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Differentials of Graphs.

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Differentials of Graphs

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by

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ABSTRACT

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Let $G = (V, E)$ be an arbitrary graph, and consider the following game. You are allowed to buy as many tokens from a bank as you like, at a cost of $1 each. For example, suppose you buy k tokens. You then place the tokens on some subset of k vertices of V. For each vertex of G which has no token on it, but is adjacent to a vertex with a token on it, you receive $1 from the bank. Your objective is to maximize your profit, that is, the total value received from the bank minus the cost of the tokens bought. Let $\mathrm{bd}(X)$ be the set of vertices in $V - X$ that have a neighbor in a set X. Based on this game, we define the *differential* of a set X to be $\partial(X) = |\mathrm{bd}(X)| - |X|$, and the *differential of a graph* to be equal to $\max\{\partial(X)\}$ for any subset X of V. In this thesis, we introduce several different variations of the differential of a graph and study bounds on and properties of these novel parameters.

DEDICATION

I would first like to thank my family for always encouraging me to pursue my dreams, especially my mother, Gail Lewis, for always pushing me to be my best and my father, Steven Lewis, for showing me the importance of an education. I would also like to thank my wonderful fiancée, Tasha Mashburn, for without her loving support I would have not been able to complete my degree. For she always stood by my side and supported me, even in the tough times, and encouraged me to never give up.

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1 Introduction

Let $G = (V, E)$ be a graph with no isolated vertices. You are allowed to buy as many tokens from a bank as you like, at a cost of \$1 each. For example, suppose that you buy k tokens. You then place the tokens on some subset of k vertices of G. For each vertex of G which has no token on it, but is adjacent to a vertex with a token on it, you receive \$1 from the bank. Your objective is to maximize your profit, that is, the total value received from the bank minus the cost of the tokens bought.

Notice that you do not receive any credit for the vertices on which you place a token. In a more generous version of this game, you also receive \$1 credit for all occupied vertices, in which case, the tokens are essentially free.

For a vertex $v \in V$, the *open neighborhood* of v is the set $N(v) = \{u \in V \mid uv \in E\}$ and the *closed neighborhood* is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, its *open neighborhood* is $N(S) = \cup_{v \in S} N(v)$ and its *closed neighborhood* is $N[S] = N(S) \cup S$. The subgraph induced by S is denoted by $\langle S \rangle$.

Thus, we can define for any subset X, the *small profit margin* of X, $\pounds(X) = |N[X] - X| - |X|$, and the *large profit margin* of X $\$(X) = |N[X]| - |X|$. Similarly, we can define the *small profit margin* of a graph G, $\pounds(G) = max\{\pounds(X) \mid X \subseteq V\}$, and the *large profit margin* of of a graph G, $\$(G) = max\{\$(X) \mid X \subseteq V\}$.

For a set $X \subseteq V$, we define:

$I(X) = X - N(X)$, the isolates in $\langle X \rangle$,

$A(X) = X \cap N(X)$, the non-isolates in $\langle X \rangle$,

$B(X) = (V - X) \cap N(X)$, the *boundary* of X, also denoted $\mathrm{bd}(X)$

$Z(X) = V - N[X]$, the vertices not dominated by X.

For a set $X \subseteq V$, $B(X)$ is the set of vertices outside of X that are dominated by X and $A(X)$ is the set of vertices in X that are dominated by another vertex in X. Intuitively, every vertex in $V - X$ which is dominated by a vertex in X gives X a differential of $+1$, while each vertex in X creates a differential of -1.

We are now ready to define several 'differentials' of a set X.

The B-*differential* of a set X is $|\mathrm{bd}(X)| = |B(X)| = \$(X)$.

The A-*differential* of a set X is $\partial_A(X) = |\mathrm{bd}(X)| - |A(X)|$.

The I-*differential* of a set X is $\partial_I(X) = |\mathrm{bd}(X)| - |I(X)|$.

The *differential* of a set X is $\partial(X) = |\mathrm{bd}(X)| - |X| = \pounds(X)$.

The definition of the A-differential of a set was first given by McRae and Parks [7], while the definition of $\partial(X)$ was given by Hedetniemi about ten years ago in terms of small profit margin $\pounds(X)$ [6]. The parameter $\partial(X)$ is also considered by Goddard and Henning [2], who called it$\eta(X)$. The minimum differential of an independent set has been considered by Zhang [9], and can be computed in polynomial time.

