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Probability for the Fifth Grade Classroom.

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Probability for the Fifth Grade Classroom

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by

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ABSTRACT

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The purpose of this thesis was to thoroughly develop probability objectives to be used by a fifth grade teacher. These probability objectives were developed in four main units. The focus of the first unit was probability vocabulary. The second unit explored the concept of fairness as determined by the probability of winning a game. The third unit's purpose was to determine sample space using tree diagrams, lists, and the Counting Principle. Unit four was designed to help the student write theoretical and experimental probability as fractions, decimals, and percentages. Each unit was written to include detailed descriptions, definitions, and probability activities that can be used in a fifth grade classroom.

DEDICATION

This thesis is dedicated to my family for their support throughout this entire process. My mom, Brenda Ray, my husband, Roger Young, and my children, Dustin, Sarah, and Chris have all made many sacrifices during my graduate studies. I will always cherish their love and encouragement. I love you all!

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1 WHAT IS PROBABILITY?

1.1 Introduction

Mathematics instruction at the fifth grade level should deepen the students' understanding of the concepts of probability. This thesis is intended to be used as a teacher's resource. It will provide detailed descriptions, definitions, and probability activities that can be used in a fifth grade classroom. The probability objectives that will be developed in the following units are the objectives that are included in the fifth grade Virginia Mathematics Standards of Learning [3]. The specific fifth grade mathematics standards addressed within this thesis are sections a, b, and c of objective 5.17. The Standards of Learning were developed by the state of Virginia to inform every one of the essential objectives that should be taught at each grade level.

This first unit will cover the basic terminology that a fifth grader needs to know in order to learn about probability. Because this thesis will focus on probability at a fifth grade level, we will use a very elementary set of terms. The following terms will be used throughout all of the probability units. Therefore, it is of utmost importance that every student demonstrates a working knowledge of probability vocabulary in order to understand and be able to actively participate in all of the following units. We will start by loosely defining probability as the chance that an event will occur. The two main types of probability that we will be using are experimental probability (also known as empirical probability) and theoretical probability. Experimental probability is found when an experiment is performed a large number of times. It is an estimate of the chance that an event will occur based on how often the chosen event occurs in relation to how many times the experiment was conducted.

$$\text{Experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of repetitions of the experiment}}$$

Theoretical probability is the number of favorable outcomes in relation to the total number of possible, equally likely outcomes. The equally likely hypothesis is critical in this case. Theoretical probability is what you expect to happen based on all of the possible outcomes. An experiment has not actually been performed.

$$\text{Theoretical probability} = \frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}}$$

These two main terms lead us to the need to define several other words. An experiment is a controlled test conducted to observe outcomes. The actual outcome is not known in advance, but the set of possible outcomes is known. An event is a collection of one or more outcomes. An outcome is a result in an experiment. Possible outcomes are all of the results that have a chance of happening in an experiment. Favorable outcomes are only the results that lead to occurrence of the event in question. Equally likely outcomes are results that have an equal chance of happening. We should stop here to illustrate the terms that have already been introduced. Figure 1 is a group of seven black balls that are exactly the same size, shape, and texture. The only difference is that four of the balls have gray paw prints and three of the balls have white paw prints. If these seven balls are placed in a paper bag and you are asked to draw one of them out without looking, what is the probability that the one that you pick will have a white paw print on it?

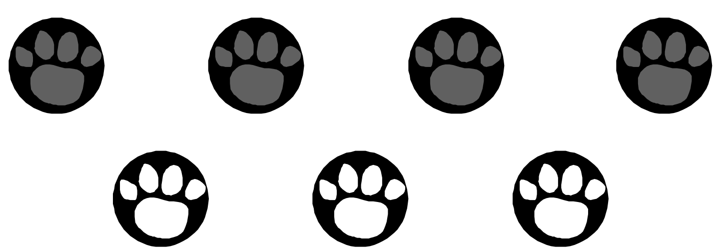


Figure 1: Balls with paw prints

If we answer this question without actually drawing any balls out of a bag (doing the experiment) our answer will be theoretical probability. First, we find the number of favorable outcomes (remember a favorable outcome is what we want to get, i.e. corresponds to the event in question). In this case we want a ball with a white paw print, so there are three favorable outcomes. Then, we find the total number of equally likely outcomes, and since all of the balls are exactly the same size, shape, and texture there are a total of seven equally likely outcomes. Therefore, the theoretical probability is 3 out of 7 picks from the bag the ball will have a white paw print on it. This can also be written as a fraction:

$$P(\text{white}) = \frac{3}{7} = \frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}}$$

If we want to test this theoretical probability to see if this is actually what happens we will conduct an experiment by drawing a ball from the bag and recording the actual outcome. We will repeat this experiment a large number of times. Then we will write the experimental probability as the number of times that we drew a ball with a white paw print in relation to the number of times that we actually drew a ball from the bag. Theoretical probability and experimental probability will not, in general, give the same answer, but the probabilities are relatively close to one another. A fifth grade class experimented by placing 4 black marbles and 3 white marbles in a paper bag. The black marbles represented the marbles with gray paw prints and the white marbles represented the white paw prints. The students drew a marble from the bag, recorded its color, and replaced the marble in the bag. They repeated this experiment 50 times. A black marble was drawn from the bag 32 times, and a white marble was drawn 18 times. Since the favorable outcome was to draw a white paw print (a white marble in this experiment), the experimental probability of picking a white paw print was 18/50 = 0.36. The theoretical probability as previously determined was 3/7 = 0.43. In this case, the 2 probabilities are not equal to one another. Students should be encouraged to try a larger number of experiments to determine if these two probabilities will become closer to equality with more experiments.

Now that we have finished reviewing the first few terms, we will complete this unit by describing the different types of outcomes with which we will be dealing. When discussing outcomes throughout the following units, we will be referring to the sample space which is the set of all possible outcomes. We will learn how to identify the sample space using lists and tree diagrams. A tree diagram is a diagram that shows all possible outcomes for an experiment. The probability of a favorable outcome occurring can be expressed with words, fractions, and decimals. Some of the words that describe probability are certain, likely, equally likely as unlikely, unlikely, and impossible. An outcome that is certain will always happen. A likely outcome is an outcome that has a greater than even chance of happening. If outcomes are equally likely as unlikely they have exactly the same chance of occurring. If an outcome has a less than even chance of occurring it is unlikely to happen. If an outcome is impossible this outcome can never happen. The fractions and decimals that represent the probability of an event occurring cannot be smaller than zero of larger than one.

We can also relate these numerical representations to the words that we just defined. If an outcome is impossible it has the numerical probability of zero. If an outcome is certain it has the numerical probability of one. If all outcomes are equally likely to occur the numerical probability will be represented as K/N where N is the number of points in the sample space and K is a whole number between 1 and N. Therefore, a likely outcome will fall between 0.5 (or ½) and one, and unlikely outcomes will fall between 0 and 0.5 (or ½). There are other ways of representing outcomes when dealing with probability, but these will be our focus when we develop the following units for fifth graders.

1.2 Classroom Activities

After introducing this new vocabulary and discussing it in class, it is important to give the students an opportunity to use these new terms to describe different probability situations.

1.2.1 Die Rolling Activity

Show the students a six-sided die and ask them to identify all of the possible outcomes. The students should be given the chance to observe the die so that they can visualize that there are six equally likely outcomes. Then have the students verbalize the outcomes as 1, 2, 3, 4, 5, and 6. Also have the students explain why each of the six outcomes is equally likely. Finally, one at a time, write several different probability questions that can be asked about this die. The students should identify the possible outcomes, favorable outcomes, and the theoretical or experimental probability of each event as a fraction. The students should also classify the probability as certain, likely, equally likely as unlikely, unlikely, or impossible.

Question 1

What is the chance that you will roll a six if you roll a regular six-sided die one time?

Answers

Possible outcomes: 1, 2, 3, 4, 5, 6

Favorable outcome: 6

Theoretical probability: $\frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}} = \frac{1}{6}$

Why did we identify theoretical probability instead of experimental probability? Answers will vary, but the answers should address that we have not actually rolled the die many times to record any actual results.

Is the outcome certain, likely, equally likely as unlikely, unlikely, or impossible? It is unlikely that we will roll a six on the first roll, but it is important for the students to understand it is possible. It is also important for the students to understand that because all of the sides are exactly the same it is equally likely to roll any one of the six numbers.

Question 2

What is the probability that you will roll a zero when you roll a regular six-sided die one time? Ask the students if they can identify theoretical or experimental probability with the information that is listed?

Answers

Possible outcomes: 1, 2, 3, 4, 5, 6

Favorable outcome: 0

Theoretical probability: $\frac{0}{6} = 0$

Theoretical probability in words: impossible because there is not a 0 on a regular six-sided die

Question 3

What is the probability that you will roll a number that is greater than 0 but less than 7 when you roll a regular six-sided die one time?

Answers

Possible outcomes: 1, 2, 3, 4, 5, 6

Favorable outcomes: 1, 2, 3, 4, 5, 6

Theoretical probability: $\frac{6}{6} = 1$

Theoretical probability in words: certain because you have to roll one of the favorable outcomes

Question 4

You have rolled your die 6 times, and after recording the data you notice that you have rolled the following:

Outcomes	Number of times rolled
1	1
2	0
3	1
4	1
5	3
6	0

Table 1: Die Roll Data

1. Using this data to record probability, are you determining experimental or theoretical probability?
2. What is the theoretical probability of rolling a 2?
3. What is your experimental probability of rolling a 2?
4. Are these two probabilities the same? Why or why not?
5. Is there a way to get these two probabilities to be closer to the same amount?
6. Are there any outcomes listed on the table above where the theoretical probability is the same as the experimental probability?
7. If there are some that are the same, which outcomes are they?

Answers

1. You are determining experimental probability because you are using actual results from rolling a die.
2. $$\text{Theoretical probability} = \frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}} = \frac{1}{6}$$
3. $$\text{Experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of experiments}} = \frac{0}{6} = 0$$

4. No because theoretically we should roll a 2 one time when we roll the die six times, and we didn't roll a 2 at all when we did the experiment.
5. We should be able to get these two probabilities closer to the same amount be actually rolling the die many more times. We didn't perform the experiment enough times.
6. Yes, there are some that both probabilities are $\frac{1}{6}$.
7. The outcomes are 1, 3, and 4.

1.2.2 Crayon Activity

To help the students to develop a deeper understanding of the terms, we are going to do a similar activity using different manipulatives so that they understand that probability can be used in many different situations. For this activity we need a brown paper bag or any other type of container so that the students cannot see the contents of the container when the manipulatives are placed inside. We also need 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon. All of these crayons should be new and exactly the same size so we cannot feel any difference when we reach into our bag. First, the students should write down how many of each color of crayon that we are going to place into our bag. Then we place all twelve crayons into the bag so we cannot see them anymore. Next, we should ask the students some theoretical probability questions. The students should identify the possible outcomes, favorable outcomes, and the theoretical probability of each event as a fraction. The students should also classify the probability as certain, likely, equally likely as unlikely, unlikely, or impossible.

Question 1

What is the chance of picking a red crayon out of the bag without looking on your first pick?

Answers

Possible outcomes: 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon = 12 possible outcomes

Favorable outcome: red crayon = 6 favorable outcome

Theoretical probability: $\frac{6}{12} = \frac{1}{2}$

Theoretical probability in words: equally likely as unlikely because you have exactly the same chance of drawing a red crayon (6 chances) as you have a drawing any other crayon color (6 chances)

Question 2

What is the probability that you will pick a black crayon out of the bag without looking on your first pick?

Answers

Possible outcomes: 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon = 12 possible outcomes

Favorable outcome: black crayon = 0 favorable outcome

Theoretical probability: $\frac{0}{12} = 0$

Theoretical probability in words: impossible because there are no black crayons inside of the bag

Question 3

What is the probability that you will pick a purple crayon out of the bag without looking on your first draw from the bag?

Answers

Possible outcomes: 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon = 12 possible outcomes

Favorable outcome: purple crayon = 1 favorable outcome

Theoretical probability: $\frac{1}{12}$

Theoretical probability in words: It is possible to draw a purple crayon from the bag because there is one in the bag, but it is unlikely because there are 11 other crayons that may also be drawn from the bag.

Question 4

What are the chances of picking a red crayon or a blue crayon from the bag without looking with you first pick?

Answers

Possible outcomes: 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon = 12 possible outcomes

Favorable outcome: red crayon or blue crayon = 8 favorable outcome

Theoretical probability: $\frac{8}{12}$

Theoretical probability in words: It is likely that we will draw a red or blue crayon out of the bag because there are more favorable outcomes (8) than unfavorable outcome (4), but it is not certain because there are still some unfavorable outcomes in the bag.

Question 5

What are the chances of picking a red, blue, purple, yellow, or green crayon from the bag without looking with your first pick?

Answers

Possible outcomes: 2 blue crayons, 6 red crayons, 2 green crayons, 1 yellow crayon, and 1 purple crayon = 12 possible outcomes

Favorable outcome: red, blue, purple, yellow, or green crayon = 12 favorable outcome

Theoretical probability: $\frac{12}{12} = 1$

Theoretical probability in words: It is certain that we will draw one of these colors out of the bag because there are no other colors in the bag.

Finally, we should actually perform the experiment. Go around the room allowing each student to pick a crayon from the bag without looking. The color of the crayon should be recorded on a chart using tally marks, and then the crayon should be replaced back into the bag before the next crayon is drawn. Continue going around the room allowing the students to pick, record, and replace the crayons until 100 crayons have been drawn from the bag. Remind the students that the more times the experiment is performed the closer our experimental data will match our theoretical data. Then answer the following questions with the data that the class recorded during their experiment.

1. What is our experimental probability of picking a red crayon out of the bag without looking?
2. Does this match the theoretical probability that we recorded earlier?
3. What is the experimental probability of picking a black crayon out of the bag without looking?
4. How does this compare with the theoretical probability of picking a black crayon?
5. What is the experimental probability of picking a purple crayon from the bag without looking?

6. Is it the same as the theoretical probability of drawing purple crayon from the bag?
7. What is our experimental probability of picking a blue crayon, red crayon, yellow crayon, purple crayon, or green crayon from the bag without looking?
8. Is our experimental probability the same as the theoretical probability? Why did this happen?
9. Were our experimental results what we expected them to be? Why or why not?
10. What could we have done differently to get different experimental results without changing any of our crayons?

1.2.3 Vocabulary Memory Game

Supplies needed:

- Vocabulary templates
- Cardstock or index cards

Preparation:

- Copy vocabulary templates onto cardstock
- Cut out into individual cards, or
- Have students write vocabulary words and definitions onto separate index cards
- Copy and send a Probability Memory Game: Vocabulary Study Sheet home with students the night prior to playing the memory game so students can become familiar with the terms

Object: Match the vocabulary word with its correct definition

Rules:

- Divide the class into 2 teams. There should be an equal number of players on each team. The teacher may need to play if teams are uneven.
- Choose enough words and definitions so each player will have one card.
- Mix up cards and pass them out, face down, to each player. Instruct everyone to leave their cards face down until their turn.
- Randomly pick a team to start the game by flipping a coin, choosing a number, etc.
- The first team member on Team 1 should then stand up, turn over his or her card, and read aloud what is on the card.
- Then, the player that is standing will pick any other player in the room to stand and read his or her card aloud.
- If the player that started this round has correctly matched a vocabulary word with its definition, then his or her team gets a point, and Team1 keeps control of the game.
- If a match was not made, no points are given and Team 2 gets control of the game. Both players should sit down and turn their cards face down again.
- If a match was made, the next player on Team 1 will stand, read his or her card aloud, and choose any other student with a card to read his or her card aloud.
- If a match was not made, the first player on Team 2 will stand, read his or her card aloud, and choose any other student to read his or her card aloud.
- If there is a match, Team 2 will get a point and the next player on Team 2 will stand and play.
- If there is not a match, play passes back to Team 1.

- When it is a player's turn that does not have a card this player will choose two players with cards, one at a time, to try to find a match.
- Play will continue in this way until all matches have been made.
- The winning team will be the team with the most points.

Rationale for playing vocabulary memory game:

- To play this game each student will need to have prior knowledge of the terms. Therefore, the game may motivate the students to study the terms the night before the class plays the game.
- Students will need to pay attention to every person's turn so they can remember where matchers are located. This causes them to be active participants throughout the game (even when it is not their turn).
- Because the object of the game is to match the word with its definition, the students will hear the words and definitions repeated many times until a match is made. This is helpful because some students commit things to memory through repetition.
- Finally, games provide a low stress, fun way of learning that some students respond to very well.

Probability Memory Game:

Vocabulary Study Sheet

1. Probability – the chance that an event will occur

2. Experimental probability - $\frac{\text{number of times event occurs}}{\text{total number of experiments}}$

3. Theoretical probability – $\frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}}$

4. Experiment – a controlled test conducted to observe outcomes

5. Event – a collection of one or more outcomes

6. Outcome – a result in an experiment

7. Possible outcomes – all of the results that have a chance of happening in an experiment

8. Favorable outcomes – only the results that “count” towards the event happening

9. Sample space - the set of all possible outcomes

10. Tree diagram – a diagram that shows all possible outcomes for an event

11. Certain (defined with words) – an outcome that will always happen

12. Certain (defined with numbers) – 1

13. Likely (words) – an outcome that has a greater than even chance of happening

14. Likely (numbers) – greater than ½ (0.5) but less than 1

15. Equally likely as unlikely (words) – an outcome that has exactly the same chance of occurring as its chance of not occurring

16. Equally likely as unlikely (numbers) – ½ (0.5)

17. Unlikely (words) – an outcome that has a less than even chance of occurring

18. Unlikely (numbers) – less than ½ (0.5) but greater than 0

19. Impossible (words) – an outcome that can never happen

20. Impossible (numbers) - 0

Probability Memory Game

Probability	**the chance that an event will occur**
Experimental Probability	$\frac{\textbf{number of times event occurs}}{\textbf{total number of experiments}}$
Theoretical Probability	$\frac{\textbf{number of favorable outcomes}}{\textbf{total number of equally likely outcomes}}$

Figure 2: Probability Memory Game templates

Experiment	**a controlled test conducted to observe outcomes**
Event	**a collection of one or more outcomes**
Outcome	**a result in an experiment**

Figure 2 (continued)

Possible outcomes	**all of the results that have a chance of happening in an experiment**
Favorable outcomes	**only the results that "count" towards the event happening**
Sample Space	**the set of all possible outcomes**

Figure 2 (continued)

Tree diagram	**a diagram that shows all possible outcomes for an event**
Certain (words)	**an outcome that will always happen**
Certain (numbers)	**1**

Figure 2 (continued)

Likely (words)	**an outcome that has a greater than even chance of happening**
Likely (numbers)	**greater than ½ (0.5) but less than 1**
Equally likely as unlikely (words)	**an outcome that has exactly the same chance of occurring as its chance of not occurring**

Figure 2 (continued)

Equally likely as unlikely (numbers)	**½ (0.5)**
Unlikely (words)	**an outcome that has a less than even chance of occurring**
Unlikely (numbers)	**less than ½ (0.5) but greater than 0**

Figure 2 (continued)

Impossible (words)	**an outcome that can never happen**
Impossible (numbers)	**0**

Figure 2 (continued)

1.3 Homework Exercises

Directions:

Identify the possible outcomes, favorable outcomes, and the theoretical probability of each event as a fraction. Classify the probability as certain, likely, equally likely as unlikely, unlikely, or impossible, and explain why you chose this classification.

