph paper to keep things straight.

1. 457

$\times$ 123

	4	5	7	
0	0/4	0/5	0/7	1
5	0/8	1/0	1/4	2
6	1/2	1/5	2/1	3
	2	1	1	

56,211

2. 5,780

$\times 360$

	5	7	8	0	
2	1/5	2/1	2/4	0/0	3
0	3/0	4/2	4/8	0/0	6
8	0/0	0/0	0/0	0/0	0
	0	8	0	0	

2,080,800

3. 50,279

$\times 1{,}567$

	5	0	2	7	9	
0	0/5	0/0	0/2	0/7	0/9	1
7	2/5	0/0	1/0	3/5	4/5	5
8	3/0	0/0	1/2	4/2	5/4	6
7	3/5	0/0	1/4	4/9	6/3	7
	8	7	1	9	3	

78,787,193

4. 9,875

$\times 9{,}582$

	9	8	7	5	
9	8/1	7/2	6/3	4/5	9
4	4/5	4/0	3/5	2/5	5
6	7/2	6/4	5/6	4/0	8
2	1/8	1/6	1/4	1/0	2
	2	2	5	0	

94,622,250

5. 9,009
$\times$5,050

	9	0	0	9	
4	4/5	0/0	0/0	4/5	5
5	0/0	0/0	0/0	0/0	0
4	4/5	0/0	0/0	4/5	5
9	0/0	0/0	0/0	0/0	0
	5	4	5	0	

45,495,450

Answer Key

Multiplication Class Exercises

Check the answers we got from the chessboard/lattice method of multiplication by excess of nines.

1. 647 /8
   $\times$548 /8

   354,556 /1

(8$\times$8=64 6+4=10 an excess of 1)

2. 891 /0
   $\times$563 /5

   501,633 /0

3. 1,597 /4
   $\times$ 326 /2

   530,662 /4

This answer does not check because the answer is not correct. The correct answer is 520,662.

4. 2,648 /2
   $\times$1,589 /5

   4,207,672 /1

5. 2,065 /4
   $\times$ 703 /1

   1,451,693 /4 (This answer does not check. The correct answer is 1,451,695.)

Answer Key

Multiplication Independent Exercises

Check the following problems using the excess of nines for multiplication method.

1. 457 /7
×123 /6
———
56,211 /6
(6×7=42 4+2= 6)
————

2. 5,780 /2
×360 /0
———
2,080,800 /0

3. 50,279 /5
×1,567 /1
————
78,787,193 /5

4. 9,875 /2
×9,582 /6
————
94,622,520 /3

The excess of nines does check for this answer; however, the numbers have been transposed. The actual answer is 94,622,250.

5. 9,009 /0
×5050 /1

45485450 /8

The correct answer is 45,495,450.

Answer Key

Division Class Exercises

Check the following answers using the excess of nines for the division algorithm.

Quotient Excess	Remainder
Divisor	Quotient Excess x Divisor + Remainder

1. 5,649 ÷ 3 = 1,883 remainder 0

2	0
3	6

5,649 /6

2. 9,876 ÷ 6 = 1,646 remainder 0

8	0
6	3

9,876 /3

3. 19,086 ÷ 52 = 367 remainder 2

7	2
7	6

19,086 /6

4. 38,148 ÷ 52 = 732 remainder 5

3	5
7	8

38148 /6

The correct answer is 733 remainder 32

5. 8,979 ÷ 11 = 815 remainder 2

5	2
2	3

8,979/6

The correct answer is 816 remainder 3.

Answer Key

Division Independent Exercises

Check the following answers using the excess of nine for division algorithm.

Quotient Excess	Remainder
Divisor	Quotient Excess x Divisor + Remainder

1. 15,216 ÷ 951 = 18 remainder 0

0	0
6	0

15,216 / 6

The correct answer is 16.

2. 627 ÷ 25 = 25 remainder 2

7	2
7	6

627 /6

3. 127,257 ÷ 169 = 753 remainder 0

6	0
7	6

127,257 /6

4. 58,105 ÷ 605 = 96 remainder 25

6	7
2	1

58,105 /1

5. 8,119 ÷ 9 = 902 reminder 1

2	1
0	1

8,119 /1

Suggested Classroom Web-sites

http://www.jimloy.com/number/nines.htm

http://www.mathpages.com/home/kmath269/kmath269.htm

http://mathforum.org/library/drmath/view/55926.html

http://iseagle.sas.edu.sg:8068/technology/enrichmentMath/arithmetic/castingoutnines.htm

http://www.wordways.com/lucky.htm

CHAPTER 5

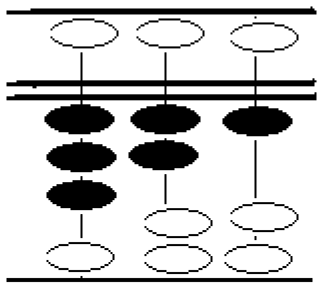
CALCULATING DEVICES

The Soroban Abacus

The abacus is the world's oldest calculating machine. It is capable of doing all the basic arithmetic operations. The abacus appeared throughout the civilized world in many different forms as early as the 5th century B.C. The common medium for abacus construction was wax and clay to make tablets. The abacus has had three basic forms. The first abacus was the dust board abacus, which was probably Babylonian in origin. Lines were drawn as place holders by a finger or by a stylus. These lines were drawn in the sand or on clay. The dust abacus was used until the close of the 9th century B.C., and was followed by the ruled table with disks or counters placed on or between lines to show numbers. The third type had a portable frame with counters or beads that moved along a wire or wooden pole. The exact origin of the abacus is unclear. The basic idea is the same in all three of the abaci. The pebbles, markers, or counters represent place value units in the base ten system.

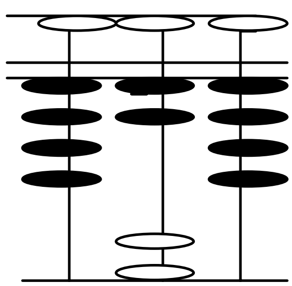
The beads on the abacus slide up and down on the rods which are divided by a horizontal beam called a reckoning bar. A cleared board has all the beads pushed away from the reckoning bar. The single rows of beads above the reckoning bar are called the Heaven Beads and are worth 5. The beads below the reckoning bar are called the Earth Beads and are worth 1. A bead only has a value if it is pushed toward the reckoning bar. A bead losses value if it is moved away from the reckoning bar. The abacus is read in the same manner as our standard place value chart. The first vertical rod of beads to the right is the ones, the second is the tens, and the third is the hundreds. The abacus number is read in the same manner as we read a traditional number. To show the number 321 push the earth beads to touch the reckoning bar as shown. In the diagram the shaded beads touching the reckoning bar have value; the white beads don't have value. (Bernazzani, 2005)

Example 1:



Addition: Numbers are added on the abacus by pushing beads towards the center bar. Regrouping may be necessary to complete the operation of addition in some examples. Number values are read from left to right in the following examples. To add 103 to the 321 already on the abacus, simply push the additional earth beads to the reckoning bar as shown. Reading from left to right the answer is 424.

Example 2:



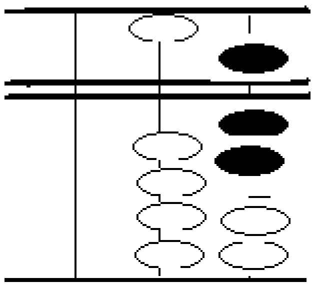
```
 321
+103
----
 424
```

In this abacus the top white beads represent 5, 50, or 500 depending on the place value. The bottom numbers represent 1, 10, and 100. Beads that are in use are touching the center horizontal pole.