Clearly, some sets have a positive differential, some sets have a negative differential and some sets have no differential. Consider therefore the collection of differentials of all subsets of vertices of a graph G.

The following proposition can easily be shown.

Proposition 1 *Any graph having a perfect matching has exponentially many sets of* *size $\frac{n}{2}$ having zero differential.*

Proposition 2 *Any graph of even order $2k$ having a vertex of degree at least k has a set of cardinality k with zero differential.*

Proof: Let v be a vertex of degree at least k. Let B be a set of k vertices in $N(v)$, and $X = V - B$. Note that $B = \mathrm{bd}(X)$ since v is in X. Thus, $\partial(X) = |\mathrm{bd}(X)| - |X| = k - k = 0$. □

Extending from a set to a graph, we define the following invariants.
The *differential* of a graph is $\partial(G) = max\{\partial(X) \mid X \subseteq V\}$.
The *A-differential* of a graph is $\partial_A(G) = max\{\partial_A(X) \mid X \subseteq V\}$.
The *I-differential* of a graph is $\partial_I(G) = max\{\partial_I(X) \mid X \subseteq V\}$.
The *B-differential* of a graph is $\Psi(G) = max\{|\mathrm{bd}(X)| \mid X \subseteq V\}$.
The parameter $\Psi(G)$ was introduced by Slater in [8] and was called the enclaveless number.

We consider the differential of arbitrary graphs in Section 2 and the differential of trees in Section 3. First, we give some more definitions. In general we will follow the notation and terminology of [5]. Given a set $S \subseteq V$ the *private neighborhood* $\mathrm{pn}[v, S]$ of $v \in S$ is defined by $\mathrm{pn}[v, S] = N[v] - N[S - \{v\}]$, equivalently, $\mathrm{pn}[v, S] = \{u \in V \mid N[u] \cap S = \{v\}\}$. Each vertex in $\mathrm{pn}[v, S]$ is called a *private neighbor* of v. The *external private neighborhood* $\mathrm{epn}(v, S)$ of v with respect to S consists of those private neighbors of v in $V - S$. Thus, $\mathrm{epn}(v, S) = \mathrm{pn}[v, S] \cap (V - S)$.

A set $S \subseteq V$ is a *dominating set* if $N[S] = V$. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set. We call a dominating set of G of minimum cardinality a $\gamma(G)$*-set*. We use similar notation for other parameters, that is, for a generic parameter $\mu(G)$, we call a set satisfying the property for the parameter and having cardinality $\mu(G)$, a $\mu(G)$-set. A set $S \subseteq V$ is a *total dominating set* if $N(S) = V$. The *total domination number* $\gamma_t(G)$ is the minimum cardinality of a minimal total dominating set. A set S is *independent* if no two vertices in S are adjacent. The *independent domination number* $i(G)$ equals the minimum cardinality of an independent dominating set.

2 Differentials in Arbitrary Graphs

Our next observation follows directly from the definitions of differentials.

Proposition 3 *For any graph G,*

(i) $\partial(G) \leqslant \partial_A(G) \leqslant \Psi(G)$.

(ii) $\partial(G) \leqslant \partial_I(G) \leqslant \Psi(G)$.

The differential values for paths and cycles are straightforward to determine, we state them without proof.

Proposition 4 *For all paths P_n and cycles C_n,*

(i) $\partial(P_n) = \partial(C_n) = \lfloor \frac{n}{3} \rfloor$.

(ii) $\partial_A(P_n) = \partial_A(C_n) = 2\lfloor \frac{n}{3} \rfloor$.

(iii) $\partial_I(P_n) = \partial_I(C_n) = \lfloor \frac{n}{2} \rfloor$.

(iv) $\partial(P_n) = \partial(C_n) = 2\lfloor \frac{n}{3} \rfloor$.

In fact the maximum boundary in G can be determined in terms of $\gamma(G)$.

Proposition 5 [8] *For any graph G of order n, $\Psi(G) = n - \gamma(G)$.*

Consequently the following problem is NP-complete, even for bipartite and chordal graphs.

BOUNDARY

INSTANCE: graph $G = (V, E)$, positive integer k

QUESTION: does G have a set $X \subseteq V(G)$ such that $|\text{bd}(X)| \geqslant k$?