1. Your mom just went to the grocery store, and she bought eight cans of soup. She bought 4 cans of chicken noodle soup, 2 cans of tomato soup, 1 can of vegetable beef soup, and 1 can of cream of chicken soup. If all of the cans of soup are exactly the same size and shape what is the chances of taking a can of tomato soup out of the bag the first time you take one can out without looking?
    a. Possible outcomes: ____
    b. Total number of possible outcomes: ________________________________
    c. Favorable outcome: ____
    d. Total number of favorable outcomes: _______________________________
    e. Theoretical probability (as a fraction): ____________________________
    f. Theoretical probability (in words): _________________________________
    g. Why did you choose this classification? ____________________________

    ____

    h. Using the cans of soup that are listed above, write a probability question where your theoretical probability of picking the one can of soup that you want is equally likely as unlikely.

    ____

    ____

2. There are 5 red marbles and 1 blue marble in a paper bag. You cannot see inside the bag. What is the theoretical probability that you will pick a red marble out of the bag on your first try?
   a. Possible outcomes: ______________________________
   b. Total number of possible outcomes: ______________________________
   c. Favorable outcome: ______________________________
   d. Total number of favorable outcomes: ______________________________
   e. Theoretical probability (as a fraction): ______________________________
   f. Theoretical probability (in words): ______________________________
   g. Why did you choose this classification? ______________________________

      ____

3. If you take the blue marble out of the bag and leave only the five red marbles what are you chances of drawing a red marble out of the bag now on your first try?
   a. Possible outcomes: ______________________________
   b. Total number of possible outcomes: ______________________________
   c. Favorable outcome: ______________________________
   d. Total number of favorable outcomes: ______________________________
   e. Theoretical probability (as a fraction): ______________________________
   f. Theoretical probability (in words): ______________________________
   g. Why did you choose this classification? ______________________________

      ____

   h. Using the bag that only has the 5 red marbles, write a question where your chances of getting what you want are impossible.

1.4 Homework Exercises Answers

Number 1

a. Possible outcomes: chicken noodle soup (4 cans), tomato soup (2 cans), vegetable beef soup (1 can), cream of chicken soup (1 can)

b. Total number of possible outcomes: 8 cans

c. Favorable outcome: tomato soup

d. Total number of favorable outcomes: 2 cans

e. Theoretical probability (as a fraction): $\frac{2}{8}$

f. Theoretical probability (in words): unlikely

g. Why did you choose this classification? It is unlikely because there are more chances of picking a can of soup that is not tomato (6) than picking tomato (2). (Answers will vary)

h. Using the cans of soup that are listed above, write a probability question where your theoretical probability of picking the one can of soup that you want is equally likely as unlikely. Answers will vary, but should include picking a can of chicken noodle soup. They could also write a question about picking tomato, vegetable beef, or cream of chicken.

Number 2

a. Possible outcomes: red marble, blue marble

b. Total number of possible outcomes: 6

c. Favorable outcome: red marble

d. Total number of favorable outcomes: 5

e. Theoretical probability (as a fraction): $\frac{5}{6}$

f. Theoretical probability (in words): Likely

g. Why did you choose this classification? It is likely because there are only 6 marbles in the bag, and five of them are want we want. It is not certain because there is a chance that we could pick the one blue marble. (Answers will vary)

Number 3

a. Possible outcomes: red marble

b. Total number of possible outcomes: 5

c. Favorable outcome: red marble

d. Total number of favorable outcomes: 5

e. Theoretical probability (as a fraction): $\frac{5}{5} = 1$

f. Theoretical probability (in words): certain

g. Why did you choose this classification? It is certain because there are only red marbles in the bag, and this is the color we want. We have to draw a red from the bag. (Answers will vary)

h. Using the bag that only has the 5 red marbles, write a question where your chances of getting what you want are impossible. (Answers will vary) The question should involve picking any color of marble except red.

2 IS IT FAIR?

2.1 Introduction

"That's not fair!" If you have ever been around a group of children that are playing together, you have definitely heard these words. So, what do they mean? As an introduction to this lesson, have your students write down their definition of fair and take these up. Then, read some of these definitions aloud. Finally, guide the class into agreeing upon one common definition of the word fair. They can use parts, all, or none of the definitions read aloud. A common definition of fair in relation to probability is having equal chances of winning or losing.

A good place to start is to evaluate a few different games to determine if the games are fair or unfair. Remind the students that a fair game is a game in which you are equally likely to win or lose. In other words, you will win half of the games you play, and you will lose half of the games you play. Set up a simple game, and ask the students if the game is fair or unfair. If the game is unfair have the students explain why it is not fair.

A simple way to demonstrate this concept quickly is to show them different ways to set up the same game. Instruct the students that you are going to place six blocks into a paper bag. They will only get one chance to pick a block from the bag without looking into the bag. If a blue block is picked from the bag they will win the game, but if they pick a red block they will lose the game.

> **Game 1:** For this game show the students that you are placing six red blocks into the paper bag. Is Game 1 fair? Why or why not?
>
> Possible answer: This game is not fair because it is impossible to draw a blue block from this bag. You cannot win this game.
>
> **Game 2:** For this game show the students that you are placing six blue blocks into the

paper bag. Is Game 2 fair? Why or why not?

Possible answer: This game is unfair because it is impossible to lose, and for a game to be fair it has to offer equal chances of winning and losing.

Game 3: Ask the students how many red blocks and blue blocks that you should place into the paper bag to have a fair game?

Possible answer: There should be three red blocks and three blue blocks to have a fair game because half of the blocks are red, which are losers, and half of the blocks are blue, which are winners. Therefore, the chances of winning and the chances of losing are equal.

After the students understand the meaning of fair, we need to help them to explore methods of determining whether a game is fair or unfair. In order to explore fairness, we need to remind the students of the two ways that probability can be determined – experimentally and theoretically. Remind them that experimental probability is an estimate that an event will happen based on how often the event occurs in an experiment. In other words, experimental probability is found by actually playing the game and seeing how many times the game is won and lost. They also need to recall that the results of experimentation are more accurate with a larger number of trials (by playing the game many times). Theoretical probability is determined by figuring out all of the possible outcomes. Then we count all of the outcomes that are favorable (winning). Finally, we divide the number of favorable outcomes by the total number of outcomes. For example, if there are three different possible ways to win the game and three different ways to lose the same game, then there are three favorable outcomes divided by six total outcomes. This means that this game is theoretically fair because half of the games will be won. Ideally, the theoretical and experimental probabilities should be very close to the same.

2.2 Classroom Activities

After discussing fairness and the different types of probability, the students need to practice using these methods to determine the fairness of a game. The game "Sweet Match" is an easy game for the students to explore fairness using experimental and theoretical probability in a classroom setting. This game is also good to use because the fairness of the games is not intuitive to the students. The only supplies needed to play this game are lunch size paper bags and three or four bags of individually wrapped candy. The candy must be wrapped in different color wrappers, such as Starburst, Hershey Kisses, Miniature Reese's Cups, etc. It is important that these candies are exactly the same size and shape. The only difference should be color of the wrapper.

First, introduce the game "Sweet Match" to the class by demonstrating how to play the game. Starburst will be used in this discussion, but remember any candy that has different colored wrappers will work.

How to Play Sweet Match

- Place three cherry Starburst and one strawberry Starburst into one paper bag and two cherry Starburst and two strawberry Starburst in the other paper bag.
- Choose a student to pick one piece of candy from one bag, and choose another student to pick one piece of candy from the other bag. Neither student should be allowed to look inside the bag before drawing the piece of candy from the paper bag.
- If the Starburst are the same flavor the students have made a "Sweet Match," and they get to eat their candy. These students have won the game.
- If they do not get a sweet match, then they have lost the game. They do not get to eat their candy.

- Replace the same colors that were picked from the bags before the next two students pick a piece of candy from them.
- Use the template and tally marks to record the results of each pair of students.

Number of Times Game Played	Number of Wins	Number of Losses

Table 2: Sweet Match Tally Chart

- Continue playing the game until all students have had one chance to play.
- After everyone in the class has had a chance to play ask the class if they think this game is fair. (Remind them that a fair game is one in which there is an equal chance of winning or losing.)
- After the class has made a prediction about the fairness of this game, guide them through determining the experimental probability of winning this game based upon the results of the games that have already been played. Remind the class how to write experimental probability as a fraction. The number of favorable outcomes (wins) should be written as the numerator, and the total number of games played should be written as the denominator. Based on these results, does the game seem fair? Is this fraction close to ½? Ask the class what they can do to make the results of an experiment more reliable. They should say that more games should be played because the experiment is better when it is performed a large number of times.
- At this time, allow the class to divide up into groups of two. Have each group play the game 25 times and record their results. They cannot eat their candy when they win this time. They can divide the candy at the end of the experiment and eat it then. When all

groups have finished their experiments, record all of the results on the chart on the board. Refigure the experimental probability with these new numbers. Is this experimental probability closer to one half? Based on this experiment, is this game fair?

- After the class has determined the experimental probability of winning this game, guide them through determining the theoretical probability of winning this game. Draw the following chart on the board or use an overhead transparency of this chart. Show that the flavors in one bag are listed across the top of the chart, and the flavors in the other bag are listed down the left side of the chart. Explain to the class that this chart will help to figure out every possible combination of flavors that can be picked out of the bags. It may also be helpful to lay the actual candy on the chart so the students can visualize the possible combinations.

Flavor or Color of Candy	Cherry	Cherry	Cherry	Strawberry
Cherry				
Cherry				
Strawberry				
Strawberry				

Table 3: Sweet Match Possible Combinations Template

Flavor or Color of Candy	Cherry	Cherry	Cherry	Strawberry
Cherry	**Cherry & Cherry**	**Cherry & Cherry**	**Cherry & Cherry**	Cherry & Strawberry
Cherry	**Cherry & Cherry**	**Cherry & Cherry**	**Cherry & Cherry**	Cherry & Strawberry
Strawberry	Cherry & Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**

Strawberry	Cherry & Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**

Table 4: Sweet Match Possible Combinations

- After the class helps fill in the chart of possible outcomes, ask the students which of these combinations are "Sweet Matches." Highlight or circle the favorable or winning outcomes.
- Then ask the students to count how many possible outcomes there are in this game. They should recognize that there are 16 different combinations that can be drawn from the bags. Next, have the students count how many winning outcomes there are and how many losing outcomes there are. They should recognize that there are 8 favorable (winning) outcomes and 8 unfavorable (losing) outcomes. Finally, remind them how to write theoretical probability:

$$\frac{\text{number of favorable outcomes}}{\text{total number of equally likely outcomes}} = \frac{8}{16} = \frac{1}{2}$$

- Therefore, based upon theoretical probability, this game of "Sweet Match" is fair. Theoretically, one half of all of the games played will be won.
- Finally, ask the students to compare the theoretical probability with the experimental probability.

2.3 Homework Exercises

Directions: Use the charts to find the theoretical probability of winning the following variations of the game "Sweet Match." Then find the experimental probability of one of the games. (Your teacher will assign the game that you should use for your experiment. Your teacher will also give you the paper bag and candy that you will need to perform your experiment.) Remember to pick one piece of candy from each bag without looking. You win if the candies are the same. Replace both pieces of candy into the correct bag before picking again. You should play your game 100 times and record your results on the chart at the bottom of this worksheet. According to your results, are any of the following games fair?

GAME 1

Bag 1: 3 orange Starburst and 1 lemon Starburst

Bag 2: 3 orange Starburst and 1 lemon Starburst

Flavor or Color of Candy	Orange	Orange	Orange	Lemon
Orange				
Orange				
Orange				
Lemon				

Table 5: Sweet Match Game 1 Template

Number of favorable outcomes (wins or matches): ____

Total number of possible outcomes: ____

Theoretical probability of winning Game 1: ____

Experimental probability of winning Game 1: ____
(We will fill this result in tomorrow in class.)

Is Game 1 fair? Explain why it is fair or why it is unfair. ______________________________

____

GAME 2

Bag 1: 4 orange Starburst

Bag 2: 3 orange Starburst and 1 strawberry Starburst

Flavor or Color of Candy	Orange	Orange	Orange	Orange
Orange				
Orange				
Orange				
Strawberry				

Table 6: Sweet Match Game 2 Template

Number of favorable outcomes (wins or matches): _____________________________________

Total number of possible outcomes: ____

Theoretical probability of winning Game 2: ____

Experimental probability of winning Game 2: ____
(We will fill this result in tomorrow in class.)

Is Game 2 fair? Explain why it is fair or why it is unfair. ______________________________

____

GAME 3

Bag 1: 2 cherry Starburst and 2 strawberry Starburst

Bag 2: 4 strawberry Starburst

Flavor or Color of Candy	Cherry	Cherry	Strawberry	Strawberry
Strawberry				
Strawberry				
Strawberry				
Strawberry				

Table 7: Sweet Match Game 3 Template

Number of favorable outcomes (wins or matches): ____

Total number of possible outcomes: ____

Theoretical probability of winning Game 3: ____

Experimental probability of winning Game 3: ____
(We will fill this result in tomorrow in class.)

Is Game 3 fair? Explain why it is fair or why it is unfair. __________________________________

____

GAME 4

Bag 1: 3 orange Starburst and 1 cherry Starburst

Bag 2: 3 cherry Starburst and 1 orange Starburst

Flavor or Color of Candy	Orange	Orange	Orange	Cherry
Cherry				
Cherry				
Cherry				
Orange				

Table 8: Sweet Match Game 4 Template

Number of favorable outcomes (wins or matches): ____

Total number of possible outcomes: ____

Theoretical probability of winning Game 4: ____

Experimental probability of winning Game 4: ____
(We will fill this result in tomorrow in class.)

Is Game 4 fair? Explain why it is fair or why it is unfair. _________________________________

____

GAME 5

Bag 1: 3 strawberry Starburst and 1 cherry Starburst

Bag 2: 2 strawberry Starburst, 1 cherry Starburst, and 1 orange Starburst

Flavor or Color of Candy	Strawberry	Strawberry	Strawberry	Cherry
Strawberry				
Strawberry				
Cherry				
Orange				

Table 9: Sweet Match Game 5 Template

Number of favorable outcomes (wins or matches): ____

Total number of possible outcomes: ____

Theoretical probability of winning Game 5: ____

Experimental probability of winning Game 5: ____
(We will fill this result in tomorrow in class.)

Is Game 5 fair? Explain why it is fair or why it is unfair. _________________________________

____

GAME 6

Bag 1: 2 cherry Starburst, 1 strawberry Starburst, and 1 lemon Starburst

Bag 2: 2 cherry Starburst, 1 strawberry Starburst, and 1 lemon Starburst

Flavor or Color of Candy	Cherry	Cherry	Strawberry	Lemon
Cherry				
Cherry				
Strawberry				
Lemon				

Table 10: Sweet Match Game 6 Template

Number of favorable outcomes (wins or matches): ____________________________________

Total number of possible outcomes: ____

Theoretical probability of winning Game 6: ____

Experimental probability of winning Game 6: _______________________________________
(We will fill this result in tomorrow in class.)

Is Game 6 fair? Explain why it is fair or why it is unfair. ______________________________

____

Experiment

Use the table below to record the results of your experiment. Remember to play your assigned game 100 times.

Game ______

Number of Times Game Played	**Number of Wins**	**Number of Losses**

Table 11: Sweet Match Experiment Template

Experimental probability: ____

Based upon your experiment, do you think that your game was fair? Explain. _______________

____

Was your experimental probability the same as the other students' experimental probabilities that played the same game? ____

2.4 Homework Exercises Answers

GAME 1

Flavor or Color of Candy	Orange	Orange	Orange	Lemon
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	Lemon & Orange
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	Lemon & Orange
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	Lemon & Orange
Lemon	Lemon & Orange	Lemon & Orange	Lemon & Orange	**Lemon & Lemon**

Table 12: Sweet Match Game 1 Answers

Favorable outcomes: 10

Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{10}{16}$

Is Game 1 fair? No

GAME 2

Flavor or Color of Candy	Orange	Orange	Orange	Orange
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**
Strawberry	Orange & Strawberry	Orange & Strawberry	Orange & Strawberry	Orange & Strawberry

Table 13: Sweet Match Game 2 Answers

Favorable outcomes: 12

Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{12}{16}$

Is Game 2 fair? No

GAME 3

Flavor or Color of Candy	Cherry	Cherry	Strawberry	Strawberry
Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**
Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**
Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**
Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**

Table 14: Sweet Match Game 3 Answers

Favorable outcomes: 8

Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{8}{16}$

Is Game 3 fair? Yes

GAME 4

Flavor or Color of Candy	Orange	Orange	Orange	Cherry
Cherry	Orange & Cherry	Orange & Cherry	Orange & Cherry	**Cherry & Cherry**
Cherry	Orange & Cherry	Orange & Cherry	Orange & Cherry	**Cherry & Cherry**
Cherry	Orange & Cherry	Orange & Cherry	Orange & Cherry	**Cherry & Cherry**
Orange	**Orange & Orange**	**Orange & Orange**	**Orange & Orange**	Cherry & Orange

Table 15: Sweet Match Game 4 Answers

Favorable outcomes: 6

Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{6}{16}$

Is Game 2 fair? No

GAME 5

Flavor or Color of Candy	Strawberry	Strawberry	Strawberry	Cherry
Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**	**Strawberry & Strawberry**	Cherry & Strawberry
Strawberry	**Strawberry & Strawberry**	**Strawberry & Strawberry**	**Strawberry & Strawberry**	Cherry & Strawberry
Cherry	Strawberry & Cherry	Strawberry & Cherry	Strawberry & Cherry	**Cherry & Cherry**
Orange	Strawberry & Orange	Strawberry & Orange	Strawberry & Orange	Cherry & Orange

Table 16: Sweet Match Game 5 Answers

Favorable outcomes: 7 Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{7}{16}$ Is Game 5 fair? No

GAME 6

Flavor or Color of Candy	Cherry	Cherry	Strawberry	Lemon
Cherry	**Cherry & Cherry**	**Cherry & Cherry**	Strawberry & Cherry	Lemon & Cherry
Cherry	**Cherry & Cherry**	**Cherry & Cherry**	Strawberry & Cherry	Lemon & Cherry
Strawberry	Cherry & Strawberry	Cherry & Strawberry	**Strawberry & Strawberry**	Lemon & Strawberry
Lemon	Cherry & Lemon	Cherry & Lemon	Strawberry & Lemon	**Lemon & Lemon**

Table 17: Sweet Match Game 6 Answers

Favorable outcomes: 6 Total number of possible outcomes: 16

Theoretical probability of winning: $\frac{6}{16}$ Is Game 6 fair? No

3 WHAT IS SAMPLE SPACE?

3.1 Introduction

The sample space is a list of all possible outcomes of an activity or experiment. It is important to know all of the possible outcomes when determining the theoretical probability of an event occurring. To compute theoretical probability, we must know the total number of possible outcomes. It is important to introduce this topic to the students in a very simple way. Therefore, we will begin this unit on sample space by asking the students to list all of the possible outcomes of a single activity or event. Some easy examples that can be used are: What are all of the possible outcomes when rolling a single die? The answer is 1, 2, 3, 4, 5, 6; therefore there are a total of 6 possible outcomes. What is the sample space of flipping one coin? This answer is heads or tails, so in this activity there are only two possible outcomes. List all of the possible outcomes if you are asked to draw one card from a standard deck of 52 cards. This answer consists of the following cards with the suit of hearts on them: 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace, and the same cards with the suits of spades, diamonds, or clubs on them. This list confirms that there are $13 \times 4 = 52$ different outcomes possible when drawing one card from a deck of 52 cards. After the students understand that sample space is simply a list of all of the possible outcomes of an event through demonstrating how to list these outcomes using a single event, explain that most experiments and activities use more than one activity at a time.