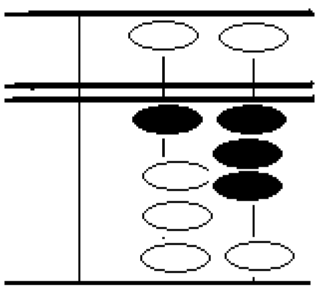
Addition with renaming: To rename with the abacus we must first understand the concept of 10 complements (the combinations that add up to the number 10). When adding 7 plus 6 we know that the answer will be greater than 10 (because 7+ 3 = 10). On the abacus, once ten of a unit is reached we must move a bead in the next greatest place value (the rod to the left) up to the counting bar. In this example we do not want to add a total of 10 units but only part, 6 units. Knowing this we must think of the number 6 as it relates to 10. Another way to make 6 is 10 -4. Using (10-4) in place of 6 we can solve the problem on the abacus as 7 + (10-4).

Example 3:

7+6 =



Add 10 subtract 4. (Subtract 4 by subtracting 5 and adding 1.)

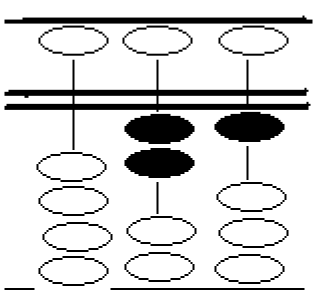


7+6= 13

Subtraction

To rename or "trade" in subtraction we must also relate numbers to 10. In the example 22-5 we convert the 5 (subtrahend) to a number that is related to 10. By subtracting 10 units and then adding 5 units, the result is the same as subtracting 5. Mathematically this would be -10 + 5 = -5. Although it may seem advanced to deal with negative and positive numbers, this process is a very simple manipulation on the abacus.

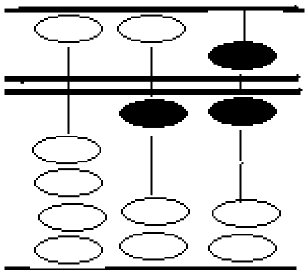
Example 4:



21 -5 = 16

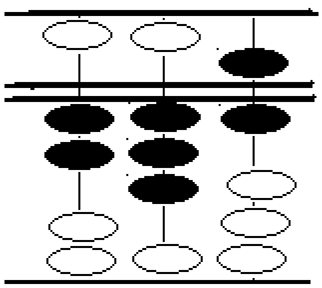
To solve this problem we must look at the 10 column. Subtract 10

(-10) and add 5 to the ones place. -10 + 5 = -5

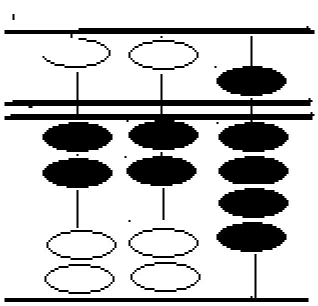


Example 5

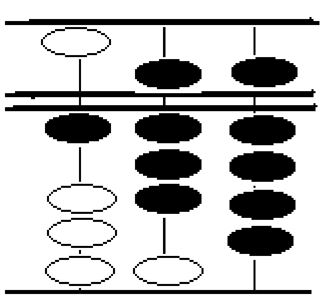
236 - 147 =



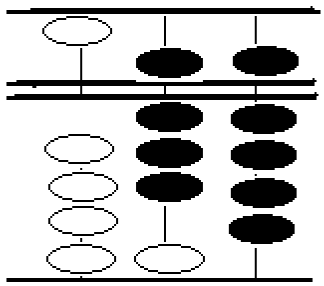
To solve this subtraction problem we need to trade in a tens bead in order to subtract the 7. Slide down a ten (-10) and add 3 to the ones rod.



Now we subtract the 4 tens from the 3 tens. To do this trade the hundreds. Slide down a hundred (-100) and add 6 to the tens rod (+60).



Now subtract the hundreds rod to find the answer.



236 - 147 = 89

Making a Japanese Soroban Abacus

There are many sites on the web to purchase a Soroban Abacus. One such site is: http://www.aph.org/products/abacus_cranmer.html . These look very nice but the cost is not practical for a classroom set. The author suggests making your own. It will be low on the budget and most supplies can be found in craft stores, discount houses, or home improvement suppliers. Because you will be using hot glue, you may wish to make these yourself or ask parent volunteers to help.Supplies:

(2) 8 inch pieces of corner molding.

(7) $\frac{3}{16}$ inch dowel rods. These need to be cut in sections of about 6-8 inches.

Make sure that the beads will move easily on the dowels!

(1) $\frac{1}{4}$ inch dowel rod. Cut about 8 inches long.

(7) beads of the same color. These beads must have holes through the center.

(28) Multi-color beads. These beads must have holes through the center.

Assembly: Place 5 beads on each of the $\frac{3}{16}$ inch rods. Make sure to place 4 of the multi-color and one bead that will be the same color at the top of each stick. The bead that is the same color will represent the fives. Hot glue the tops and bottoms of these dowels to the corner molding. Space the sticks about one inch apart. After all seven of the beaded dowels are glued to the corner molding, glue the $\frac{1}{4}$ inch dowel about 2 inches down from the top of the corner molding. Make sure to keep the five digit beads (the same color beads) above this dowel.

Abacus Class Exercises A

Using your abacus to solve the following problems.

1. 321
 +103
----

2. 5,724
 + 160
----

3. 623
 + 364
----

4. 3,425
 + 2,243
----

5. 796
 + 482

Abacus Independent Exercises A

Using your abacus to solve the following problems.

1. 47,836

+78,892

----

2. 1,346

4,120

+ 2,012

----

3. 3,079

5,413

+ 862

----

4. 93,467

+5,204

----

5. 24,836

+ 62,045

31,907

Abacus Class Exercises B

Using your abacus to solve the following problems.

1. 7,458
 +2,127
----

2. 4,069
 5,293
 +316
----

3. 40,735
 13,802
 + 24,083
----

4. 68,319
 59,628
+791,275
----

5. 7,185
 - 4,361

Abacus Independent Exercises B

Use your abacus to solve the following problems.

1. 5,820
   - 3,913
----

2. 29,037
 - 13,648
----

3. 735
   +462
   -133
----

4. 268
  + 830
   -724
----

5. 2,847
     -301
     -459

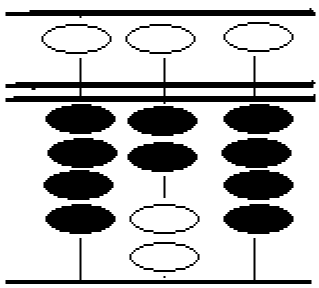
Answer Key

Abacus Class Exercise

Use your abacus to solve the following problems.