Proposition 6 *For any graph G of order n without isolated vertices,*

(i) $n - 2\gamma(G) \leqslant \partial(G) \leqslant n - \gamma(G) - 1$,

(ii) $n - \gamma_t(G) \leqslant \partial_I(G)$,

(iii) $n - i(G) \leqslant \partial_A(G)$, and

(iv) $n - 2\gamma_t(G) \leqslant \partial_A(G)$.

Proof:

(i) Let X be a $\gamma(G)$-set. Notice that,

$$\partial(G) \geqslant \partial(X) = |\mathrm{bd}(X)| - |X| = (n - \gamma(G)) - \gamma(G) = n - 2\gamma(G).$$

Let X be a $\partial(G)$-set. Then it follows from Proposition 5 that

$$\begin{array}{rcl}\partial(G) = \partial(X) & = & |\mathrm{bd}(X)| - |X| \\ & \leqslant & \Psi(G) - |X| \\ & = & n - \gamma(G) - |X|.\end{array}$$

Since $|X| \geqslant 1$, the result holds.

(ii) Let X be a $\gamma_t(G)$-set. We note that $|\mathrm{bd}(X)| = n - \gamma_t(G)$. Thus,

$$\begin{array}{rcl}\partial_I(G) \geqslant \partial_I(X) & = & |\mathrm{bd}(X)| - |I(X)| \\ & = & n - \gamma_t(G) - 0.\end{array}$$

(iii) Let X be an $i(G)$-set. Then $|\mathrm{bd}(X)| = n - i(G)$. Thus

$$\begin{array}{rcl}\partial_A(G) \geqslant \partial_A(X) & = & |\mathrm{bd}(X)| - |A(X)| \\ & = & n - i(G) - 0.\end{array}$$

(iv) Let X be a $\gamma_t(G)$-set. Then $|\mathrm{bd}(X)| = n - \gamma_t(G)$. Thus,

$$\begin{array}{rcl}\partial_A(G) \geqslant \partial_A(X) & = & |\mathrm{bd}(X)| - |A(X)| \\ & = & (n - \gamma_t(G)) - \gamma_t(G) \\ & = & n - 2\gamma_t(G). \quad \square\end{array}$$

Proposition 7 *For any graph G with maximum degree $\Delta(G)$,*

(i) $\Delta(G) - 1 \leqslant \partial(G)$,

(ii) $\Delta(G) \leqslant \partial_A(G)$.

Proof: Let $X = \{v\}$, where v is a vertex of maximum degree $\Delta(G)$. We have

$$\begin{array}{rcl}\partial(G) \geqslant \partial(X) & = & |\mathrm{bd}(X)| - |X| \\ & = & \Delta(G) - 1, \text{ and}\end{array}$$

$$\begin{array}{rcl}\partial_A(G) \geqslant \partial_A(X) & = & |\mathrm{bd}(X)| - |A(X)| \\ & = & \Delta(G) - 0 \\ & = & \Delta(G). \quad \square\end{array}$$

Proposition 8 *For every k-regular graph G with diam(G) ⩾ 3,*

(i) $2k - 2 \leqslant \partial(G)$,

(ii) $2k \leqslant \partial_A(G)$.

Proof: Since $\text{diam}(G) \geqslant 3$, there exists two vertices, u and v, such that $N[u] \cap N[v] = \emptyset$. Let $X = \{u, v\}$. Since G is k-regular, $|\text{bd}(X)| = 2k$ and hence,

$$\partial(G) \geqslant \partial(X) = |\text{bd}(X)| - |X| = 2k - 2, \text{ and}$$

$$\partial_A(G) \geqslant \partial(X) = |\text{bd}(X)| - |A(X)| = 2k - 0 = 2k. \qquad \square$$

3 Differentials in Trees

We have seen from Proposition 6(i) and 7 that for any graph G,

$$n - 2\gamma(G) \leq \partial(G) \leq n - \gamma(G) - 1, \text{ and}$$

$$\Delta(G) - 1 \leq \partial(G).$$

In Section 3.1, we consider realizability of the differential values from $n - 2\gamma(T)$ to $n - \gamma(T) - 1$ for trees T. In Section 3.2, we characterize the trees T that attain the upper bound of $n - \gamma(T) - 1$, and in Section 3.3 we investigate the trees T that attain the lower bound of $n - 2\gamma(T)$. Finally in Section 3.4, we give a characterization of the trees T having $\partial(T) = \Delta(T) - 1$.