Sometimes it will not be necessary to make a list of all of the possible outcomes, because we also can find the total number of possible outcomes using the Counting Principle. The Counting Principle allows us to quickly find the total number of possible outcomes by multiplying the number of choices you have in one category with the number of choices you have in the other category of the task. You will have a factor for each category in which there is a choice to be made. For example, if you bought a red shirt, a blue shirt, tan shorts, black shorts, denim shorts, a pair of tennis shoes, and a pair of sandals; how many different outfits did you buy? This answer can be found using the Counting Principle by multiplying the number of shirts choices x the number of shorts choices x the number of shoe choices (2 x 3 x 2 = 12). Therefore, by using the counting principle we know that there are a total of 12 different ways that these clothes can be worn together, or in other words, a total of 12 possible outcomes.

When performing an experiment, assessing the fairness of an activity, or computing probability it is sometimes helpful to be able to make a list of all of the possible outcomes. We will focus on two main ways to list and organize our outcomes. The methods we will use are making an organized list or drawing a tree diagram. An organized list can be started by writing a heading for each of the categories where choices need to be made. If we use the example that we used to illustrate the Counting Principle, our three category headings will be shirt, shorts, and shoes. Then, start by writing down the first color of shirt listed, which is red, the first color of shorts listed, which is tan, and the first pair of shoes listed, which is tennis shoes. It will be easier for the students to understand this example if the example is written on the board as it is discussed. Therefore, the list should look like the following:

Shirt	Shorts	Shoes
Red	Tan	Tennis shoes

It will be easier to keep track of all of the possible choices if you only change one category at a time, starting with the last category on the right. Therefore, we will start by writing a new outcome by changing only tennis shoes to sandals.

Shirt	Shorts	Shoes
Red	Tan	Tennis shoes
Red	Tan	Sandals

Now that we have no other choices to make in the last category, we will change the choice of tan shorts in the next category to black shorts. If we make this new choice of shorts, then we must make sure we list this new choice with both of our choices of shoes.

Shirt	Shorts	Shoes
Red	Tan	Tennis shoes
Red	Tan	Sandals
Red	Black	Tennis Shoes
Red	Black	Sandals

We will repeat this same process when we change our choice of shorts to denim.

Shirt	Shorts	Shoes
Red	Tan	Tennis shoes
Red	Tan	Sandals
Red	Black	Tennis Shoes
Red	Black	Sandals
Red	Denim	Tennis Shoes
Red	Denim	Sandals

After we finish listing our outfit choices with the denim shorts, help the students to see that they have made all of their possible choices in the shorts and shoes categories while choosing the red shirt in the shirt category. Then ask them if there are any other choices that can be made in any of the categories. They should quickly realize that they haven't chosen the blue shirt. Help them to realize that their choices of shorts and shoes will be exactly the same as they were with the red shirt. Therefore, they can quickly write all of the choices with the blue shirt by rewriting their list with only changing the red shirt to the blue shirt. Finally, the final list of all of the possible outfit choices should look like the following:

Shirt	Shorts	Shoes
Red	Tan	Tennis shoes
Red	Tan	Sandals
Red	Black	Tennis Shoes
Red	Black	Sandals
Red	Denim	Tennis Shoes
Red	Denim	Sandals
Blue	Tan	Tennis shoes
Blue	Tan	Sandals
Blue	Black	Tennis Shoes
Blue	Black	Sandals
Blue	Denim	Tennis Shoes
Blue	Denim	Sandals

If we count the number of outfits we listed, then we will notice that we listed 12 possible choices which matches the total number of possible outcomes we found using the Counting Principle.

Another way that we can make a list of all of the possible outcomes of an experiment or activity is to draw a tree diagram. A tree diagram is an organizer that looks like a tree branching out. It will help students illustrate all of the possible outcomes of an experiment or activity. We will use our outfit example to demonstrate how to draw a tree diagram. Since there are 3 categories where choices need to be made, there will be 3 branches, or columns, of our tree. We begin drawing a tree diagram by writing all of the possible choices in one category in a column. The choices should have space between them. These choices are the first branch of our tree.

After completing the first branch of our tree, we need to draw the second branch of our tree. First, we need to draw lines that will connect our first branch choices to our second branch choices. Since our second category, shorts, includes 3 different choices, we need to draw 3 lines from the red shirt and 3 lines from the blue shirt. The students need to realize that you have to give all choices to each and every choice in the column or branch directly in front of it. Then, we will write our 3 choices of shorts at the ends of the lines that we just drew.

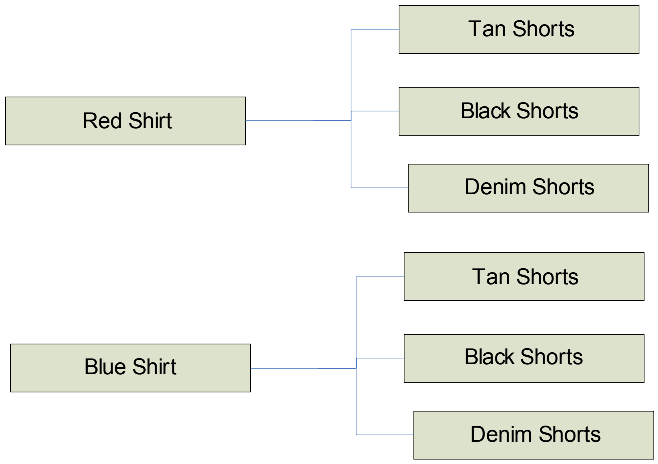


Figure 3: Tree Diagram of Outfit Choices (Part A)

It is important for the students to realize that we have written these second choices twice – once connected to the red shirt and once connected to the blue shirt. Now, we are ready to form the last branch of our tree using the 2 choices of shoes. We start this last branch of our tree by drawing 2 lines from every choice that we wrote in the second branch or column. Reinforce that we are drawing 2 lines from each choice because there are 2 choices of shoes. Finally, we finish our tree diagram by writing each type of shoes at the end of these lines.

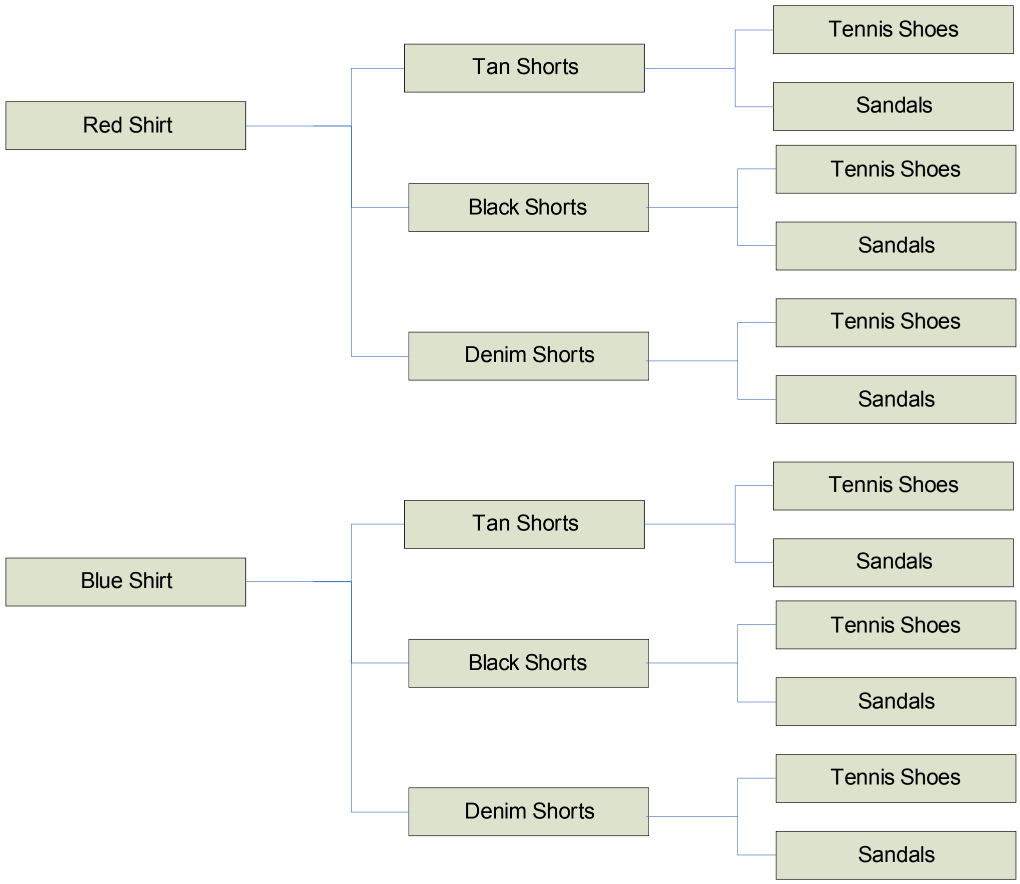


Figure 4: Tree Diagram of Outfit Choices (Part B)

When our tree diagram is complete, we can see all of our possible outcomes by following the paths that we made with the lines that we have drawn. Show the students that they can also count the total number of possible outcomes by counting the entries of the last branch of the tree. Explain that they do not count the other branches because they are connected to the last branch, and this makes them all part of the last branch.

Students will wonder if the order in which they build their tree diagram is important. To answer this question, divide the class into small groups and have them draw this same event in a different order. Assign the following variations of this tree diagram to different groups:

1) Branch 1 – Shirts Branch 2 – Shoes Branch 3 – Shorts
2) Branch 1 – Shoes Branch 2 – Shirts Branch 3 – Shorts

3) Branch 1 – Shoes Branch 2 – Shorts Branch 3 – Shirts

4) Branch 1 – Shorts Branch 2 – Shirts Branch 3 – Shoes

5) Branch 1 – Shorts Branch 2 – Shoes Branch 3 – Shirts

Each group's tree diagram should look like one of the following:

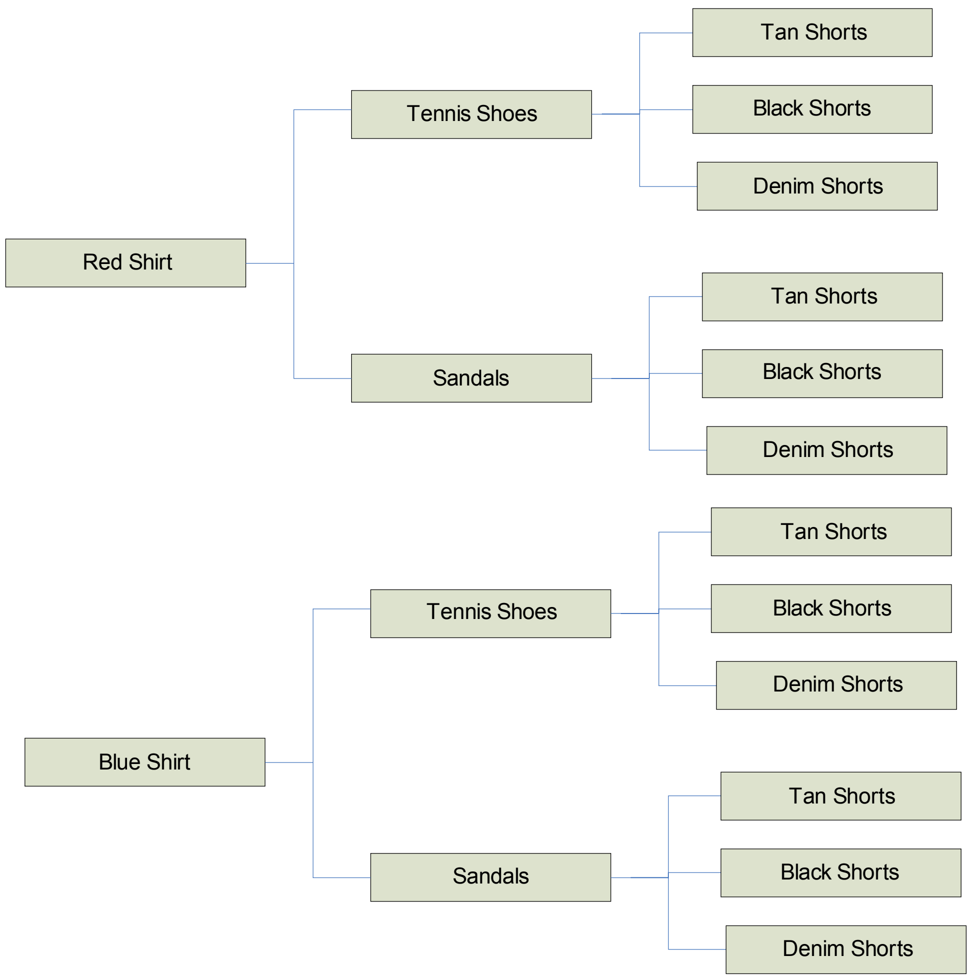


Figure 5: Group 1 Outfit Tree Diagram

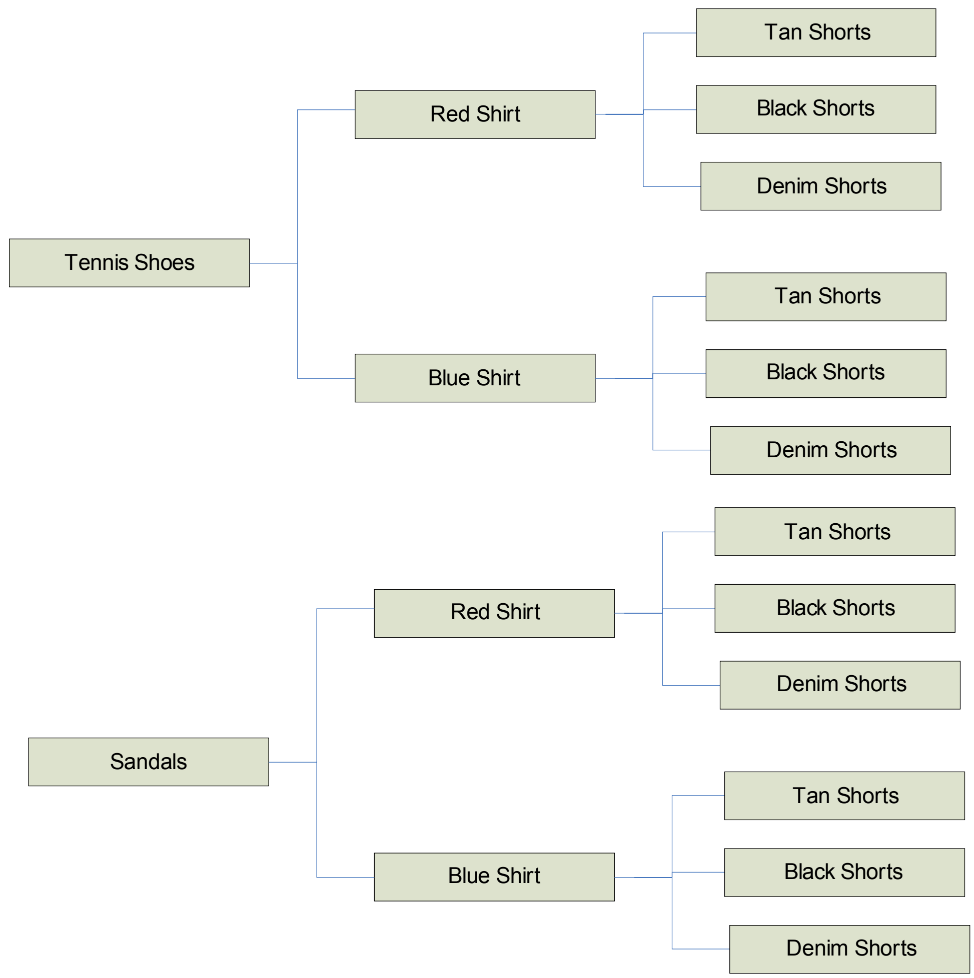


Figure 6: Group 2 Outfit Tree Diagram

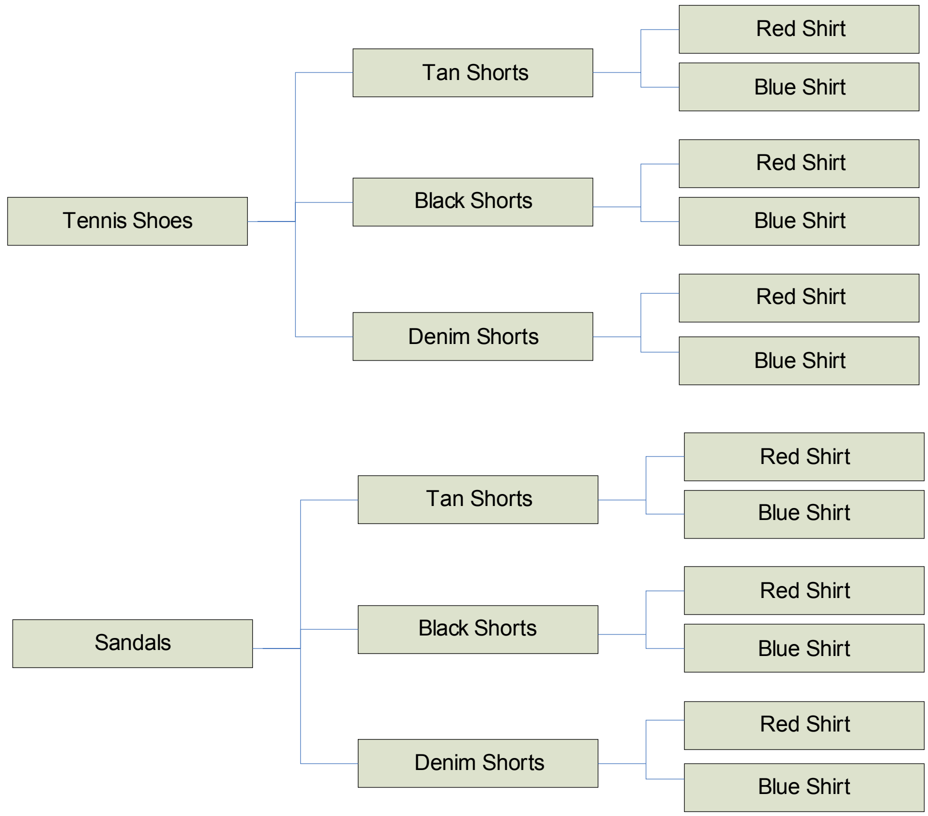


Figure 7: Group 3 Outfit Tree Diagram

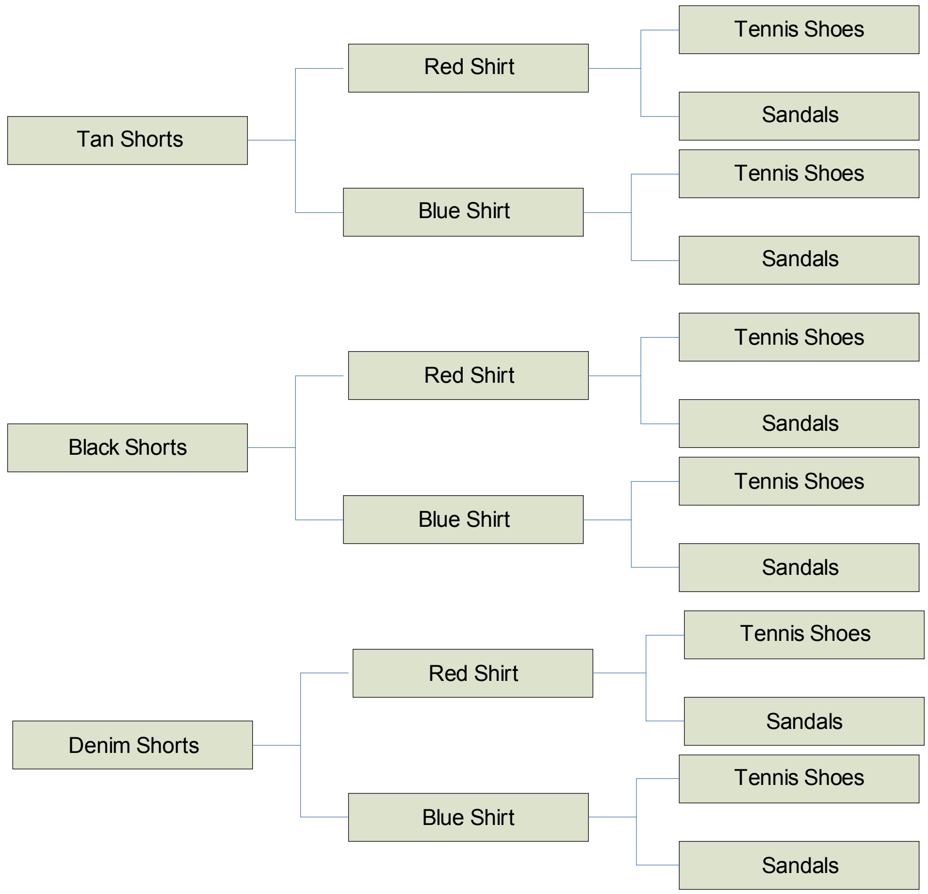


Figure 8: Group 4 Outfit Tree Diagram

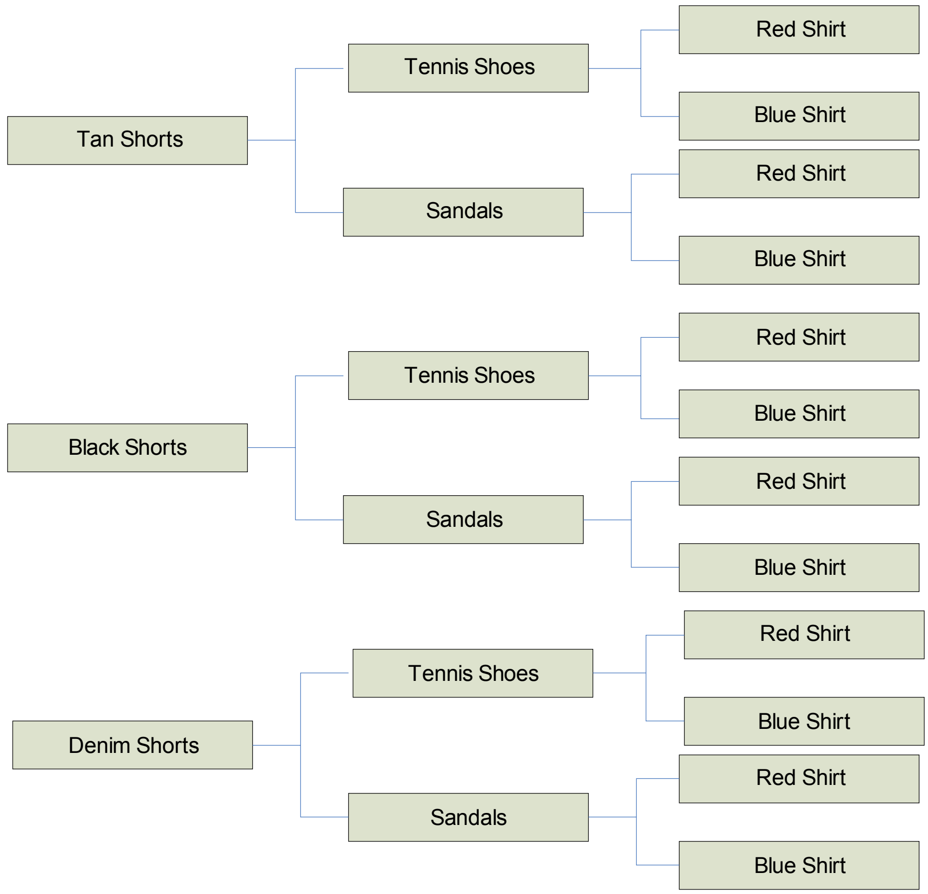


Figure 9: Group 5 Outfit Tree Diagram

After they finish drawing their trees have them follow all of the paths in their diagram to make a list of all of the possible outcomes (sample space). Then, read each outfit on the list that the class made earlier, and have the groups check that they have the exact same choices on their lists. This should help the students to see and remember that the order in which the tree diagram is drawn will not change the number of outcomes.

3.2 Classroom Activities

3.2.1 The Great M&M Showdown

In order to demonstrate the usefulness of knowing the sample space of an activity, play "The Great M&M Showdown" with the class. "The Great M&M Showdown" was created by Diana Freeman [1]. The directions for "The Great M&M Showdown are as follows:

- Give each student 12 M&Ms and a copy of "The Great M&M Showdown" graph. (The graph template is included at the end of the directions.)
- Explain the rules of the game.
    1) Each student will place all 12 M&Ms on any 12 of the 84 blocks on his/her graph.
    2) The teacher will then roll 2 dice and announce the sum of this roll to the class.
    3) If the student has an M&M on the sum of the 2 dice, the student removes only one M&M beside of the rolled sum on the graph. The student has to wait until the sum is rolled again before he/she can remove another M&M that has been placed beside that number on the graph.
    4) Play will continue until a student removes all of his/ her M&Ms from the graph. This student is the winner.
- Now, allow the students to place their M&Ms on their graphs, and play the game until there is a winner.
- After the game is over, ask the students if any of the sums occurred move often than the others. Then, ask if knowing the sample space would help them to play the game better. They should be able to realize that knowing the sample space will help them identify the probability of rolling each sum.

* Next, help the class identify all of the possible outcomes (sample space) of rolling 2 dice by drawing a tree diagram.

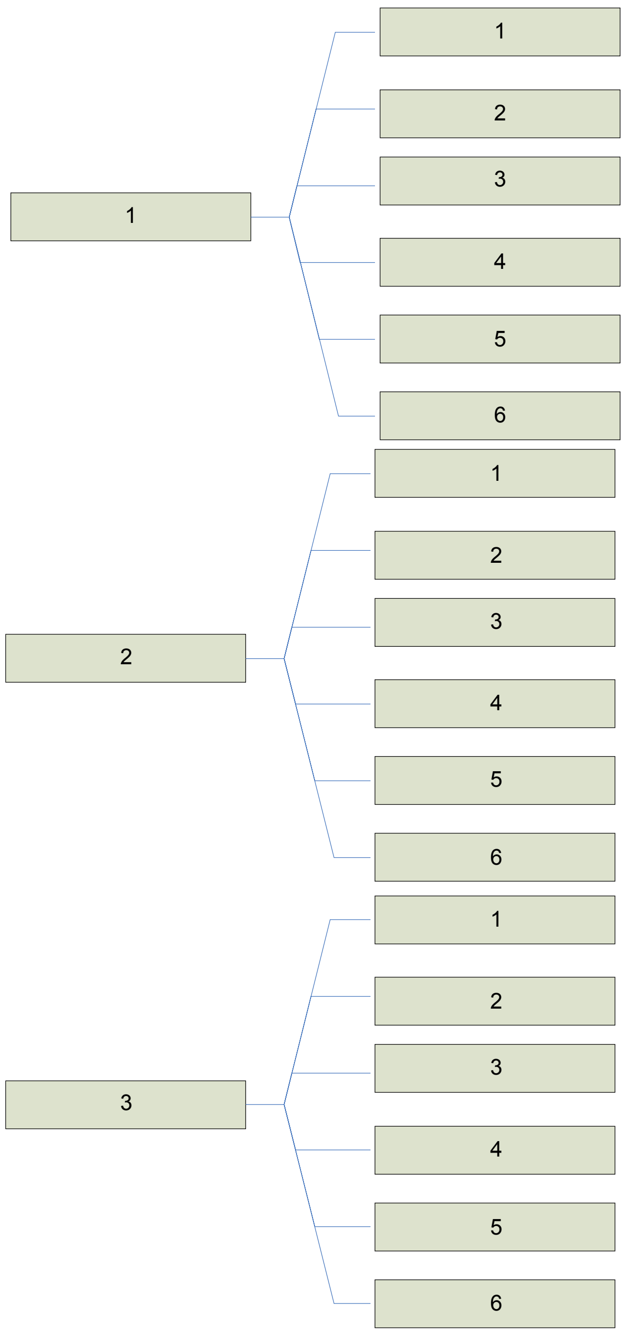


Figure 10: Sums of 2 Dice Tree Diagram

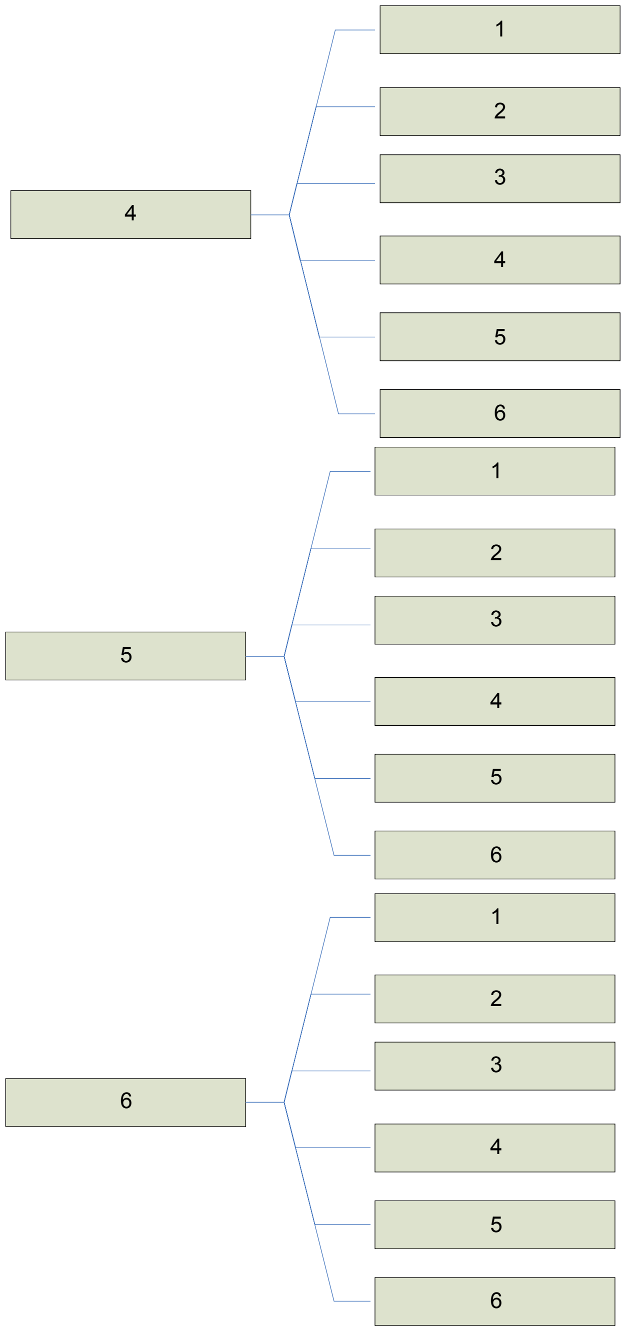


Figure 10 (continued)

- Then, using the tree diagram list all of the sums that are possible, and count how many different ways each sum can be rolled. It may be easier for the class to count the total number of each sum by using tally marks on the following chart.

Possible Sums	Number of Ways Sums Can Occur
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

Table 18: Number of Ways Sums Occur Rolling 2 Dice

- After the sample space has been identified, replay the game.
- At the end of the second game, ask the students if knowing the sample space helped them make better predictions of where to place their M&Ms. How did the knowledge of sample space change the game?

The Great M&M Showdown Graph

Place your 12 M&Ms beside the sums that you think will be rolled. More than one M&M may be placed beside of each number. The first to remove all of the candy is the winner!

1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							

Table 19: The Great M&M Showdown Graph Template

3.2.2 Restaurant Menus and Advertising

To introduce this activity, we need to discuss restaurant menus and advertising. Help the students realize that customers want to go to a restaurant that offers them choices. Therefore, many restaurants focus their advertising on the fact that they allow the consumer to make choices. Burger King's current advertising slogan is "Have it Your Way." Wendy's Kid's Meal "lets you be choosy." These are just two specific examples of how restaurants are using choices to influence people to choose their restaurant over all of the others. Other restaurants include in their advertising how many items are on their menus, or how many choices their customers are offered.

Now that the students are focused on the reason restaurants mention menu choices in their advertising, ask them if they think that they are ever misled or tricked by advertising. Do they think that advertising sometimes makes something seem better than it really is? This question is what we will be answering with our second classroom activity.

The class will be divided into small groups, and each of these groups will design an advertising poster for their new restaurants. Each group's poster must include the restaurant's name and the number of meal choices that are offered by their restaurant. The rest of the design of the poster is up to the group, but the students need to remember that they cannot put false facts on their advertisement.

Before they begin, assign each group their menu items. Instruct the students that their restaurants serve meals only. The customer must choose a complete meal, which includes one choice from each of the restaurants food and drink categories (main course, side item, drink, and dessert – if available). The group's first task should be to figure out how many meal choices that their restaurant will be able to offer by using the Counting Principle. Their next task will be to make a list of all of their different meal choices using a tree diagram. Then they should design their poster to advertise their new restaurant. They do not have to list menu items on their poster, but they can if they think it will help draw people to their restaurant.

Group 1 menu items:

Sandwiches: ham, turkey, or tuna salad

Chips: plain or barbeque

Drinks: milk, chocolate milk, or water

Total number of meal choices: 3 x 2 x 3 = 18

Group 1 List of Meal Choices

	Sandwiches	Chips	Drinks
1.	Turkey	Plain	Milk
2.	Turkey	Plain	Chocolate Milk
3.	Turkey	Plain	Water
4.	Turkey	Barbeque	Milk
5.	Turkey	Barbeque	Chocolate Milk
6.	Turkey	Barbeque	Water
7.	Ham	Plain	Milk
8.	Ham	Plain	Chocolate Milk
9.	Ham	Plain	Water
10.	Ham	Barbeque	Milk
11.	Ham	Barbeque	Chocolate Milk
12.	Ham	Barbeque	Water
13.	Tuna Salad	Plain	Milk
14.	Tuna Salad	Plain	Chocolate Milk
15.	Tuna Salad	Plain	Water
16.	Tuna Salad	Barbeque	Milk
17.	Tuna Salad	Barbeque	Chocolate Milk
18.	Tuna Salad	Barbeque	Water

Group 2 menu items:

Pizza: Pepperoni, sausage, or cheese

Sizes: Small, medium, or large

Crust: Thin or thick

Drinks: Lemonade, tea, or water

Total number of meal choices: 3 x 3 x 2 x 3 = 54

Group 2 List of Meal Choices

	Pizza	Sizes	Crust	Drinks
1.	Pepperoni	Small	Thin	Lemonade
2.	Pepperoni	Small	Thin	Tea
3.	Pepperoni	Small	Thin	Water
4.	Pepperoni	Small	Thick	Lemonade
5.	Pepperoni	Small	Thick	Tea
6.	Pepperoni	Small	Thick	Water
7.	Pepperoni	Medium	Thin	Lemonade
8.	Pepperoni	Medium	Thin	Tea
9.	Pepperoni	Medium	Thin	Water
10.	Pepperoni	Medium	Thick	Lemonade
11.	Pepperoni	Medium	Thick	Tea
12.	Pepperoni	Medium	Thick	Water
13.	Pepperoni	Large	Thin	Lemonade
14.	Pepperoni	Large	Thin	Tea
15.	Pepperoni	Large	Thin	Water
16.	Pepperoni	Large	Thick	Lemonade
17.	Pepperoni	Large	Thick	Tea
18.	Pepperoni	Large	Thick	Water
19.	Sausage	Small	Thin	Lemonade
20.	Sausage	Small	Thin	Tea
21.	Sausage	Small	Thin	Water
22.	Sausage	Small	Thick	Lemonade
23.	Sausage	Small	Thick	Tea
24.	Sausage	Small	Thick	Water
25.	Sausage	Medium	Thin	Lemonade
26.	Sausage	Medium	Thin	Tea
27.	Sausage	Medium	Thin	Water
28.	Sausage	Medium	Thick	Lemonade
29.	Sausage	Medium	Thick	Tea
30.	Sausage	Medium	Thick	Water
31.	Sausage	Large	Thin	Lemonade
32.	Sausage	Large	Thin	Tea
33.	Sausage	Large	Thin	Water
34.	Sausage	Large	Thick	Lemonade
35.	Sausage	Large	Thick	Tea
36.	Sausage	Large	Thick	Water

37. Cheese	Small	Thin	Lemonade
38. Cheese	Small	Thin	Tea
39. Cheese	Small	Thin	Water
40. Cheese	Small	Thick	Lemonade
41. Cheese	Small	Thick	Tea
42. Cheese	Small	Thick	Water
43. Cheese	Medium	Thin	Lemonade
44. Cheese	Medium	Thin	Tea
45. Cheese	Medium	Thin	Water
46. Cheese	Medium	Thick	Lemonade
47. Cheese	Medium	Thick	Tea
48. Cheese	Medium	Thick	Water
49. Cheese	Large	Thin	Lemonade
50. Cheese	Large	Thin	Tea
51. Cheese	Large	Thin	Water
52. Cheese	Large	Thick	Lemonade
53. Cheese	Large	Thick	Tea
54. Cheese	Large	Thick	Water

Group 3 menu items:

Peanut Butter: Smooth or chunky

Jelly: Grape, strawberry, or peach

Bread: Wheat or white

Drinks: Pepsi, Mt. Dew, Dr. Pepper, or Water

Total number of meal choices: 2 x 3 x 2 x 4 = 48

Group 3 List of Meal Choices

	Peanut Butter	Jelly	Bread	Drinks
1.	Smooth	Grape	Wheat	Pepsi
2.	Smooth	Grape	Wheat	Mt. Dew
3.	Smooth	Grape	Wheat	Dr. Pepper
4.	Smooth	Grape	Wheat	Water
5.	Smooth	Grape	White	Pepsi
6.	Smooth	Grape	White	Mt. Dew
7.	Smooth	Grape	White	Dr. Pepper
8.	Smooth	Grape	White	Water
9.	Smooth	Strawberry	Wheat	Pepsi
10.	Smooth	Strawberry	Wheat	Mt. Dew