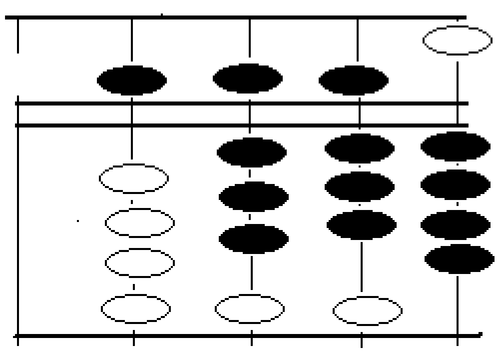
1. 321

+103



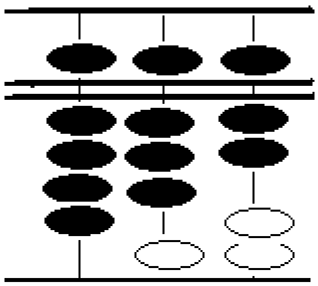
2. 5,724

\+ 160



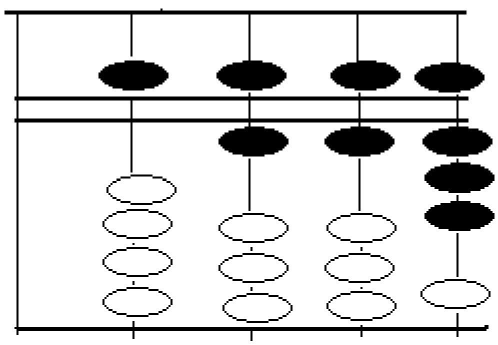
3. 623

\+ 364



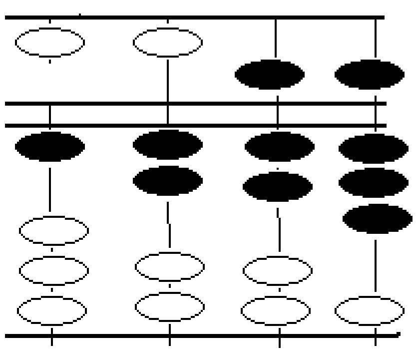
4. 3,425

 + 2,243



5. 796

 + 482

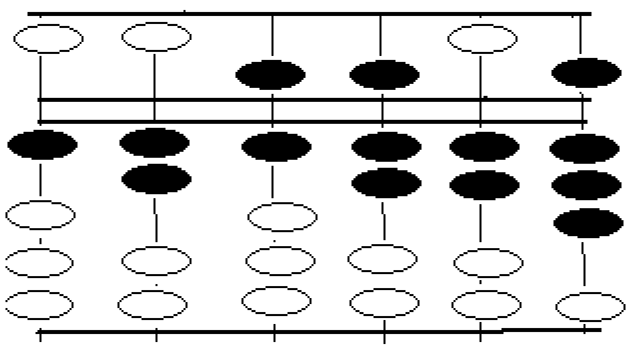


Answer Key

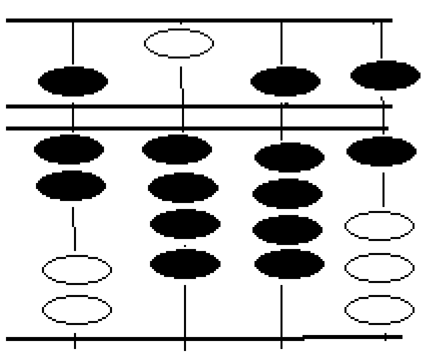
Abacus Independent Exercise

Use your abacus solve the following problems.

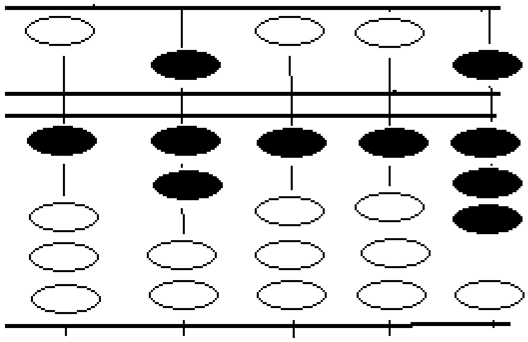
1. 47,836
 +78,892



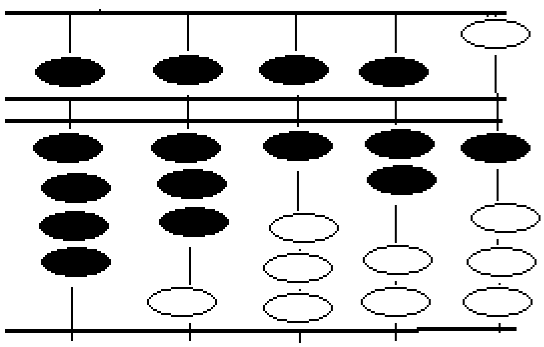
2. 1,364
 4,120
 + 2,012



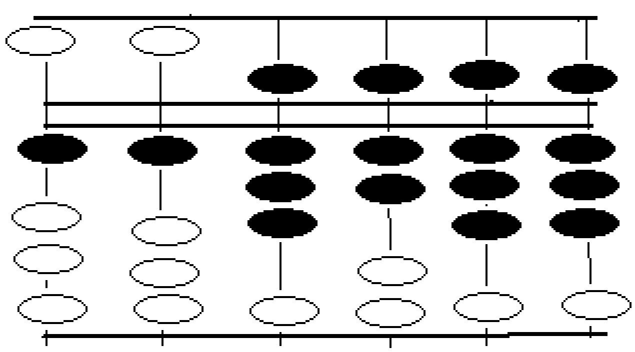
3. 3,079
 5,413
 + 8,626



4. 93,467
   +5,204



5. 24,836
 + 62,045
   31,907



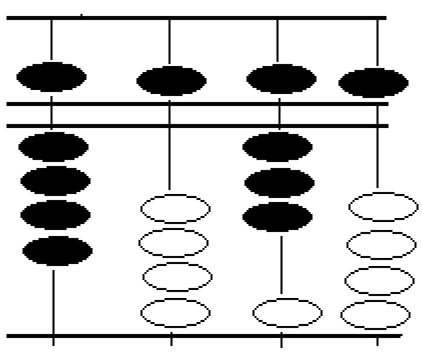
Answer Key

Abacus Class Exercise

Use your abacus solve the following problems.