3.1 Realizability

From Proposition 6i, we have $n - 2\gamma(G) \leqslant \partial(G) \leqslant n - \gamma(G) - 1$. Next we show that there exists a tree T having $\gamma(T) \leqslant \partial(T) = k$ for each k, $n - 2\gamma(T) \leqslant k \leqslant n - \gamma(T) - 1$.

Theorem 9 *For any triple (a, b, c) of positive integers such that $a \leqslant b \leqslant c$ and $c - 2a \leqslant b \leqslant c - a - 1$, there exists a tree T having order $n = c$, $\gamma(T) = a$, and $\partial(T) = b$.*

Proof: Let a, b, and c be integers where $a \leqslant b \leqslant c$ and $c - 2a \leqslant b \leqslant c - a - 1$, we will show that the tree in Figure 1 has order $n = c$, $\gamma(T) = a$, and $\partial(T) = b$.

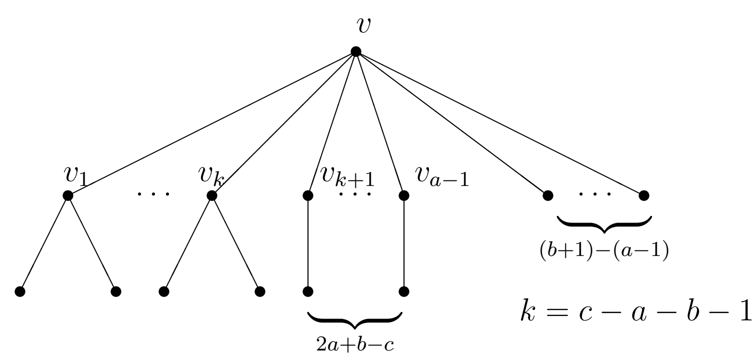

Figure 1: A tree T with $n = c$, $\gamma(T) = a$, and $\partial(T) = b$.

Notice that $\deg(v) = b + 1$ and excluding v, there are $a - 1$ support vertices. It is straightforward to show that $\gamma(T) = a$ and $n = c$.

Next we will show that $\partial(T) = b$. Now $\partial(T) \geqslant \partial(\{v\}) = deg(v) - 1 = b$. To see that $\partial(T) \leqslant b$, let D be a $\partial(T)$-set. If $v \in D$, then we may assume that no leaf is in D and the vertices of degree two in $N(v)$ are not in D (if one was the net effect on the differential is 0). If a strong support vertex $v_i \in N(v)$, for $1 \leqslant i \leqslant k$, is in

D, then its two leaf neighbors add 2 to the differential but v_i no longer adds one and also one more must be subtracted because $v_i \notin \mathrm{bd}(D)$. Hence, the net effect is 0. Therefore, $\partial(D) = \partial(\{v\}) = b$. If $v \notin D$, then since $(b+1) - (a-1) \geqslant 2$, it follows that $\partial(D \cup \{v\}) > \partial(D) = \partial(T)$, a contradiction. Thus, $\partial(T) = b$ as desired. □

3.2 Trees T with $\partial(T) = n - \gamma(T) - 1$

Domke, Dunbar, and Markus [1] characterized the trees T for which $\Delta(T) = n - \gamma(T)$. We shall show that these trees are precisely the trees obtaining the upper bound of Proposition 6(i). A *subdivision* of an edge uv is obtained by removing edge uv, adding a new vertex w, and adding edges uw and vw. A *wounded spider* is the graph formed by subdividing at most $t-1$ of the edges of a star $K_{1,t}$ for $t \geq 0$. Note that the star $K_{1,n-1}$ is a wounded spider. See Figure 2 for another example.

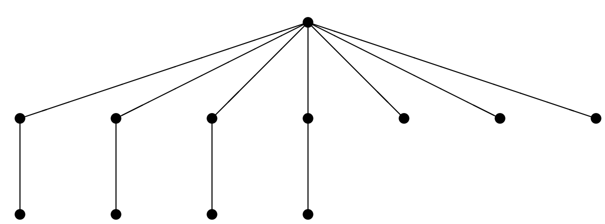

Figure 2: A wounded spider.