11. Smooth	Strawberry	Wheat	Dr. Pepper
12. Smooth	Strawberry	Wheat	Water
13. Smooth	Strawberry	White	Pepsi
14. Smooth	Strawberry	White	Mt. Dew
15. Smooth	Strawberry	White	Dr. Pepper
16. Smooth	Strawberry	White	Water
17. Smooth	Peach	Wheat	Pepsi
18. Smooth	Peach	Wheat	Mt. Dew
19. Smooth	Peach	Wheat	Dr. Pepper
20. Smooth	Peach	Wheat	Water
21. Smooth	Peach	White	Pepsi
22. Smooth	Peach	White	Mt. Dew
23. Smooth	Peach	White	Dr. Pepper
24. Smooth	Peach	White	Water
25. Chunky	Grape	Wheat	Pepsi
26. Chunky	Grape	Wheat	Mt. Dew
27. Chunky	Grape	Wheat	Dr. Pepper
28. Chunky	Grape	Wheat	Water
29. Chunky	Grape	White	Pepsi
30. Chunky	Grape	White	Mt. Dew
31. Chunky	Grape	White	Dr. Pepper
32. Chunky	Grape	White	Water
33. Chunky	Strawberry	Wheat	Pepsi
34. Chunky	Strawberry	Wheat	Mt. Dew
35. Chunky	Strawberry	Wheat	Dr. Pepper
36. Chunky	Strawberry	Wheat	Water
37. Chunky	Strawberry	White	Pepsi
38. Chunky	Strawberry	White	Mt. Dew
39. Chunky	Strawberry	White	Dr. Pepper
40. Chunky	Strawberry	White	Water
41. Chunky	Peach	Wheat	Pepsi
42. Chunky	Peach	Wheat	Mt. Dew
43. Chunky	Peach	Wheat	Dr. Pepper
44. Chunky	Peach	Wheat	Water
45. Chunky	Peach	White	Pepsi
46. Chunky	Peach	White	Mt. Dew
47. Chunky	Peach	White	Dr. Pepper
48. Chunky	Peach	White	Water

Group 4 menu items:

Main Course: Hamburger, cheeseburger, or chicken nuggets

Side item: French fries or fruit cup

Drink: Mt. Dew or water

Desert: Chocolate ice cream or vanilla ice cream

Total number of meal choices: 3 x 2 x 2 x 2 = 24

Group 4 List of Meal Choices

	Main Course	Side Item	Drinks	Dessert
1.	Hamburger	French fries	Mt. Dew	Chocolate ice cream
2.	Hamburger	French fries	Mt. Dew	Vanilla ice cream
3.	Hamburger	French fries	Water	Chocolate ice cream
4.	Hamburger	French fries	Water	Vanilla ice cream
5.	Hamburger	Fruit cup	Mt. Dew	Chocolate ice cream
6.	Hamburger	Fruit cup	Mt. Dew	Vanilla ice cream
7.	Hamburger	Fruit cup	Water	Chocolate ice cream
8.	Hamburger	Fruit cup	Water	Vanilla ice cream
9.	Cheeseburger	French fries	Mt. Dew	Chocolate ice cream
10.	Cheeseburger	French fries	Mt. Dew	Vanilla ice cream
11.	Cheeseburger	French fries	Water	Chocolate ice cream
12.	Cheeseburger	French fries	Water	Vanilla ice cream
13.	Cheeseburger	Fruit cup	Mt. Dew	Chocolate ice cream
14.	Cheeseburger	Fruit cup	Mt. Dew	Vanilla ice cream
15.	Cheeseburger	Fruit cup	Water	Chocolate ice cream
16.	Cheeseburger	Fruit cup	Water	Vanilla ice cream
17.	Chicken nuggets	French fries	Mt. Dew	Chocolate ice cream
18.	Chicken nuggets	French fries	Mt. Dew	Vanilla ice cream
19.	Chicken nuggets	French fries	Water	Chocolate ice cream
20.	Chicken nuggets	French fries	Water	Vanilla ice cream
21.	Chicken nuggets	Fruit cup	Mt. Dew	Chocolate ice cream
22.	Chicken nuggets	Fruit cup	Mt. Dew	Vanilla ice cream
23.	Chicken nuggets	Fruit cup	Water	Chocolate ice cream
24.	Chicken nuggets	Fruit cup	Water	Vanilla ice cream

Group 5 menu items:

Main Course: Hotdog, corndog, or popcorn chicken

Side item: Curly fries, French fries, or onion rings

Drinks: Water or tea

Total number of meal choices: 3 x 3 x 2 = 18

Group 5 List of Meal Choices

	Main Course	Side item	Drink
1.	Hotdog	Curly fries	Water
2.	Hotdog	Curly fries	Tea
3.	Hotdog	French fries	Water
4.	Hotdog	French fries	Tea
5.	Hotdog	Onion rings	Water
6.	Hotdog	Onion rings	Tea
7.	Corndog	Curly fries	Water
8.	Corndog	Curly fries	Tea
9.	Corndog	French fries	Water
10.	Corndog	French fries	Tea
11.	Corndog	Onion rings	Water
12.	Corndog	Onion rings	Tea
13.	Popcorn chicken	Curly fries	Water
14.	Popcorn chicken	Curly fries	Tea
15.	Popcorn chicken	French fries	Water
16.	Popcorn chicken	French fries	Tea
17.	Popcorn chicken	Onion rings	Water
18.	Popcorn chicken	Onion rings	Tea

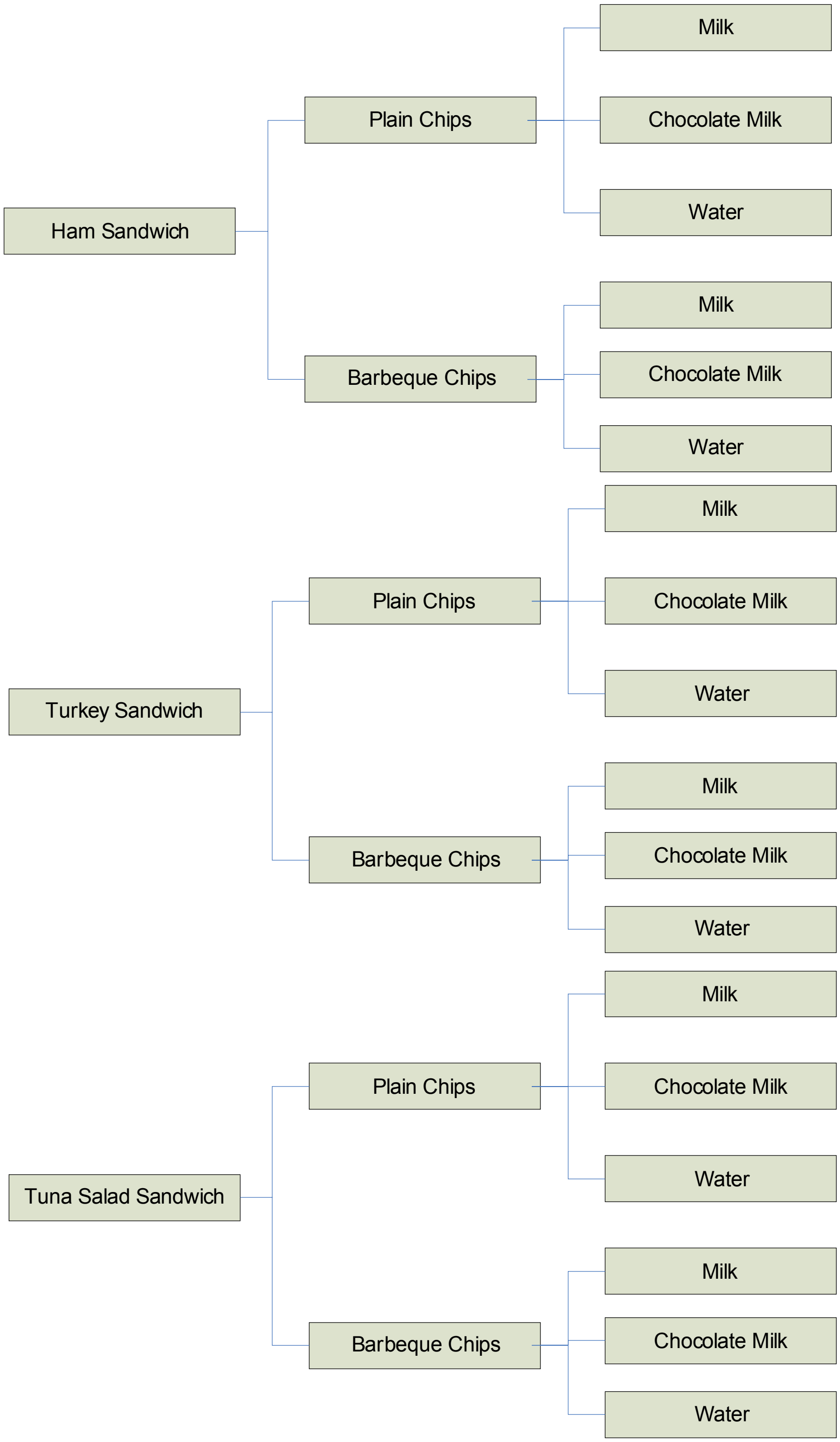


Figure 11: Group 1 Tree Diagram of Meal Choices

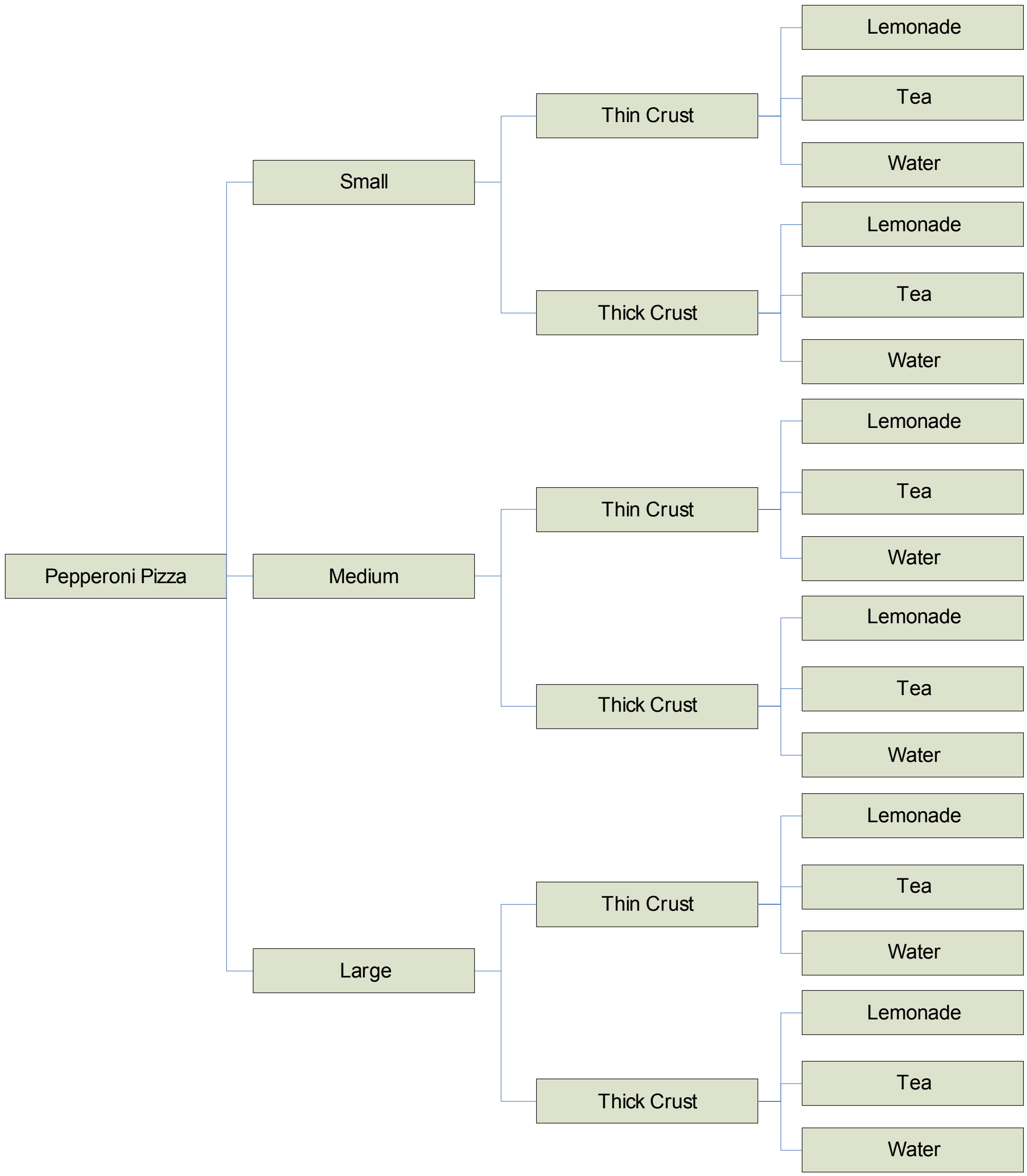
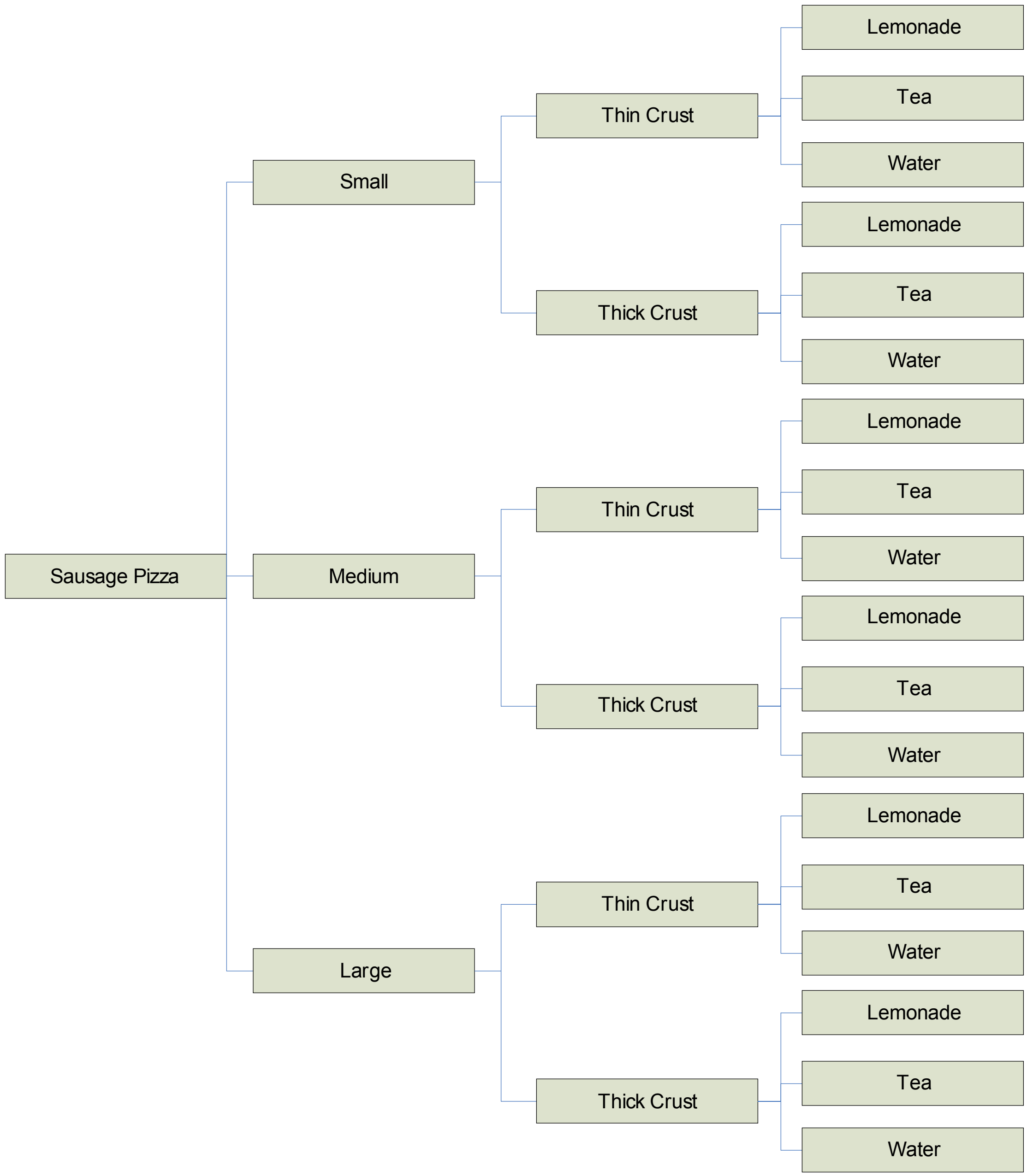


Figure 12: Group 2 Tree Diagram of Meal Choices



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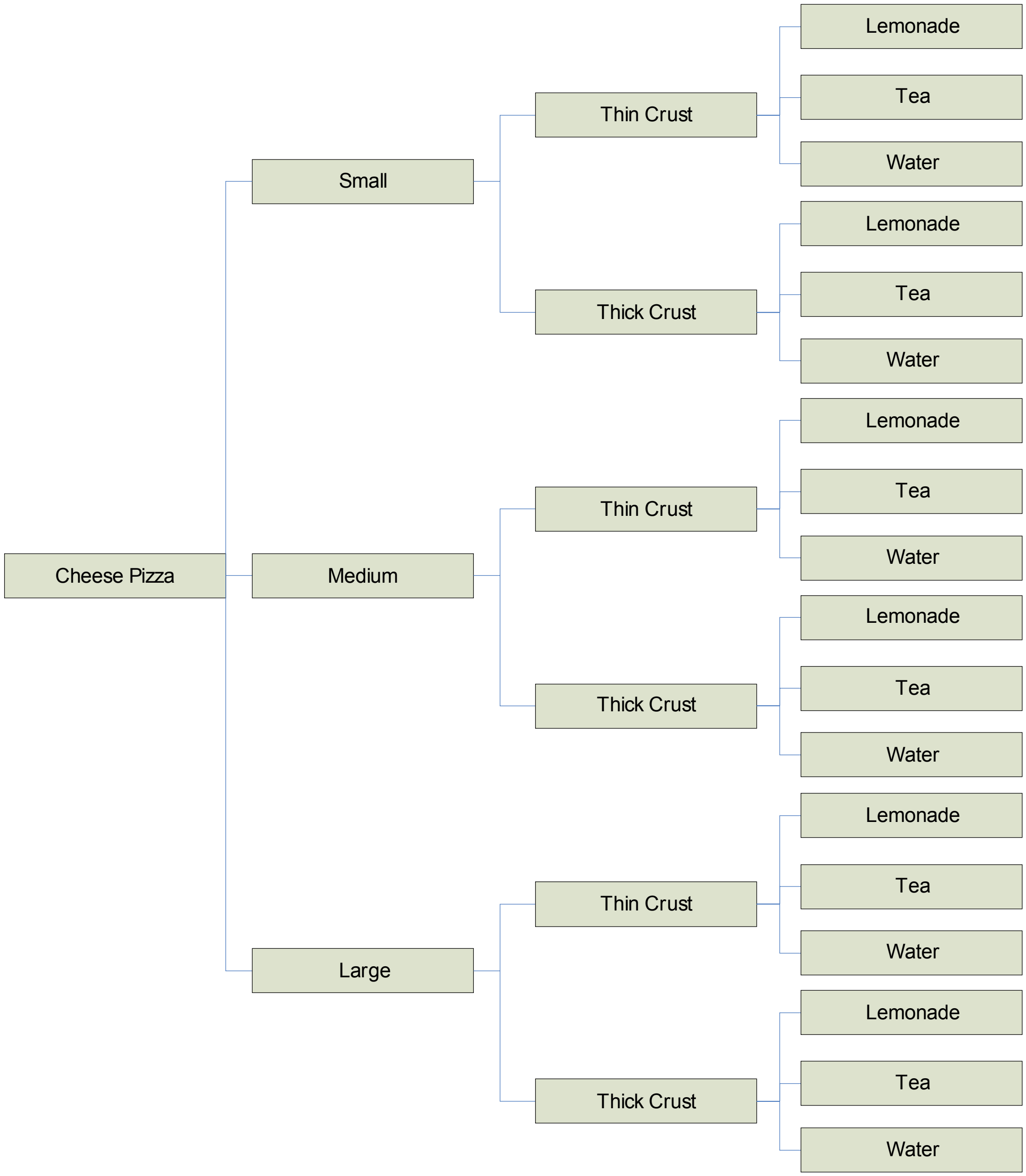