1. 7,458

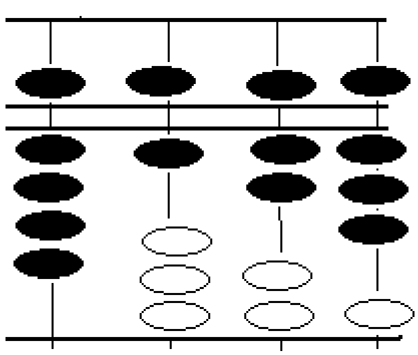
+2,127



2. 4,069

5,293

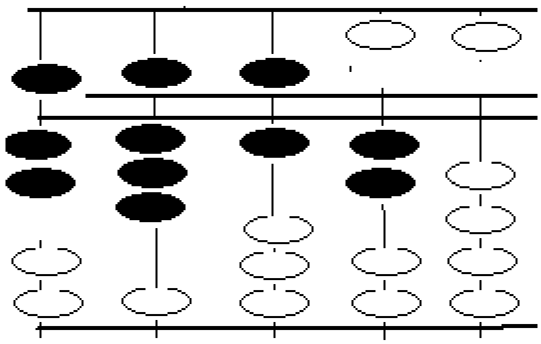
+316



3. 40,735

13,802

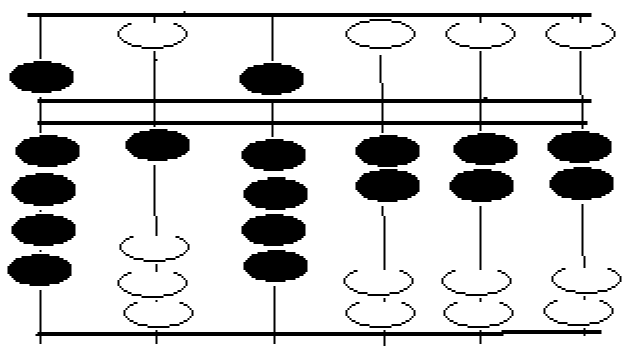
\+ 24,083



4. 68,319

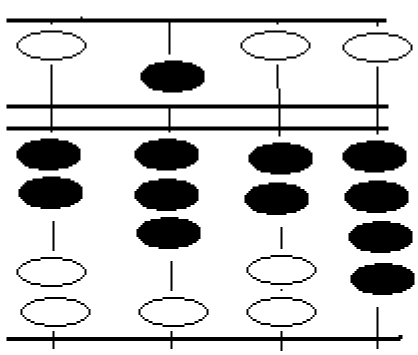
59,628

+791,275



5. 7,185

- 4,361



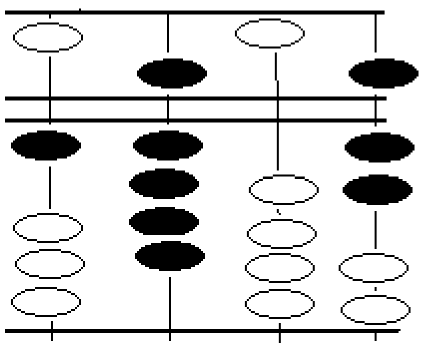
Answer Key

Abacus Independent Exercise

Use your abacus solve the following problems

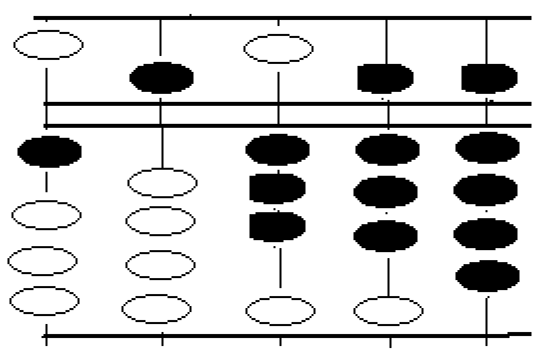
1. 5,820

   -3,913



2. 29,037

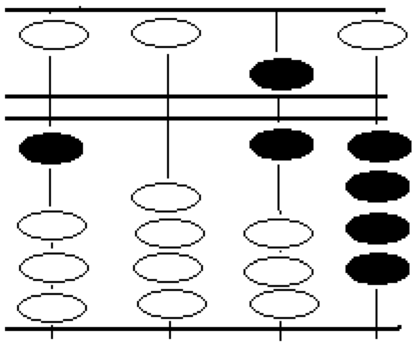
 - 13,648



3. 735

   +462

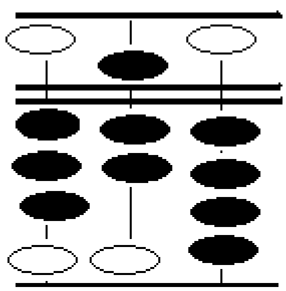
   -133



4. 268

+ 830

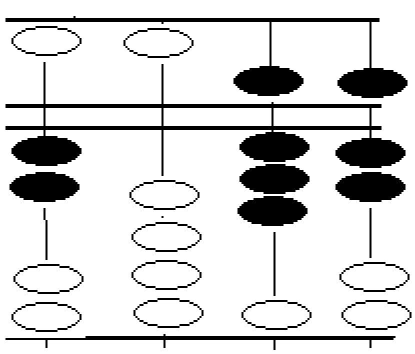
-724



5. 2,847

-301

-459



Suggested Classroom Web-sites

http://www.soroban.com/howto_eng.html (use the soroban by click/drag)

http://www.ee.ryerson.ca:8080/~elf/abacus/history.html (Wonderful history of the Abacus)
http://www.ee.ryerson.ca:8080/~elf/abacus/intro.html

http://fenris.net/~lizyoung/abacus.html

http://fusionanomaly.net/abacus.html (Worlds largest abacus)

http://webhome.idirect.com/~totton/abacus/pages.htm

http://www.mandarintools.com/abacus.html (examples of different types)

http://www.gis.net/~daveber/Abacus/Abacus.htm

http://infusion.allconet.org/virtual/Petronella/Practice.html

http://www.jimloy.com/arith/abacus.htm

http://www.typoscriptics.de/soroban/index.html

http://www.soroban.com/index_eng.html

http://www.ee.ryerson.ca:8080/~elf/abacus/

http://www.aph.org/products/abacus_cranmer.html (purchase)

CHAPTER 6

JOHN NAPIER

John Napier was a 16th century Scottish mathematician, philosopher, and inventor. He was the son of a wealthy judge and land owner. He discovered logarithms and decimal notation for complex fractions. Such mathematicians as Galileo, Kepler, and Newton were aided by his discoveries. (Hodges, 2000)

For decades Napier's superstitious neighbors grew convinced that he was involved in sorcery and witchcraft. His appearance in a long black gown to match his black beard only furthered their illusions. Walking around in a long black cloak with a large dog during a time of civil war in Scotland could draw attention!

He married Elizabeth Sterling in 1573 at Gartness, a family estate where he sought refuge from the civil war that raged around him. They had two children before her death in 1579. Later he married Agnes Chisholm and had five sons and five daughters. (Hodges, 2000)

He achieved great mathematical discoveries while Scotland around him wrestled through one of the most violent and turbulent periods of its history. His home town of Edinburg was embroiled in civil war and the plague. Napier was best known in his day for being a protestant theological author, who prophesied about the Apocalypse to prove the Pope was the Antichrist. (Hodges, 2000) John Napier is a fascinating Scotsman! Students should be encouraged to study his life!

Napier's Rods

John Napier's rods of 1617 (Napier Rechenstabchen), also known as Napier's bones, is one of his most important contributions to the world of mathematics. Along with William Ougthred's invention of the Slide Rule in 1615, it represents one of the most revolutionary advances in calculating devices since the abacus. (Hodges, 2000) The rods are basically multiplication tables inscribed on sticks of wood or bone. In addition to multiplication, the rods were also used in taking square roots and cube roots. Notice that the rods are just the multiplication tables for numbers 0-9. The rod to the extreme left has numbers 0 to 9. By placing the rods side by side you are able to multiply large numbers very easily.

So to multiply $9 \times 6{,}497$:

	6	4	9	7
1	0/6	0/4	0/9	0/7
2	1/2	0/8	1/8	1/4
3	1/8	1/2	2/7	2/1
4	2/4	1/6	3/6	2/8
5	3/0	2/0	4/5	3/5
6	3/6	2/4	5/4	4/2
7	4/2	2/8	6/3	4/9
8	4/8	3/2	7/2	5/4
9	5/4	3/6	8/1	6/3

When multiplying 9 ×6,000 go across from the 9 to see the answer is 54. The answer is of course 54,000. 9×400 go across from the 9 to see the answer is 36. Because 4 is in the hundreds place, it makes the answer 3,600. 9×90 go across from the 9 to the 9. Because 9 is in the tens place, it makes the answer 810. 9×7=63. To find the final answer add the third column of numbers (54,000+ 3,600 + 810 + 63 = 58,474). The answer can also be found by adding along the diagonals and carrying as needed.

9	5/4	3/6	8/1	6/3
5	8	4	7	3

9	X	6,000	=	54,000
9	X	400	=	3,600
9	X	90	=	810

9	X	7	=	+ 63
9	X	6,479	=	58,474

Making Napier's Rods

To make a set of Napier's Rods run copies of the following table on adhesive foam or glue to thicker paper. Cut these apart in columns and store in a zip-lock bag or envelope to use when needed.

1	2	3	4	5	6	7	8	9	
0/1	0/2	0/3	0/4	0/5	0/6	0/7	0/8	0/9	1
0/2	0/4	0/6	0/8	1/0	1/2	1/4	1/6	1/8	2
0/3	0/6	0/9	1/2	1/5	1/8	2/1	2/4	2/7	3
0/4	0/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6	4
0/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5	5
0/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	6
0/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3	7
0/8	1/6	2/4	3/2	4/0	4/8	5/4	6/4	7/2	8
1/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1	9

To make a more a permanent set of rods, enlarge and glue onto sticks. Or copy the charts with permanent marker onto sticks, wooden rulers, or paint stir sticks.

Another great way to make the rods is to use foam sheets with adhesive backs. Enlarge and copy the chart. Peel the adhesive back and stick. Cut with scissors or exacta knife.

Napier's Rods Exercises

Solve the following multiplication problems using Napier's Rods. Record your solutions showing the expanded form (5,400+630+81+2). Check your work at the following site: http://www.cee.hw.ac.uk/~greg/calculators/napier/great.html

1. 525 x 6 =

2. 678 x 4 =

3. 482 x 7=

4. 1,874 x 4 =

5. 4,589 x 26 =

Enrichment: Research John Napier. Find out about the story of the stolen rooster. Why did his neighbors think he was a sorcerer? What other interesting things did you learn about his life? Check out Napier's Chessboard.

http://www.electricscotland.com/history/other/john_napier.htm

http://www.comphist.org/HistoricMachines/Napier/Chessboard.multiply.html

Additional are links listed at the end of this section.

Napier's Rods Exercises

Answer Key

1. $525 \times 6 =$

	5	2	5
1	0/5	0/2	0/5
2	1/0	0/4	1/0
3	1/5	0/6	1/5
4	2/0	0/8	2/0
5	2/5	1/0	2/5
6	3/0	1/2	3/0
7	3/5	1/4	3/5
8	4/0	1/6	4/0
9	4/5	1/8	4/5

6	X	500	=	3,000
6	X	20	=	120
6	X	5	=	30
6	X	525	=	3,150

$525 \times 6 = 3{,}150$ or

6	3/0	1/2	3/0
3	3	5	0

2. 678 x 4

	6	7	8
1	0/6	0/7	0/8
2	1/2	1/4	1/6
3	1/8	2/1	2/4
4	2/4	2/8	3/2
5	3/0	3/5	4/0
6	3/6	4/2	4/8
7	4/2	4/9	5/6
8	4/8	5/4	6/4
9	5/4	6/3	7/2

4	X	600	=	2,400
4	X	70	=	280
4	X	8	=	32
4	X	678	=	2,712

678 x 4= 2,712 or

4	2/4	2/8	3/2
2	7	1	2

3. 482 x 7=

	4	8	2
1	0/4	0/8	0/2
2	0/8	1/6	0/4
3	1/2	2/4	0/6
4	1/6	3/2	0/8
5	2/0	4/0	1/0
6	2/4	4/8	1/2
7	2/8	5/6	1/4
8	3/2	6/4	1/6
9	3/6	7/2	1/8

7	X	400	=	2,800
7	X	80	=	560
7	X	2	=	14
7	X	482	=	3,374

482 x 7= 3,374 or

7	2/8	5/6	1/4
3	3	7	4

4. 1,874 x 4 =

	1	8	7	4
1	0/1	0/8	0/7	0/4
2	0/2	1/6	1/4	0/8
3	0/3	2/4	2/1	1/2
4	0/4	3/2	2/8	1/6
5	0/5	4/0	3/5	2/0
6	0/6	4/8	4/2	2/4
7	0/7	5/6	4/9	2/8
8	0/8	6/4	5/4	3/2
9	1/0	7/2	6/3	3/6

4	X	1,000	=	4,000
4	X	800	=	3,200
4	X	70	=	280
4	X	4	=	16
4	X	1,874	=	7,496

1,874 x 4 = 7,496

4	0/4	3/2	2/8	1/6
	7	4	9	6

5. 4,589 x 26 =

		4	5	8	9
1	1	0/4	0/5	0/8	0/9
2	2	0/8	1/0	1/6	1/8
3	3	1/2	1/5	2/4	2/7
4	4	1/6	2/0	3/2	3/6
5	5	2/0	2/5	4/0	4/5
6	6	2/4	3/0	4/8	5/4
7	7	2/8	3/5	5/6	6/3
8	8	3/2	4/0	6/4	7/2
9	9	3/6	4/5	7/2	8/1

20	X	4,000	=	80,000
20	X	500	=	10,000
20	X	80	=	1,600
20	X	9	=	180
6	X	4,000	=	24,000
6	X	500	=	3,000
6	X	80	=	480
6	X	9	=	54
6	X	4,589	=	119,314

4,589 x 26 = 113,314

Answers may be entered and checked on line at:

http://www.cee.hw.ac.uk/~greg/calculators/napier/great.html

It is worth noting again that many students struggle with multiplication facts. This method of multiplying is a great way to reinforce multiplication facts and reduce the stress of the multiplication algorithm. Napier's Rods have great benefit for the special education student because of the ability to concentrate on the parts of the multiplication problem without the constant frustration of returning to a table of facts.

Napier's fascinating life can't be overlooked. It is believed that students will be greatly motivated by the strange behavior of this extraordinary man. The author suggests that students dig into the life of John Napier and report on the various stories that make the man both famous and elusive. Perhaps students could gain a great deal by acting out various stories from his life.

Suggested Classroom Web-sites

http://www.cee.hw.ac.uk/~greg/calculators/napier/great.html (on-line calculations)

http://www.electricscotland.com/history/other/john_napier.htm

http://www.ualr.edu/%7Elasmoller/napier.html

http://infohost.nmt.edu/~borchers/napier/napier.html (also square and cube roots)

http://www.comphist.org/HistoricMachines/Napier/Bones.html

http://www.electricscotland.com/history/other/john_napier.htm

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Napier.html

http://www.johnnapier.com/

CHAPTER 7

COMMERCIAL APPLICATION
TREVISO ARITHMETIC 1478

The next section of this document is devoted to some of the applications found in the *Treviso Arithmetic*. It is important to note that the entire text of the Treviso is written as a practical book on arithmetic. This book was designed to support the need of a growing commercial society in Italy. The problems were a reflection of situations that merchants would face in the 15th century. Many of the problems have great problem-solving benefits for middle school students. (Swetz, 1987)

The applications of the *Treviso Arithmetic* can be found in Frank Swetz's 1987 translation (chapter 6). They are listed as the following eight sections:

1. The Rule of Three
2. Tare and Tret
3. Partnership
4. Barter
5. Alligation
6. Rule of Two
7. Pursuit
8. Calendar Reckoning.