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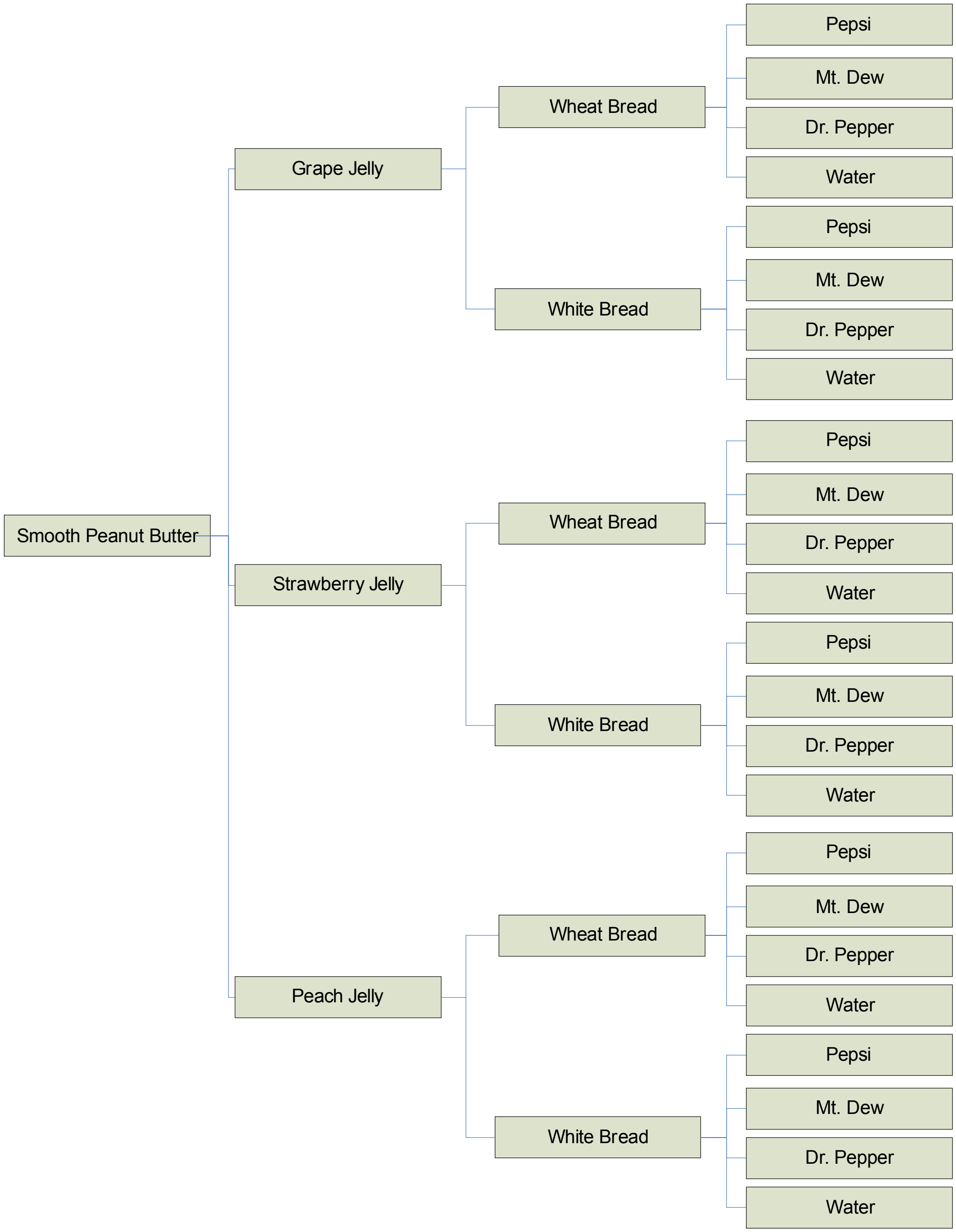
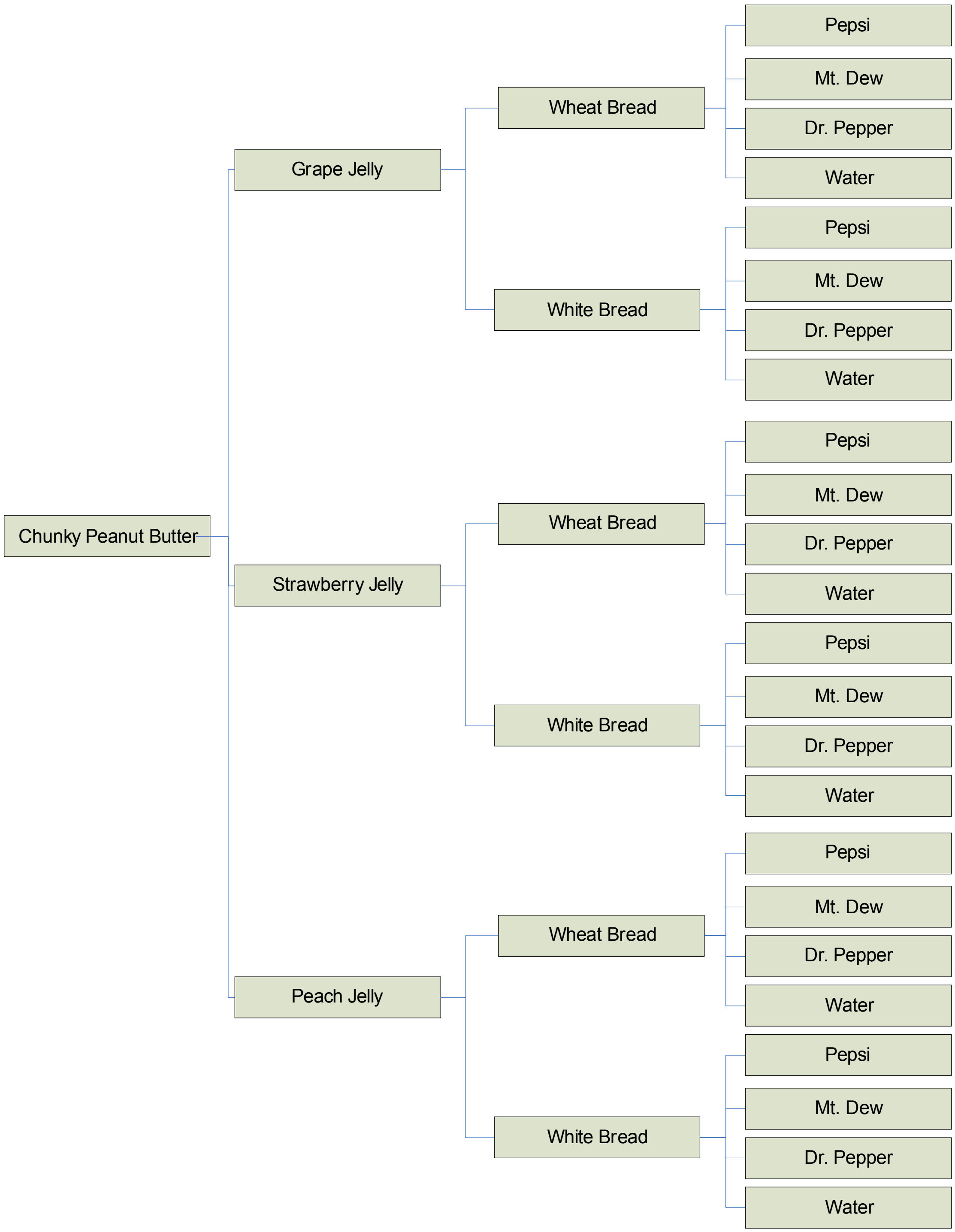


Figure 13: Group 3 Tree Diagram of Meal Choices



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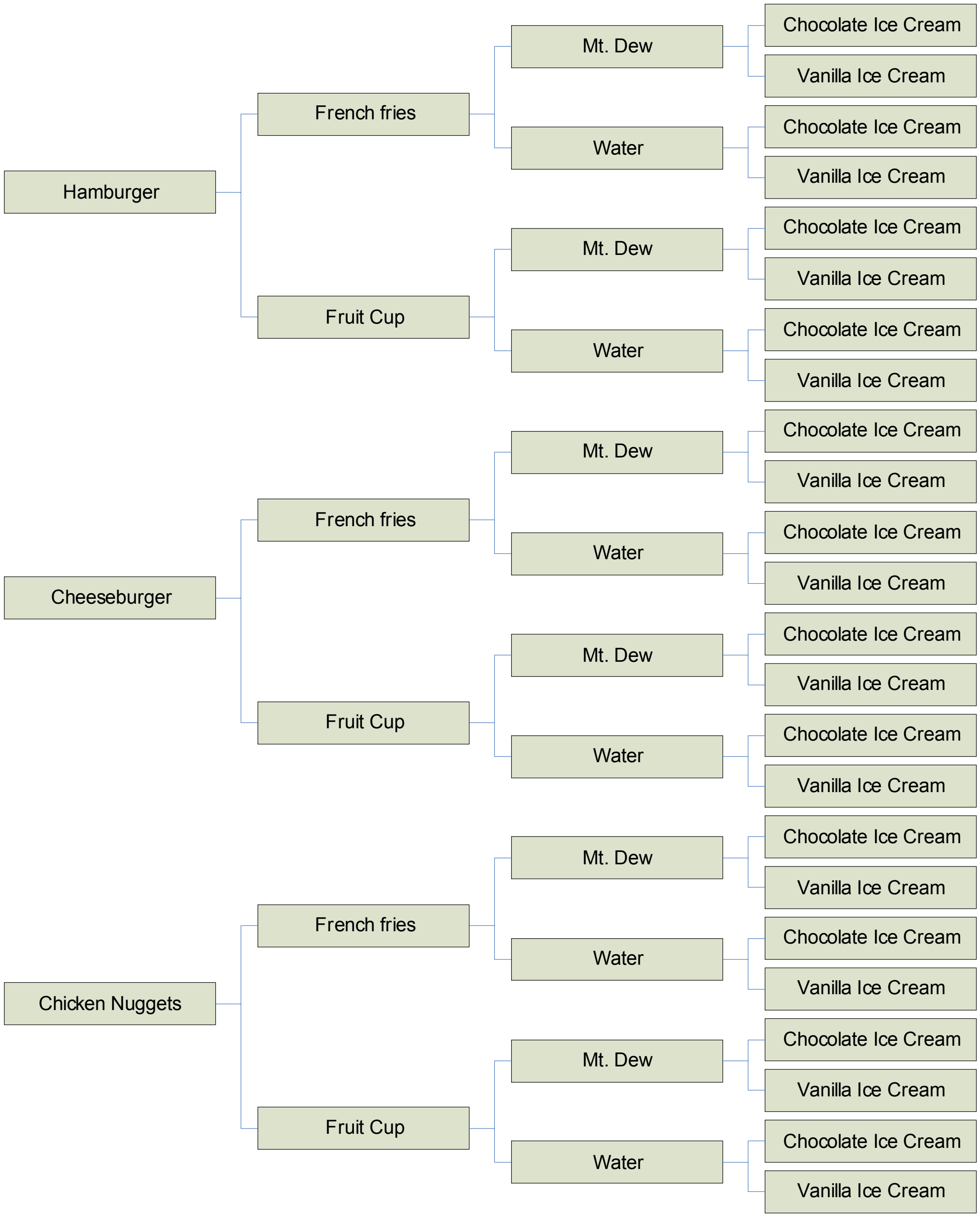

Figure 14: Group 4 Tree Diagram of Meal Choices

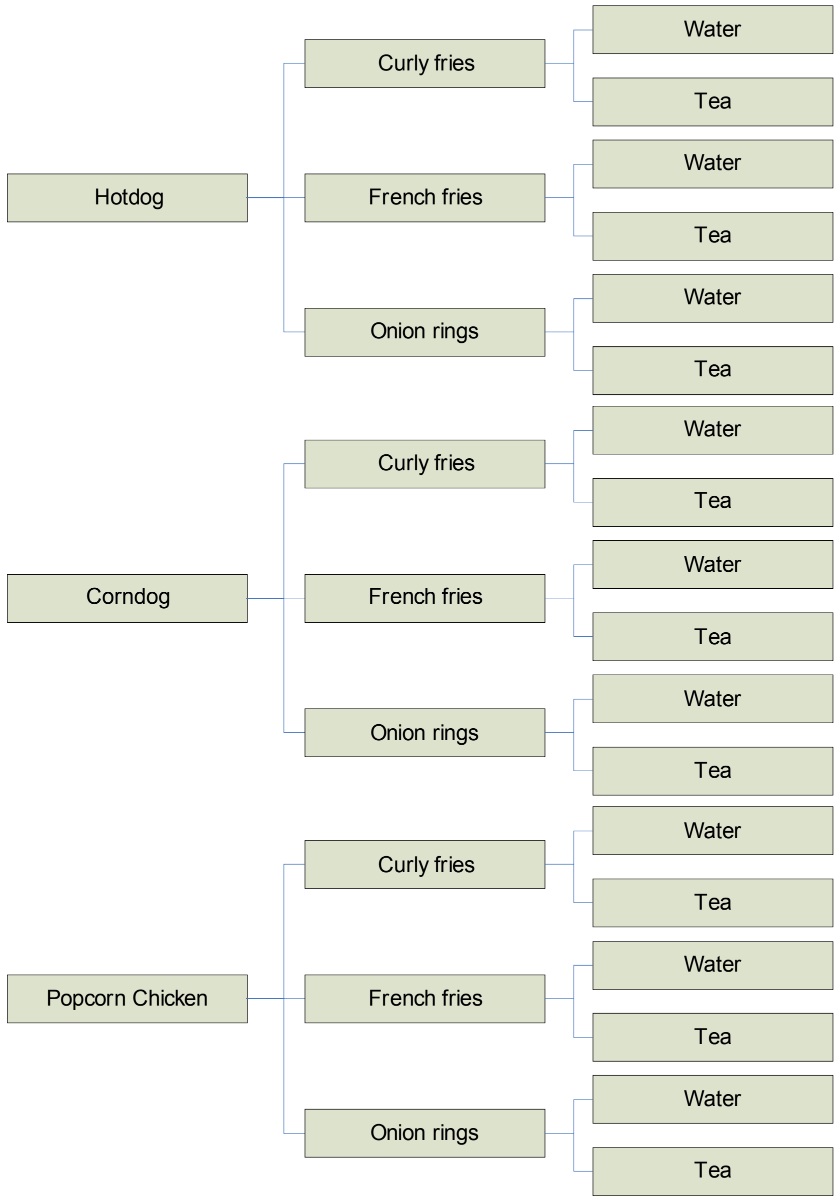

Figure 15: Group 5 Tree Diagram of Meal Choices

3.3 Homework Exercises

Directions:

Identify the sample space of each event. Use the Counting Principle to identify the total number of possible outcomes. Then draw a tree diagram and make a list of all of the possible outcomes for event.

1. You are going on vacation for 10 days, and you are only allowed to pack one small suitcase. There will be a place to wash your clothes. If you pack 3 pairs of shorts (black, tan, and white) and 4 shirts (red, pink, yellow, and blue), will you have enough clothes to wear a different outfit each day of your vacation? How many outfit choices do you have?
2. Your class has decided to elect a King and a Queen to represent the class in the fall festival. Chris, Dustin, and Trent are running for King and Sarah and Abby are running for Queen. If each student has an equal chance of being picked, how many different King and Queen combinations does your class have?
3. Your teacher has asked you to design a poster for an art competition. The rules state that you can use one color of poster (white or beige) and two colors of markers (one color must be black or brown, and the other color must be red, orange, blue, or purple). How many different poster and marker combinations are possible?
4. The local movie theater is showing four different movies – comedy, cartoon, action, and drama. They are showing these movies at 1:00, 3:00, and 5:00. You also have enough money to buy one snack – popcorn, nachos, or a candy bar. How many different movie/time/snack combinations do you have from which to choose?