This unit will address four of the eight principles featured in the Treviso. While these problems were commonplace in Italy during the fifteenth century they are certainly non-routine to sixth graders. This is precisely why they have such tremendous potential. They provide the element of great problem solving, understanding what the problem is asking, and not seeing an immediate answer. All levels of learners will find some seed of learning here. Not only do they provide problem solving at many levels, but they provide a vision of the culture of 15th century Italy. Mathematics is more than a pencil and paper task; rather, it becomes a powerful tool to solve the situations that arose in the Renaissance.

The Rule of Three

This mathematical technique pops up in many societies dating back as far as the *Rhind Papyrus*. It has had many titles, such as Merchant's Rule, Backer Rule, Chiu chang suan shu, and the Golden Rule as it was referred in Europe. Whatever the name, we will be most accustomed to looking at it as a ratio. It is a method of obtaining a piece of information if we know three of the other pieces of information. (Swetz, 1987)

Ratio as used today $\frac{a}{b} = \frac{c}{d}$ or d = $\frac{cb}{a}$

Example: $\frac{3}{9} = \frac{2}{x}$. We have x= 6. We find x by multiplying and dividing.

This rule in the Treviso is presented for the standpoint of commercial use by reckoning clerks. They, of course, are not interested in the proof of the mathematics, so the Treviso lacks explanations. The Treviso gives the following abstract for the Rule of Three (page 224-232). For the following a, b, and c, multiply b and c, then divide by a.

a b c 1. b x c

2. b x c/a

Example: 3 9 2 1. 9 x 2

2. 9 x 2 / 3

The resulting quotient will provide us with the x we seek. In this case 6. (224-232)

Rule of Three Class Exercises

1. If 10 men can dig a trench in 4 days, how many days will 7 men take to dig the trench?

2. If 5 Venetian glass blowers can make 100 glass containers, how long will it take for 10 glass blowers to make the bottles?

3. If 4 pounds of brass cost 36 grossi, what will 12 pounds of brass cost?

4. If a 2 braccia of cloth are worth 5 grossi and you have 1 ducat, how much cloth can you buy? (1 ducat = 24 grossi).

Tare and Tret

Although we don't use this exact term in the Twenty-first Century, Tare and Tret was the principle used to determine the cost of shipping paid by a merchant of Venice. The transportation of merchandise involved large casks, barrels, and containers. Tare is the determination of the weight of a cargo with a deduction made for the weight of the container that carried it. (Tare might be 4 pounds per hundredweight. The word tare evolved from the Arabic traha, which is to throw away.) The Treviso referred to this weight as tara. Tret comes from the Italian tratto, allowance for transportation, and actually was an allowance, usually in money, made to a buyer of certain goods for waste, damage, or deterioration during the shipping. (Swetz, 1987)

In this problem the reader is required to find the total tare allowed on 4562 pounds at 4 pounds per hundredweight. This is the allowed deduction for the shipping cost. This problem would be a great way to bring in percent to a classroom as seen in the following explanation.

The Treviso taught to multiply 4,562 by 4 to get 18,248. Then divide the result by 100 giving an answer of 182.48.

The suggestion is made that many students would feel more comfortable in first finding the part of the shipment deduction by dividing 4,562 by 100 equaling 45.62. Then multiply 45.62 by 4 to find the discount of 182.48. The Treviso translator (Swetz, 1987) suggests that the phrase by the hundred or hundredweight in the Italian was per cento or percent. (232-234)

The author also reminds the reader that the use of decimals as we know them today would have been unavailable to the reader of the Treviso Arithmetic. While it is hard to pinpoint the exact origin of decimals, many mathematicians feel that decimals were used in ancient China, medieval Arabia, and Renaissance Europe. In the 1500s researchers and mathematicians were using some form of decimals. However, most agree that Simon Stevin of Bruges made the idea of decimals understandable to the common people. His system dealt with tenths, hundredths, and thousandths. Stevin's form of decimals was improved upon by John Napier who used the decimal point to separate the whole number from the decimal part. Students could benefit from research into the history of the decimal point. Here is another opportunity for enrichment.

When solving the problems in this section it may be beneficial to allow students to solve some of these applications using the method of fractions. The author believes that it will certainly provide the student with great appreciation for the decimal point. The author does feel that if a student is overburdened by the use of fractions then the decimal point is appropriate.

Tare and Tret Class Exercises

1. Find the total tare allowed on 6,578 pounds at 5 pounds per hundredweight. This is the allowed deduction for the shipping cost.

2. Find the total tare allowed on 18,679 pounds at 6 pounds per hundredweight. This is the allowed deduction for the shipping cost.

3. Find the total tare allowed on 7,813 pounds at 4 pounds per hundredweight. This is the allowed deduction for the shipping cost.

4. Find the total tare allowed on 9,345 pounds at 7 pounds per hundredweight. This is the allowed deduction for the shipping cost.

Partnership (Rule of Fellowship)

Partnership was the principle of a mercantile institution by which a person could lend money. The actual lending of money as we know it today would never have been allowed by the church in Italy at the time of the Treviso. Merchants did find a way around this sticky point by using a contract of partnership. A merchant would earn capital by investing in a business venture and expect to be paid out of the profits an agreed percent based on the total of his investment. The Treviso explains this plan in two ways: all money grows at the same rate without consideration of time, and with consideration of time. (Swetz, 1987)

The banking practices of the Italian Renaissance are considered by many in the business world to be a building block in the evolution of our present-day stock market. A brief history of commercial Italy during the renaissance is at: http://www.learner.org/exhibits/renaissance/florence.html. Many sites have very in-depth information on banking during the renaissance. The study of the stock market is a Virginia Standard of learning in Social Studies for sixth grade. (VDOE, 2005)

This lesson on profit sharing is from the Treviso Arithmetic (pp.234-238):

Two merchants, Sebastiano and Jacomo, enter into a partnership. Sebastiano put in 350 ducats on the first day of January, 1472. Jacomo put in 500 ducats 14 grossi on the first day of July, 1472. On the 1st day of January 1474 they find that they have gained 622 ducats. How do they determine their share of the partnership?

Thus Sebastiano puts up 350 ducats for a period of two years, and Jacomo invests 500 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided?

This will make an excellent problem for the middle school, because it is easy to understand, yet complex in its steps. It also gets the students thinking about consumer applications, which are a Virginia Standard.

1. To work this problem a student needs a lesson on currency conversion. It will help to place all monetary amounts in the same denomination. We should begin by converting ducats to grossi. It is known that 1 ducat = 24 grossi, while 1 grossi = 32 piccoli. Some problems on these types of conversions will be helpful.

Sebastiano's 350 ducats = 8,400 grossi

Jacomo's 500 ducats, 14 grossi = 12,014 grossi

2. Each share is multiplied by the time of the investment period.

Sebastiano's 8,400 grossi x 24 months = 201,600

Jacomo's 12,014 grossi x 18 months = 216,252

3. The total shares in terms of grossi are 417,852.

4. The Rule of Three again is used to complete the problem

Sebastiano's profit: $\frac{201{,}600}{417{,}852} = \frac{x}{622}$. By multiplying and dividing we find his share of the profit is 300.10 ducats.

Students may also write the profit in the other previously mentioned currencies.

5. Jacomo's profit: $\frac{216{,}252}{417{,}852} = \frac{x}{622}$. Again by multiplying and dividing we find his share of the profit to be 321.90 ducats.

6. The calculations can be checked by adding the two profits together to see if they equal the total profit: 621.90 + 300.10 = 622.

Decimals were not known at the time the _Treviso Arithmetic_ was written, so it is interesting to see how the Sebastiano and Jacomo problem was originally solved.

Two merchants, Sebastiano and Jacomo, enter into a partnership. Sebastiano put in 350 ducats on the first day of January 1472. Jacomo put in 500 ducats 14 grossi on the first day of July 1472. On the 1st day of January 1474 they find that they have gained 622 ducats. How do they determine their share of the partnership?

Let us recall that Sebastiano has contributed 8,400 grossi for 24 months, while Jacomo has contributed 12,014 grossi for 18 months. Then Sebastiano's contribution amounts to 8,400 x 24 = 201,600 while Jacomo's is 12,014 x 18 = 216,252. We have that 201,600 + 216,252 = 417,852.

Now we can establish the proportion:

$$\frac{S}{622} = \frac{201{,}600}{417{,}852}$$

where S denotes the share of Sebastiano. Then:

S= $\frac{622x201{,}600}{417{,}852}$ = 300 + $\frac{39{,}600}{417{,}852}$ ducats. But $\frac{39{,}600}{417{,}852}$ ducats =

$2 \text{ grossi} + \dfrac{114{,}696}{417{,}852} \text{grossi, while } \dfrac{114{,}696}{417{,}852} \text{grossi} = \dfrac{3{,}670{,}272}{417{,}852} \text{ piccoli} =$

$8 \text{ piccoli} + \dfrac{327{,}456}{417{,}852}$ piccoli. So S = 300 ducats, 2 grossi, and $8\dfrac{324{,}456}{417{,}852}$ piccoli.

In similar fashion we reach J = 321 ducats, 21 grossi, $13\ \dfrac{90{,}396}{417{,}852}$ piccolo.

(1 ducat = 24 grossi, 1 grossi = 32 piccoli)

Class Partnership Problem

State answers in decimals.

1. Sebastiano puts up 250 ducats for a period of two years, and Jacomo invests 400 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided?

2. Sebastiano puts up 100 ducats for a period of two years, and Jacomo invests 500 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided?

3. Sebastiano puts up 350 ducats for a period of two years, and Jacomo invests 350 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided?

Enrichment Activities:

1. Divide the class into groups to discuss and report the answers to these problems.
2. Set up classroom partnerships and determine the profit. Use monopoly money to make up partnerships that reflect profits from a business that the students create.
3. Research the history and development of the present stock market and how it relates to commercial Renaissance Italy. Trace the idea of “shares” and “investment partnerships”. How does this relate to ownership in current companies in the S & P 500.

4. After working these class exercises did you notice a relationship between the amount invested and the time of investment? Which increases profit growth the most: the time of investment or the amount of shares? Is it ever better to invest more money for a shorter time? Explain.

The Rule of Two

The Rule of Two is given in many situations old and new to show a ratio that will reflect the product of two numbers divided by a third. It sounds like a close cousin to the Rule of Three.

In the Treviso (243-247) we are given that a courier embarks to Rome from Venice and intends to be there in 9 days. At the same time, a messenger from Rome is traveling to Venice and will reach his destination in 7 days. We are further told that the distance between the cities is 250 miles, and asked in how many days these travelers will meet. Let us solve the problem.

The Rome to Venice courier is traveling at a rate of $\frac{250}{7}$ m/d.

The Venice to Rome courier is traveling at a rate of $\frac{250}{9}$ m/d.

Because they will meet in a fixed number of days, d:

$$\frac{250}{7}d + \frac{250}{9}d = 250$$

Note that $\frac{250}{7}d$ is the distance traveled by one courier in d days, while $\frac{250}{9}d$ is the distance traveled by the other courier in d days. Thus

$$d\left(\frac{1}{7}+\frac{1}{9}\right) = 1$$

Hence $d\frac{16}{63} = 1$

$$d = \frac{63}{16}$$

So $d = 3\frac{15}{16}$ days

A middle school student might have difficulty understanding this approach right away, so it is advisable to draw a picture and make a chart of the distances that the couriers cover.

Working from the concrete first and then going into the mathematics behind it would be a greater benefit. Here again it would be wise to allow the students to think quietly at their seats for several minutes, making notes and drawing their own conclusions. After this reflective thinking, pair or group the students to work together. Remind the students of the amount of time they will have to complete the problem. Also require that the student groups have a suggested strategy/answer to report to the class in this given amount of time (Not more than 40 minutes).

Possible solutions will include but are not limited to:

1. Draw a picture.
2. Make a chart:

Example Chart

Day	Venice Courier	Rome Courier	Total Miles
1	35.7	27.77	63.47
2	71.4	55.54	126.94
3	107.1	83.31	109.41
4	142.8	111.08	253.88

The couriers will obviously meet sometime between days 3 and 4. Students should be able to draw the conclusion that the actual meeting will be close to the end of day 3.

Again allow group work and collaboration among the students; this can become noisy, but it is often very valuable! Let students explain their methods to the class.

One approach might use a proportion, as follows: let us recall that Velocity = distance/time. Let x be the distance traveled by the courier from Venice to Rome. They will meet when respective times are equal. Then $\dfrac{x}{\frac{250}{9}} = \dfrac{250-x}{\frac{250}{7}}$. So x = $\dfrac{1{,}750}{16}$. Thus

time = $\dfrac{\frac{1{,}750}{16}}{\frac{250}{9}} = \dfrac{63}{13} = 3.9375$ or $3\ \dfrac{15}{16}$

Other problems should be made available to the students, or they should make their own problems. For instance, consider two students walking to the library from different locations. Set up a ratio of the number of steps to be covered. Allow students to measure the distances, draw pictures, charts, and experiment with their results.

Answer Key

Rule of Three Class Exercises

1. If 10 men can dig a trench in 4 days how many days will 7 men take to dig the trench.

$$\frac{10}{4} = \frac{7}{x}$$

x = 2.8 or $2\frac{4}{5}$

2. If 5 Venetian glass blowers can make 100 glass bottles, how long will it take for 10 glass blowers to make the bottles.

$$\frac{5}{100} = \frac{10}{x}$$

x = 200

3. If 4 pounds of brass cost 36 grossi, what will 12 pounds of brass cost?

$$\frac{4}{36} = \frac{12}{x}$$

x = 108

4. If a 2 braccia of cloth are worth 5 grossi and you have 1 ducat, how much cloth can you buy? (1 ducat = 24 grossi).

$$\frac{2}{5} = \frac{x}{24}$$

x = 9.6 or $9\frac{3}{5}$

Answer Key

Tare and Tret Class Exercises

1. Find the total tare allowed on 6,578 pounds at 5 pounds per hundredweight. This is the allowed deduction for the shipping cost.

$$\left(\frac{6{,}578}{100}\right)\times 5 = 328.9$$

or $328\frac{9}{10}$

2. Find the total tare allowed on 18,679 pounds at 6 pounds per hundredweight. This is the allowed deduction for the shipping cost.

$$\left(\frac{18{,}679}{100}\right)\times 6 = 1{,}120.74$$

or $1{,}120\frac{37}{50}$

3. Find the total tare allowed on 7,813 pounds at 4 pounds per hundredweight. This is the allowed deduction for the shipping cost.

$$\left(\frac{7{,}813}{100}\right)\times 4 = 312.52$$

or $312\frac{13}{25}$

4. Find the total tare allowed on 9,345 pounds at 7 pounds per hundredweight. This is the allowed deduction for the shipping cost.

$$\left(\frac{9{,}345}{100}\right)\times 7 = 654.15$$

or $645\frac{3}{20}$

Answer Key

Partnership Class Exercises

1. Sebastiano puts up 250 ducats for a period of two years, and Jacomo invests 400 ducats, 14 grossi, for a period of 18 months; how should their profit of 622 ducats be divided?