3.4 Homework Exercises Answers

1. Total number of outfit choices: 3 x 4 =12 (yes, there is enough clothes to wear a different outfit each day.

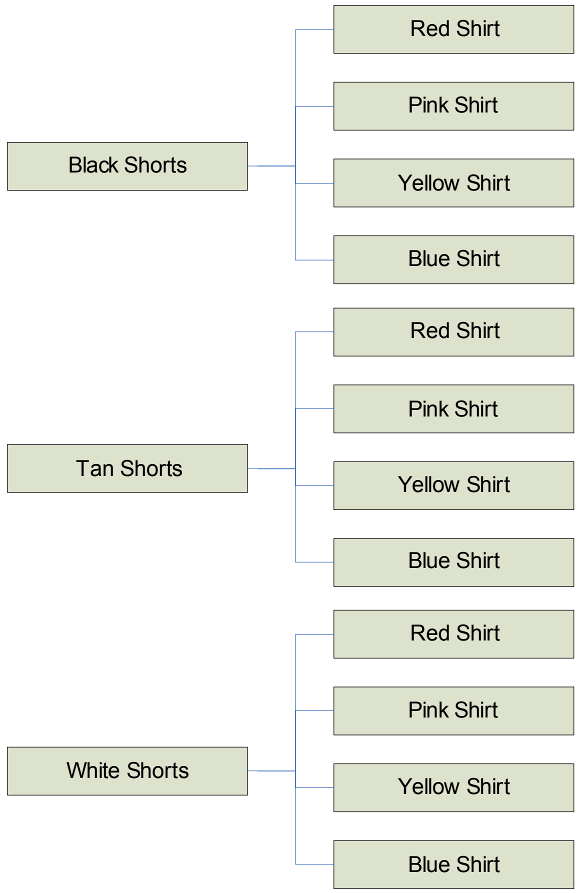

Figure 16: Homework #1 Tree Diagram of Outfit Choices

<u>List of Outfit Choices</u>

Shorts	Shirts
Black	Red
Black	Pink
Black	Yellow
Black	Blue
Tan	Red
Tan	Pink
Tan	Yellow
Tan	Blue
White	Red
White	Pink

White	Yellow
White	Blue

2. Total Number of King and Queen combinations: 3 x 2 = 6

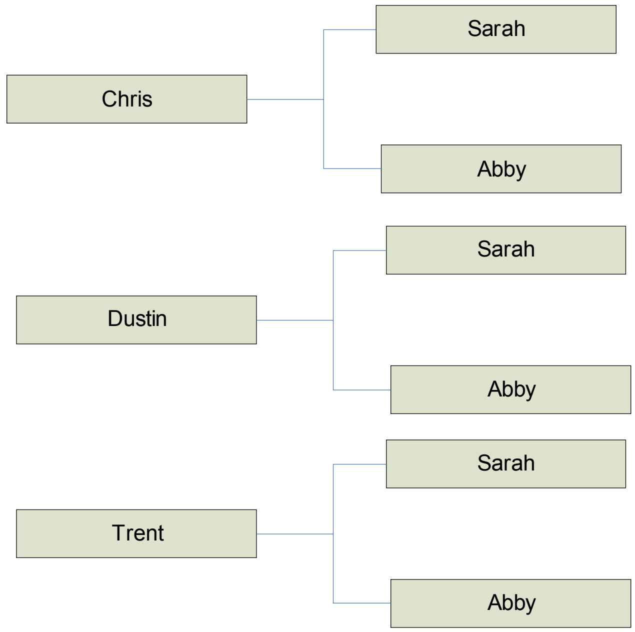

Figure 17: Homework #2 Tree Diagram of King/Queen Combinations

List of King/Queen Combinations

King	Queen
Chris	Sarah
Chris	Abby
Dustin	Sarah
Dustin	Abby
Trent	Sarah
Trent	Abby

3. Total number of poster and marker combinations: 2 x 2 x 4 = 16

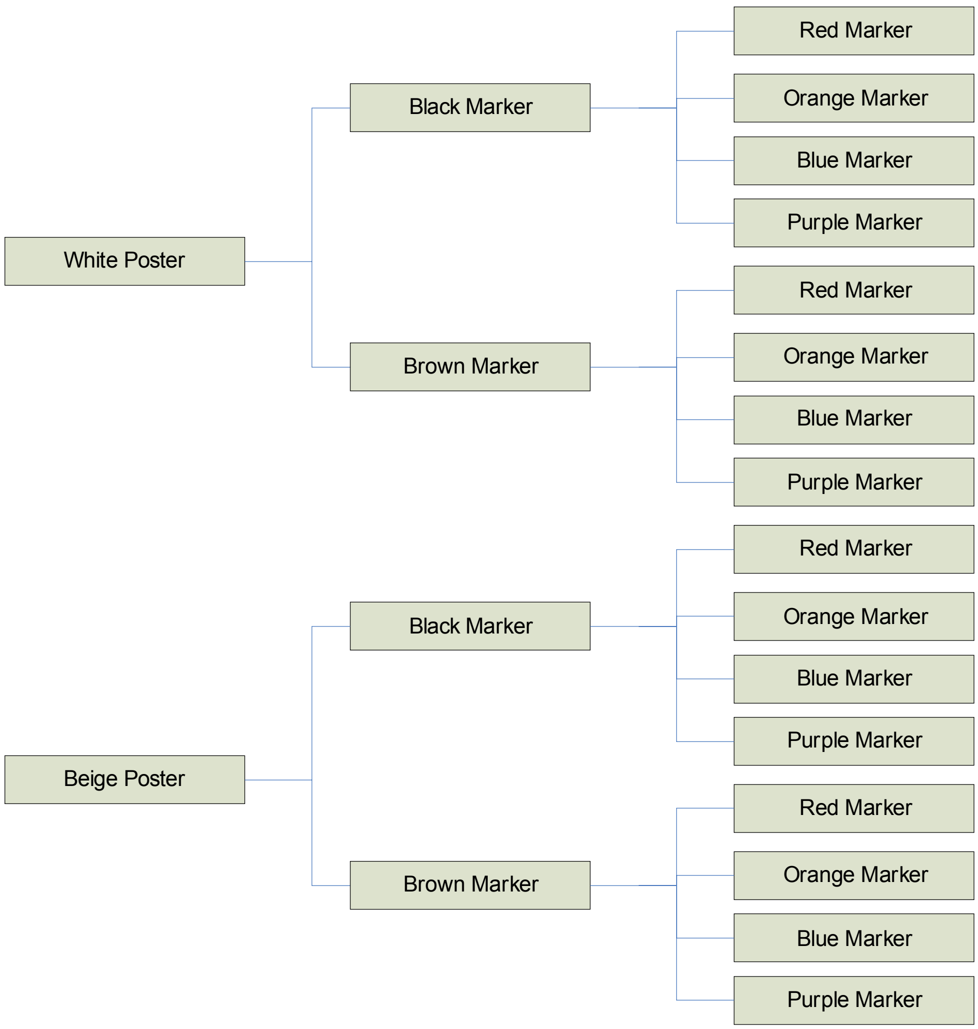

Figure 18: Homework #3 Tree Diagram of Poster & Marker Choices

Poster & Marker List

Poster	1st Marker	2nd Marker
White	Black	Red
White	Black	Orange
White	Black	Blue
White	Black	Purple
White	Brown	Red
White	Brown	Orange
White	Brown	Blue
White	Brown	Purple
Beige	Black	Red

Beige	Black	Orange
Beige	Black	Blue
Beige	Black	Purple
Beige	Brown	Red
Beige	Brown	Orange
Beige	Brown	Blue
Beige	Brown	Purple

4. Total number of movie, time, and snack combinations: 4 x 3 x 3 = 36

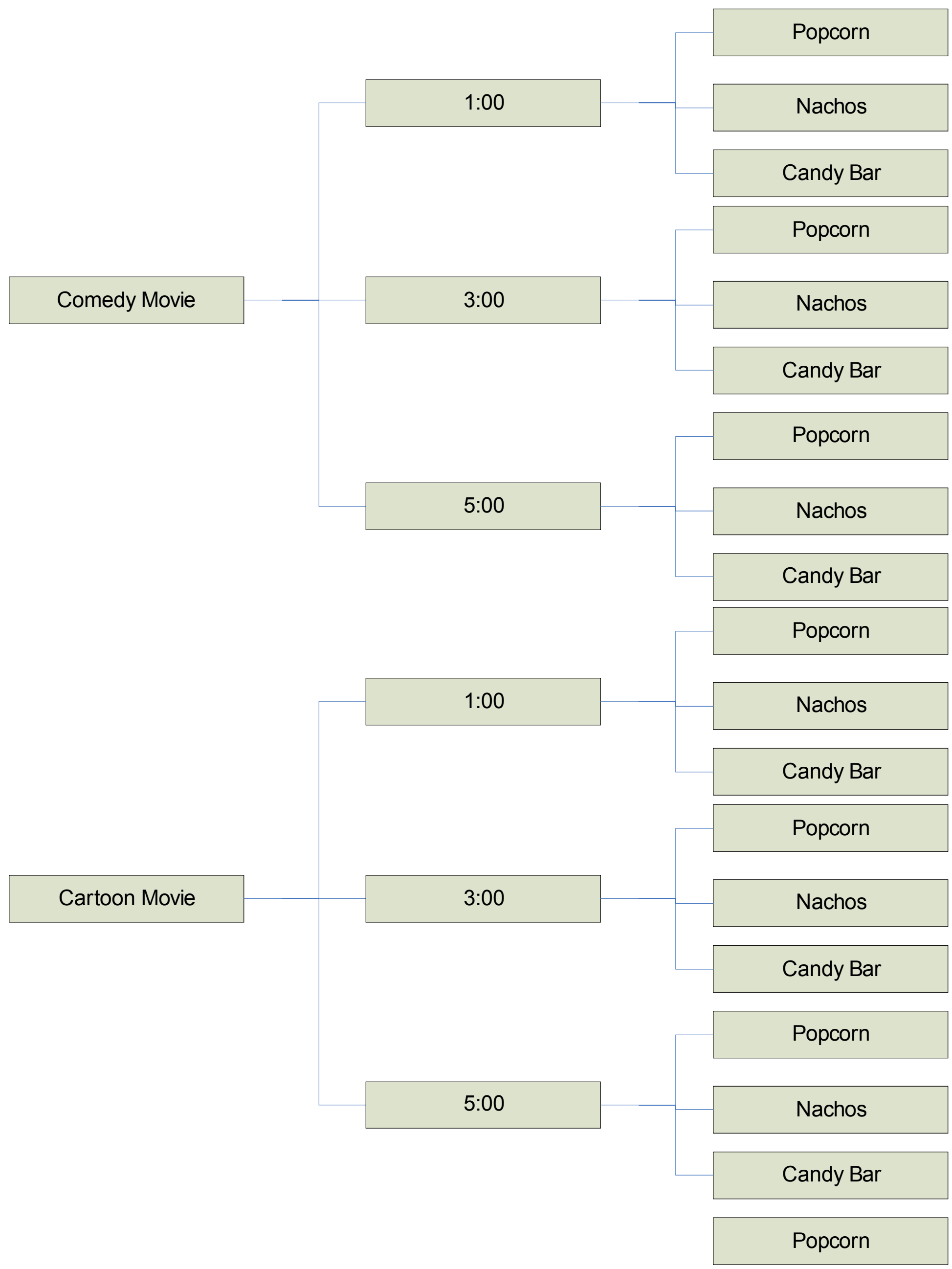

Figure 19: Homework #4 Tree Diagram of Movie Choices

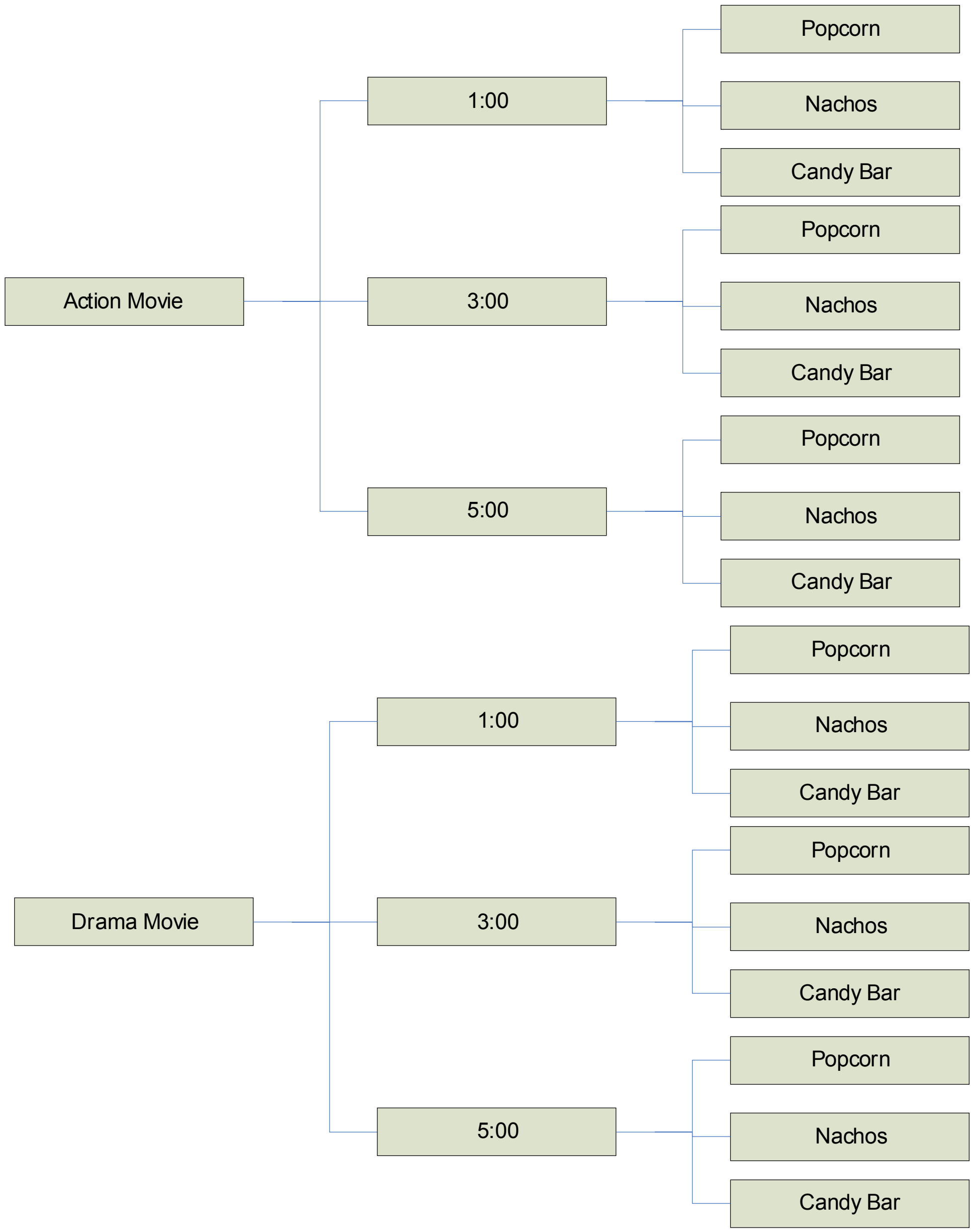

Figure 19 (continued)

List of Movie Choices

Movie	Time	Snack
Comedy	1:00	Popcorn
Comedy	1:00	Nachos
Comedy	1:00	Candy Bar
Comedy	3:00	Popcorn
Comedy	3:00	Nachos
Comedy	3:00	Candy Bar
Comedy	5:00	Popcorn
Comedy	5:00	Nachos
Comedy	5:00	Candy Bar
Cartoon	1:00	Popcorn
Cartoon	1:00	Nachos
Cartoon	1:00	Candy Bar
Cartoon	3:00	Popcorn
Cartoon	3:00	Nachos
Cartoon	3:00	Candy Bar
Cartoon	5:00	Popcorn
Cartoon	5:00	Nachos
Cartoon	5:00	Candy Bar
Action	1:00	Popcorn
Action	1:00	Nachos
Action	1:00	Candy Bar
Action	3:00	Popcorn
Action	3:00	Nachos
Action	3:00	Candy Bar
Action	5:00	Popcorn
Action	5:00	Nachos
Action	5:00	Candy Bar
Drama	1:00	Popcorn
Drama	1:00	Nachos
Drama	1:00	Candy Bar
Drama	3:00	Popcorn
Drama	3:00	Nachos
Drama	3:00	Candy Bar
Drama	5:00	Popcorn
Drama	5:00	Nachos
Drama	5:00	Candy Bar

4 DETERMINING PROBABILITY

4.1 Introduction

We hear and see probability used every day, and we make decisions based upon these probabilities. In this unit, we will discuss a few ways that probability is used in our lives. Then, we will determine the probability of a variety of different events using fraction, decimal, and percentage representations.

First, ask the students if they heard probability used this morning before coming to school. The first use of probability that the students think of will probably be the weather forecast. What did the weatherman predict today's weather to be? Is the weather forecast ever wrong? Why do you think that the forecast is sometimes wrong? Ask the students what it means if the forecast is predicting a 20 percent chance of rain today and an 80 percent chance of rain tomorrow. Is it definitely going to rain today or tomorrow? Is it more likely to rain on either of these two days? If it doesn't rain either day, was the forecast wrong?

After the students have had a chance to discuss the weather forecast, they should recognize that probability plays a major role in forecasting the weather. Meteorologists use the weather that has occurred in the past to predict what the weather will be like in the future. This is an example of empirical probability, which is defined as an estimate that an event will occur based on how often the event has occurred in the past. The meteorologists first examine the day's weather characteristics such as temperature, humidity, and barometric pressure, and then they search weather databanks for days that have had these same characteristics. They base their weather predictions upon what the weather was like on these similar days in the past. For example, if the meteorologist found ten days with the same weather characteristics as today, and it rained on three of those days, then the weatherman would predict that there is a 30% chance of rain today.

Before discussing other ways that we see probability used in the real world, let's stop and review how to write probability as a fraction. Then, we need to demonstrate how to convert this fraction to a decimal and a percentage. In order to write probability as a fraction, we need a numerator, which is the number of favorable outcomes, and a denominator, which is the total number of possible outcomes. For example, what is the probability that you will roll a 6 if you are rolling a die one time? The probability is 1/6 because there is only one favorable outcome out of 6 possible outcomes. To change this fraction to a decimal we need to divide the numerator by the denominator. We will round this decimal to the hundredths place because it will make it easier to change the decimal to a percentage (1 ÷ 6 = 0.17). To change this decimal to a percentage, we will move the decimal two places to the right and add the percent sign, so 0.17 will become 17%. Now, ask the students to write the probability as a fraction, decimal, and percent for the following situations.

Situation 1: Chris has 3 red marbles, 2 blue marbles, and 4 yellow marbles in a bag.

- What is the probability of picking a red marble without looking?

  Fraction: 3/9 Decimal: 0.33 Percentage: 33%

- What is the probability of picking a blue marble without looking?

  Fraction: 2/9 Decimal: 0.22 Percentage: 22%

- What is the probability of picking a yellow marble without looking?

  Fraction: 4/9 Decimal: 0.44 Percentage: 44%

Situation 2: Sarah is rolling one regular six sided die.

- What is the probability that the first roll will be an even number?

  Fraction: 3/6 Decimal: 0.50 Percentage: 50%

- What is the probability that the first roll will be a number greater than 4?

  Fraction: 2/6 Decimal: 0.33 Percentage: 33%

- What is the probability that the first roll will be an odd number?

  Fraction: 3/6 Decimal: 0.50 Percentage: 50%

Another way that many students are familiar with probability is in its use in sports. We are going to discuss how probability is used in baseball and basketball. First, most students have heard about a baseball player's batting average, but do they know what it really means? One of the best hitters in Major League Baseball history was Hank Aaron. His career batting average was .305 [5]. This means that out of 1000 times at bat he was able to hit the ball 305 times, or in simpler terms, if he batted 10 times, he would only hit the ball about 3 times. As of April 26, 2006, the player with the highest batting average in the 2006 season was Miquel Tejada [4]. His batting average on this date was .434. Everyone else's batting averages were below .400. This means that the best batters in Major League Baseball are not able to hit the ball even half of the times they are at bat. Does this mean that these baseball players are not very good batters, or does it mean that the Major League pitchers are extremely good?

Let's take this opportunity to figure out some batting averages on our own. We want to write the batting averages as fractions, decimals, and percentages. To write the probability of getting a hit as a fraction, the number of hits is written as the numerator, and the total number of at bats becomes the denominator. To change this fraction to a decimal, the numerator (number of hits) should be divided by the denominator (total number of at bats). The quotient should be rounded to the thousandths place to write the batting average as a decimal. To change the decimal to a percentage, round the decimal to the hundredths place and then move the decimal point two places to the right (or drop the decimal point) and add the percentage sign (%). Use these techniques to find the batting averages of the following players.

Batter	**At Bats**	**Hits**	**Batting Average: Fraction**	**Batting Average: Decimal**	**Batting Average: Percentage**
Player A	119	43			
Player B	89	25			
Player C	75	15			
Player D	65	20			
Player E	88	28			
Player F	83	36			
Player G	89	33			
Player H	71	28			
Player I	80	30			
Player J	2	1			

Table 20: Batting Averages

Which of the players has the best batting average? Does this mean that Player J is the best batter? Why or why not? The students should realize that although Player J does have the best batting average, this does not mean that Player J is the best batter. Player J has not been up to bat as many times as the other batters, therefore Player J's statistics cannot be fairly compared to the other players. Out of the other nine players, who has the best batting average? Is the batter likely to get a hit the next time the batter is at bat? Do you think the pitchers know the batting averages of the players on the opposing team before they pitch to them? The following are the answers to the above table.