1. Investment:

Sebastiano: 250 ducats = 6,000 grossi

Jacomo: 440 ducats + 14 grossi = 9614 grossi

2. Shares:

6,000 x 24 months = 144,000 shares

9,614 x 18 months = 173,052 shares

3. Total shares:

144,000 + 173,052 = 317,052 shares

4. Sebastiano's Profit:

$$\frac{144{,}000}{317{,}052} = \frac{x}{622} = 282.5 \text{ or } 282\ \frac{4{,}426}{8{,}807}$$

Jacomo's Profit:

$$\frac{173{,}052}{317{,}052} = \frac{x}{622} = 339.5 \text{ or } 339\frac{4{,}381}{8{,}807}$$

5. Check: 339.5+282.5 = 622

2. Sebastiano puts up 100 ducats for a period of two years, and Jacomo invests 500 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided.

1. Investment:

Sebastiano: 100 ducats = 2,400 grossi

Jacomo: 500 ducats + 14 grossi = 12,014 grossi

2. Shares:

2,400 x 24 months = 57,600 shares

12,014 x 18 months = 216,252 shares

3. Total shares:

57,600 + 216,252 = 273,852 shares

4. Sebastiano's Profit:

$$\frac{57{,}600}{273{,}852} = \frac{x}{622} = 130.83 \text{ or } 130\ \frac{6{,}290}{7{,}607}$$

Jacomo's Profit:

$$\frac{216{,}252}{273{,}852} = \frac{x}{622} = 491.17 \text{ or } 491\frac{1{,}317}{7{,}607}$$

5. Check: 130.83 + 491.17 = 622

3. Sebastiano puts up 350 ducats for a period of two years, and Jacomo invests 350 ducats, 14 grossi, for a period of 18 months: how should their profit of 622 ducats be divided.

1. Investment:

Sebastiano: 350 ducats = 8,400 grossi

Jacomo: 350 ducats + 14 grossi = 8,414 grossi

2. Shares:

8,400 x 24 months = 201,600 shares

8,414 x 18 months = 151,452 shares

3. Total shares:

210,600 + 151,452 = 353,052 shares

4. Sebastiano's Profit:

$$\frac{201{,}600}{353{,}052} = \frac{x}{622} = 355.17 \text{ or } 355\ \frac{1{,}715}{9{,}807}$$

Jacomo's Profit:

$$\frac{151{,}452}{353{,}052} = \frac{x}{622} = 266.83 \text{ or } 266\frac{8{,}092}{9{,}807}$$

5. Check: 266.83 + 355.17 = 622

Suggested Classroom Web-sites

http://www.d.umn.edu/~aroos/jensen2.htm

http://www.wsu.edu:8080/~dee/REN/BACK.HTM

http://www.metmuseum.org/toah/ht/08/eustc/ht08eustc.htm

http://www.learner.org/exhibits/renaissance/florence.html

http://en.wikipedia.org/wiki/Italian_Renaissance

http://extranet.edfac.unimelb.edu.au/DSME/decimals/SLIMversion/backinfo/metric.shtml#theev
o

http://www.computersmiths.com/chineseinvention/decimal.htm

CHAPTER 8

SUMMARY

The goal of this thesis has been to encourage the teaching of historical mathematics in middle school mathematics. It is certainly a proper and fitting course for this age group. The situation currently in mathematics is often teaching in isolation with no consideration of the unity of mathematics and the civilization of its origin. That means little if any thought is given to how mathematics is shaped by a culture, or how it drives the development of ancient or modern nations. Mathematics when taught in isolation is a subject of algorithms, rules of operations, and formulas that are only memorized for a short time in order to pass a standardized test.

The implementation of history of mathematics into a curriculum can be achieved in two ways. First it can be accomplished by the teaching of original source documents, as shown by the *Treviso Arithmetic* and the *Rhind Papyrus*. The second way is to choose topics that fit into the curriculum such as the abacus or Napier's rods. Either of these ways is effective. Using original source documents does require more preparation and some understanding of the civilization at the time of the document. It is a great way to teach students how mathematics played a key role in the formation of a society. Short historical stories and references in mathematics during the ongoing teaching takes less preparation yet will keep the students thinking about the mathematics they are learning and enhance the understanding of mathematics as a vital tool in learning and understanding.

Mathematics teachers need to be trained in the topic of history of mathematics. Currently it is the author's experience that the training is sparse. It usually consists of courses taught by professors who have a personal interest in the topic and is not a required area of study. While there is a vast number of resources that are wonderful and usually inexpensive, many educators do not realize the significance of the topic. Unfortunately, classroom teachers, who have little time to independently pursue this area of study, may view it as "fluff", consequently believing that it is not necessary to their curriculum. Educators must be encouraged by other professionals to seek out the historical connections that relate to their standards. The encouragement from college mathematics professors in forming partnerships with classroom teachers will enhance the historical component in mathematics.

Another wonderful place to add teacher training has been mathematical conferences and county in-services. Since the implementation of the Standards of Learning in Virginia, addressing the historical topics in mathematics has almost vanished. Its worth cannot be evaluated on a multiple choice state test and is; therefore, not considered worthy of the time and effort. Sadly, we are moving away from thematic units that develop and incorporate this topic into other subject areas.

If standardized testing continues to drive our curriculum, historical topics in mathematics will be overlooked. The swing of the pendulum over the last 20 years has pushed educators to use drill and practice of algorithms because this drives up scores. Most devoted instructors welcome evaluation. Evaluation is a necessary part of learning for everyone. The point in mathematics being: scores are wonderful, but true understanding of a topic is priceless. Mathematics isn't only a set of rules memorized for a test; it is a tool used by every society past and present to achieve their needs. The author encourages other educators to be brave enough to add this topic to their teaching.

The teaching of historical mathematics will deepen the students' understanding of mathematics and its role in civilizations. It is best when taught along with the current curriculum. It is not a cure for fear of mathematics; however, it does remove some of the mystery. It is a fantastic way to provide differentiation of instruction in the classroom. There are limitless opportunities to provide gifted and talented students with enrichment activities as well as to provide many opportunities for students with learning differences to understand mathematics.

Teaching historical mathematics is great fun in middle school. Students are curious learners who are often bored with the traditional teaching and respond well to studying math from this perspective. Beginning to collect and read historical references will only excite a classroom. Educators who constantly seek out ways to motivate students to learn will love this topic! Teaching of historical mathematics in middle school will serve to ignite mathematical classrooms. It is exciting to the teacher as well as the student. Seeing ourselves as part of a greater world developing over thousands of years inspires students to learn and impacts them with the love of discovery. An inspired learner is a life long learner. A curious mind looks for answers and does not wait on an instructor to hand them a rule.

This thesis has focused chiefly on the *Rhind Papyrus*, the *Treviso Arithmetic*, the Abacus, and Napier's Rods as examples of early materials that can easily be incorporated into existing middle-school curriculum. There are hundreds of materials that could be used with the current curriculum. Designing classrooms around these and other topics will deliver liveliness to students, which is contagious to the teacher and all those nearby. It is a wonderful way to bring life to a dead curriculum and the gift of seeing beyond oneself into a greater realm.

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