Batter	At Bats	Hits	Batting Average: Fraction	Batting Average: Decimal	Batting Average: Percentage
Player A	119	43	$\frac{43}{119}$	.361	36%
Player B	89	25	$\frac{25}{89}$	.281	28%
Player C	75	15	$\frac{15}{75}$	.200	20%
Player D	65	20	$\frac{20}{65}$	.308	31%
Player E	88	28	$\frac{28}{88}$	.318	32%
Player F	83	36	$\frac{36}{83}$	.434	43%
Player G	89	33	$\frac{33}{89}$	.371	37%
Player H	71	28	$\frac{28}{71}$	.394	39%
Player I	80	30	$\frac{30}{80}$	.375	38%
Player J	2	1	$\frac{1}{2}$	.500	50%

Table 21: Batting Averages Answers

Another sport that relies heavily on probability to make decisions throughout the game is basketball. In basketball, we know each player's field goal percentage, free throw percentage, and three point percentage. How do these percentages help coaches make game decisions? Let's examine two game situations where each player's shot percentages play a major role in the coach's play decisions. In the first game, your team is losing by three points, and there are three seconds left in the game. Does the coach want Player A, a 60% three point shooter, or Player B, a 30% three point shooter, to take the shot? Who will the defense try to keep from getting the ball? Why? The students should quickly realize that the 60% shooter is more likely to make the shot, but neither player will definitely make the shot. How many three point shots is the 60% shooter likely to make out of 10 shots? This player is likely to make the shot 6 times out of 10. In the second game, your team is losing by 5 points with one minute left in the game. The other team has the ball, and they want to keep the ball as long as they can to run down the clock. Your team doesn't have any more time outs to stop the clock, so your team has to foul a player on the other team to stop the clock and get the ball back. The player you foul will shoot foul shots before your team gets the ball back, and your team doesn't want the other team to score any more. Which player do you want to foul – Player 1, a 74% free throw shooter, or Player 2, a 63% free throw shooter? You want to foul Player 2 because this player is less likely to make the free throw. To reinforce how to find probability as a percentage, have the students find the field goal percentage (FG %), free throw percentage (FT %), and three point percentage (TP %) for each player on the following basketball team.

Player	**Field Goal Made**	**Field Goal Attempt**	**FG%**	**Three Point Made**	**Three Point Attempt**	**TP%**	**Free Throw Made**	**Free Throw Attempt**	**FT%**
1	199	349		2	4		187	253	
2	145	306		45	118		108	135	
3	158	297		39	92		45	72	
4	81	187		27	76		42	53	
5	70	160		64	145		18	25	
6	68	182		27	87		34	44	
7	70	172		10	41		44	61	

Table 22: Basketball Stats

The following chart includes the percentages that the students should have computed.

Player	Field Goal Made	Field Goal Attempt	FG%	Three Point Made	Three Point Attempt	TP%	Free Throw Made	Free Throw Attempt	FT%
1	199	349	57%	2	4	50%	187	253	74%
2	145	306	47%	45	118	38%	108	135	80%
3	158	297	53%	39	92	42%	45	72	63%
4	81	187	43%	27	76	36%	42	53	79%
5	70	160	44%	64	145	44%	18	25	72%
6	68	182	37%	27	87	31%	34	44	77%
7	70	172	41%	10	41	24%	44	61	72%

Table 23: Basketball Stats Answers

Using the percentages in this table, ask the students the following questions.

1. Which player is most likely to make a field goal?
2. Which player has the best three point percentage?
3. Do you think this is the best three point shooter on the team? Why or why not? If you do not think this is the best three point shooter, then which player is? Why?
4. Who has the best free throw percentage?
5. Are any of the players unlikely to make a free throw? Why?
6. Which player is least likely to make a field goal?
7. Which player do you think is least likely to attempt a three point shot? Why?
8. Which 5 players are most likely the five players that start the game? Why?

Answers:

1. Player 1
2. Player 1 has the best three point percentage.
3. No because this player has only tried to make a three point shot 4 times. Player 5 is the best three point shooter because he makes this shot 44% of the time, and he has attempted to make this shot many times.
4. Player 2
5. No because they all make more than 50% of their free throws.
6. Player 6
7. Player 1 because he has only attempted 4 three point shots which is a much lower number of attempted shots than the rest of the team.
8. Players 1, 2, 3, 4, and 5 because they have the best averages.

4.2 Classroom Activities

In order to give the students an opportunity to practice writing probability as fractions, decimals, and percentages, we are going to play "Laffability." "Laffability" is a game that was adapted from Diana Freeman's "Sweet Probability" [1]. During this game, the students will be determining the probability of picking their favorite flavor of Laffy Taffy out of a paper bag without looking. The game "Laffability" will be played according to the following rules.

1. Each student will choose his or her favorite flavor of Laffy Taffy and place this piece of candy into the paper bag at the front of the room.
2. The teacher will then count the number of each flavor in the bag and write these totals on the board.
3. Each student should receive one copy of the "Laffability" chart and a calculator.
4. The students should fill in their chosen favorite flavor in the blank on the chart. Then, they need to write the total number of that flavor in the bag in the favorite flavor column. They also need to write the total number of pieces of candy in the bag in the total column.
5. Now, they are ready to calculate the probability of picking their favorite flavor. Help them fill in the probabilities if needed. Here is an example for a student that picked cherry as her favorite flavor. For this example, 10 students picked cherry as their favorite and there are a total of 25 students in the class.

Number in the Bag		Probability of Picking Your Favorite Flavor			
My Favorite Flavor: Cherry	Total of all Flavors	Fraction	Decimal	Percentage	Likelihood of picking favorite flavor
10	25	10/25	0.40	40%	

Table 24: Laffability Chart Example

6. After everyone has calculated their probability of drawing their favorite flavor from the bag, instruct them to describe their chances of drawing their favorite flavor in the last column. They should use the following descriptions: Certain- if their favorite flavor is the only flavor in the bag, Likely- if at least half, 50%, of the candy left in the bag is their favorite flavor, Unlikely- if their favorite flavor is less than half of the total number of pieces of candy in the bag, Impossible- only if there aren't any of their favorite flavor left in the bag. So in the example used above, this student should list her probability as unlikely.
7. After everyone has filled in the first row of the chart, ask the first student to describe his probability of picking his favorite flavor. Then let this student pick a piece of Laffy Taffy from the bag without looking. Did he get his flavor? Is this what we thought would happen?
8. This student gets to keep and eat the candy he drew out of the bag. Therefore, totals and probability will need to be recalculated after each student picks a piece of Laffy Taffy from the bag.
9. The game will continue until every student has drawn a piece of candy from the bag. Everyone should continue filling in their charts, even if they have already picked from the bag.

LAFFABILITY CHART

Number in the Bag		Probability of Picking Your Favorite Flavor			
My Favorite Flavor: ______________	Total of all Flavors	Fraction	Decimal	Percentage	Likelihood of picking favorite flavor

Table 25: Laffability Chart Template

4.3 Homework Exercises

Directions:

Write the probability of the following situations as a fraction, a decimal rounded to the hundredths place, and a percentage. Answer any questions related to these probabilities.

Situation 1:

Dustin's socks are all the same except for the color. He puts them all in his drawer without putting them in pairs. There are 6 white socks, 4 black socks, and 2 navy socks in the drawer.

1. What is the probability that the first sock he picks from the drawer in a dark room will be white? Fraction: __________ Decimal: __________ Percentage: _________
2. What is the probability that the first sock he picks from the drawer in a dark room will be black? Fraction: __________ Decimal: __________ Percentage: __________
3. What is the probability that the first sock he picks from the drawer in a dark room will be navy? Fraction: __________ Decimal: __________ Percentage: __________
4. If the first sock Dustin picks is white and he keeps it out of the drawer, what are his chances of picking another white sock with his very next pick? Fraction: __________ Decimal: __________ Percentage: __________
5. If the first sock Dustin picks is black and he keeps it out of the drawer, what are his chances of picking another black sock with his very next pick? Fraction: __________ Decimal: __________ Percentage: __________
6. If the first sock Dustin picks is navy and he keeps it out of the drawer, what are his chances of picking the other navy sock with his very next pick? Fraction: __________ Decimal: __________ Percentage: __________

7. Based on these probabilities is Dustin likely to have a pair of matching socks after picking only 2 socks from the drawer in a dark room? Explain.

Situation 2:

Trent and Abby are playing "Rock, Paper, Scissors." In this game each player forms a rock (fist), paper (flat hand), or scissors (2 outstretched fingers) at the same time (after the count of 3). A rock beats scissors but loses to paper, paper beats rock but loses to scissors, and scissors beat paper but lose to rock. If the players both choose the same thing, neither player wins.

1. Draw a tree diagram listing the sample space for this game: Trent has 3 possible choices (rock, paper, or scissors) and Abby has 3 possible choices (rock, paper, or scissors).
2. If Trent is represented by the first branch of the tree diagram, how many different ways can he win the game? _____ How many different ways can he lose the game? _____
3. What is Trent's probability of winning the game? Fraction: _____ Decimal: _____ Percentage: _____
4. If Abby is represented by the second branch of the tree diagram, how many different ways can she win the game? ____ How many different ways can she lose the game? ____
5. What is Abby's probability of winning the game? Fraction: _____ Decimal: _____ Percentage: _____
6. Do both players have the same chances of winning this game? ________________
7. Is this game fair? Explain.

Situation 3:

Brandon and Stephen have one bag of M&Ms and they have decided to play a game to decide who gets to keep this bag of candy. There are 7 yellow M&Ms, 16 brown M&MS, 6 red M&Ms,

and 5 green M&Ms in the bag. They have each decided to choose one color, and whoever picks their chosen color first from the bag without looking gets to keep the bag of candy.

1. What is the probability of picking each of the following colors from the bag?

   Yellow: Fraction: __________ Decimal: __________ Percentage: __________

   Brown: Fraction: __________ Decimal: __________ Percentage: __________

   Red: Fraction: __________ Decimal: __________ Percentage: __________

   Green: Fraction: __________ Decimal: __________ Percentage: __________

2. Which color should neither player be allowed to pick? Why?

3. Which two colors will give both players an almost equal chance of winning?

4.4 Homework Exercises Answers

Situation 1

1. Fraction: 6/12 Decimal: 0.5 Percentage: 50%
2. Fraction: 4/12 Decimal: 0.33 Percentage: 33%
3. Fraction: 2/12 Decimal: 0.17 Percentage: 17%
4. Fraction: 5/11 Decimal: 0.45 Percentage: 45%
5. Fraction: 3/11 Decimal: 0.27 Percentage: 27%
6. Fraction: 1/11 Decimal: 0.09 Percentage: 9%
7. Dustin is not likely to have a pair of matching socks after picking only 2 socks from the drawer because all of the probabilities of picking a match with the second pick are less than 50%.

Situation 2

1.

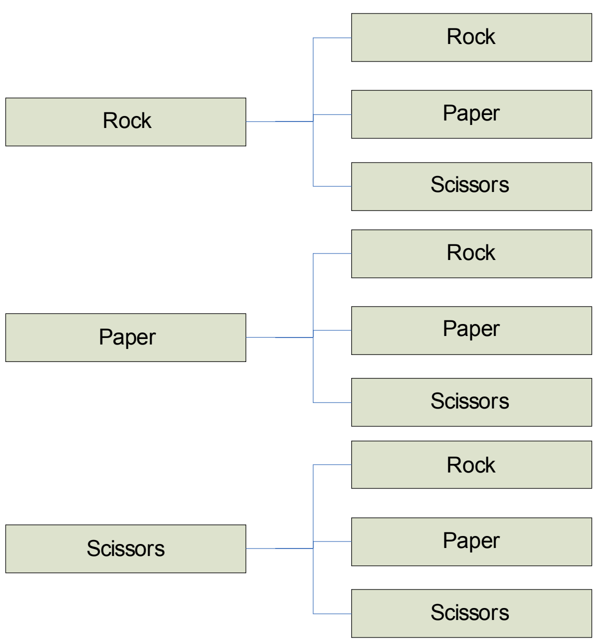

Figure 20: Rock, Paper, Scissors Tree Diagram

2. He has 3 different ways to win the game, 1 if he chooses rock, 1 if he chooses paper, and 1 if he chooses scissors. He also has 3 different ways that he can lose the game.
3. Fraction: 3/9 Decimal: 0.33 Percentage: 33%
4. She also has 3 different ways to win the game.
5. Fraction: 3/9 Decimal: 0.33 Percentage: 33%
6. Yes, the do have the same chances of winning the game.
7. The game is fair because both players have the exact same chances of winning and losing.

Situation 3

1. Yellow: Fraction: 7/34 Decimal: 0.21 Percentage: 21%

   Brown: Fraction: 16/34 Decimal: 0.47 Percentage: 47%

   Red: Fraction: 6/34 Decimal: 0.18 Percentage: 18%

   Green: Fraction: 5/34 Decimal: 0.15 Percentage: 15%
2. Neither player should be allowed to choose brown because the probability of picking brown is much better than the probability of choosing any of the other colors.
3. The closest probabilities are yellow and red or green and red.

5 SUMMARY

The four units developed in this thesis were designed as a resource for a fifth grade mathematics teacher. The units were developed to include all of the probability objectives that are included in Virginia's fifth grade mathematics Standards of Learning [3]. The activities that were used in all of the units have been successfully used in the author's fifth grade classroom. Due to the time constraints of the school year, the author has not had the opportunity to completely follow the units as developed in this thesis. Since there may not be enough time to use every detail contained within these units during the normal school year, the author selected activities that can be used successfully together or as individual activities. This thesis has only begun to explore probability at an elementary level. As a conclusion to these units on probability it is important to let the students explore a more difficult and interesting probability problem to illustrate what they can expect in their future mathematics classes.

The birthday problem [2] is a good example to use because it can be understood by a fifth grader. We want to find the probability that at least 2 people share the same birthday in this mathematics class. First, ask the students if they think that there are enough people in the room to have a 50% chance of having 2 people with the same birthday. If they don't think there are enough people, ask them how many people should be in the room to have a 50% chance of having 2 people with the same birthday. The students will probably think that there needs to be 183 people in the room because this is a little over 50% of 365, which is all of the possible birthdays. They will be completely surprised that the number of people needed is much smaller than this. To find this probability, we need to initially explain that it is easier to work out some problems, like this one, by calculating the probability that the event will not happen. Therefore, we are going to find the probability that no one in this mathematics class has the same birthday.

We are not going to count February 29, leap year, so there are a total of 365 possible outcomes. To find the probability as a decimal, we need to take the number of favorable outcomes, in this problem the number of ways that no one in the class shares the same birthday, and divide it by the total number of possible outcomes.

Let's start by writing the first student's birthday on the board and finding the probability that our first student has a birthday that doesn't match anyone else's birthday that has been written on the board. Since we haven't written any other birthdays on the board at this time, this student can have any of the 365 birthdays as a favorable outcome. This means that the first student has a probability of 365/365 of not matching anyone's birthday because there are no other birthdays written on the board to match. The second student now has 364 possible birthdays that doesn't match the first student's birthday, so the probability that the second student doesn't have the same birthday as student one is 364/365. The probability that the third student has a different birthday than the first two students is 363/365. Notice that the numerator is decreasing by 1 with each new student. Therefore, the fourth student's probability is 362/365, and the fifth student's probability is 361/365. The next part of this birthday problem is to find the combined probability of the entire class of 5 students, and we do this by multiplying all 5 numerators and then multiplying all 5 denominators. After we get these products, we will then divide the numerator by the denominator.

$$\frac{365 \times 364 \times 363 \times 362 \times 361}{365 \times 365 \times 365 \times 365 \times 365} = 0.973$$

Remember, this is the probability that there will not be anyone with the same birthday. We can find the probability that at least 2 people share the same birthday by subtracting this probability from 1 because the two probabilities, having the same birthday and not sharing a birthday, must be equal to 1. Therefore, if 0.973 is the probability that no one shares the same birthday, then 1 – 0.973 = 0.027 is the probability that at least 2 people share the same birthday in a classroom with 5 students. This rounds to a 3% chance of sharing a birthday with someone else in this classroom. This same method can be used in a classroom of any size. Let's find the probability that at least 2 people share the same birthday in classrooms with 10, 20, 23, and 30. How many students have to be in the classroom to have at least a 50% chance of at least 2 students sharing the same birthday?

Number of students in the classroom	Probability of not sharing a birthday	Probability of sharing a birthday
10	$\frac{365 \times 364 \times 363 \times \ldots \times 356}{365 \times 365 \times 365 \times \ldots \times 365} = 0.883$ (88%)	1 – 0.883 = 0.117 (12%)
20	$\frac{365 \times 364 \times 363 \times \ldots \times 346}{365 \times 365 \times 365 \times \ldots \times 365} = 0.589$ (59%)	1 – 0.589 = 0.411 (41%)
23	$\frac{365 \times 364 \times 363 \times \ldots \times 343}{365 \times 365 \times 365 \times \ldots \times 365} = 0.493$ (49%)	1 – 0.493 = 0.507 (51%)
30	$\frac{365 \times 364 \times 363 \times \ldots \times 336}{365 \times 365 \times 365 \times \ldots \times 365} = 0.294$ (29%)	1 – 0.294 = 0.706 (71%)

Table 26: Birthday Problem

As we can see with our table, there only needs to be 23 students in the room to have at least a 50% chance of at least 2 students sharing the same birthday. There is a formula that can be used to calculate this probability, but it is above an average fifth grader's level of understanding. The formula is:

$$1 - \frac{365!}{(365 - n)!\ 365^n}$$

The variable *n* can be replaced with any number of students. The (!) is a factorial notation. A factorial is the product of all the positive integers less than or equal to it. For example, 3! = 3 x 2 x 1 = 6, 4! = 4 x 3 x 2 x 1 = 24, and 5! = 5 x 4 x 3 x 2 x 1 = 120. If there are any advanced mathematics students in the classroom they may be able to experiment with this formula and a statistics calculator.

To find empirical data to compare to this theoretical probability, classroom birthdays were researched in a Virginia elementary school. To match the probabilities in this thesis, only classes with 20 or 23 students were selected. There were 8 classes found that had an enrollment of 20 students. Two of those classes had at least 2 students that shared the same birthday. According to this data, 25% of the classes with 20 students in this school had at least 2 students that shared a common birthday. The theoretical probability of having a class with 20 students containing at least 2 students that share a common birthday is 41%. These probabilities are actually very close considering the small data size. Out of the 7 classes that had 23 students in the class, 4 of these classes contained at least 2 students that shared a common birthday. According to this data, 57% of the classes with 23 students in this school had at least 2 students that shared a common birthday. The theoretical probability of having a class with 23 students containing at least 2 students that share a common birthday is 51%. These probabilities are also very close considering the small data size.

This is only one problem that is more difficult to figure out, but easy enough to explain and experiment with in a fifth grade classroom. In conclusion, this thesis focused on teaching probability in fun and engaging ways. Many students refer to math as “hard” and “boring.” We, as teachers, need to overcome this obstacle by teaching the mathematics content in various ways. We also need to make an effort to show students that math can be fun, and that it is definitely important to their daily lives. Mathematics doesn’t have to be “hard” if we make sure our students understand previously taught concepts and then build upon them.

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