Theorem 10 [1] *For a tree T, $\gamma(T) = n - \Delta(T)$ if and only if T is a wounded spider.*

Theorem 11 *A tree T has $\partial(T) = n - \gamma(T) - 1$ if and only if T is a nontrivial wounded spider.*

Proof: Assume that T has $\partial(T) = n - \gamma(T) - 1$, and let X be a $\partial(T)$-set. Hence, $n - \gamma(T) - 1 = \partial(T) = \partial(X) = |\text{bd}(X)| - |X| \leq \Psi(T) - |X| = n - \gamma(T) - |X| \leq n - \gamma(T) - 1$. Thus, equality applies throughout implying that $\Psi(T) = n - \gamma(T)$ and $|X| = 1$. Then $X = \{x\}$ and $deg(x) = |\text{bd}(X)| = n - \gamma(T)$. If $deg(x) < \Delta(T)$, then $\partial(\{y\}) > \partial(X)$ where y is a vertex of maximum degree, a contradiction. Hence, $deg(x) = \Delta(T) = n - \gamma(T)$, and these trees are precisely the trees characterized in Theorem 10 (with the exception of the trivial tree which we exclude). $\square$

3.3 Trees T with $\partial(T) = n - 2\gamma(T)$

We next consider trees that achieve the lower bound of Proposition 6(i).

A graph G has *property* $\mathcal{EPN}$ if for every $\gamma(G)$-set S and for every $v \in S$, $\text{epn}(v, S) \neq \emptyset$. We call a tree with property $\mathcal{EPN}$ an $\mathcal{EPN}$-tree.

Lemma 12 *If G does not have property $\mathcal{EPN}$, then $\partial(G) \geqslant n - 2\gamma(G) + 1$.*

Proof: If G has a $\gamma(G)$-set S and $u \in S$ such that $\text{epn}(u, S) = \emptyset$, then

$$\begin{aligned} \partial(G) &\geqslant \partial(S - \{u\}) \\ &= n - \gamma(G) - (\gamma(G) - 1) \\ &= n - 2\gamma(G) + 1. \quad \square \end{aligned}$$

Figure 3 is an example of an $\mathcal{EPN}$-tree. Clearly, $\gamma(T) = 2$ and

$$\begin{aligned} \partial(T) &= \partial(\{u, v\}) \\ &= 6 - 2 \\ &= 4 \\ &= n - 2\gamma(T). \end{aligned}$$

Notice that $\gamma(P_{3k}) = 2k$ and $\partial(P_{3k}) = k$. Furthermore, P_{3k} is an $\mathcal{EPN}$-tree.

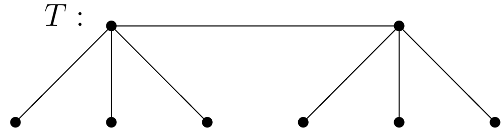

Figure 3: An $\mathcal{EPN}$-tree T.

However, the converse of Lemma 12 is not true, as can be seen with the subdivided star $K_{1,t}$, $t \geqslant 3$.

We are continuing to work on the problem of characterizing the trees T for which $\partial(T) = n - 2\gamma(T)$.

3.4 Trees T with $\partial(T) = \Delta(T) - 1$

Finally we characterize the trees having $\partial(T) = \Delta(T) - 1$.

We define a family $\mathcal{T}$ of trees T that can be obtained from a nontrivial star $K_{1,t}$ and $rK_1 \cup sK_2$, $r \geqslant 0$ and $s \geqslant 0$, by adding edges such that each of the following hold (note that r and s are restricted by the rules for adding edges).

1. T is a tree,
2. for each $u \in N(v)$, $\deg(v) \leqslant 3$, and
3. $N(v)$ contains at least two vertices, each of which is either a leaf or a support vertex of degree 2, or

   $N(v)$ contains exactly one leaf and no vertex of $N(v)$ has degree 3.

For an example, see Figures 4 and 5.

Theorem 13 *A tree T has $\partial(T) = \Delta(T) - 1$ if and only if $T \in \mathcal{T}$.*

Proof: Let $\partial(T) = \Delta(T) - 1$, and let v be a vertex of maximum degree in T. Note that $D = \{v\}$ is a $\partial(T)$-set. If any vertex $x \in V - N[v]$ has at least two neighbors in $V - N[v]$, then $\partial(\{v, x\}) \geqslant \Delta(T) + 2 - 2 > \Delta(T) - 1 = \partial(T)$, a contradiction. Hence $V - N[v]$ induces $rK_1 \cup sK_2$ where $r \geqslant 0$ and $s \geqslant 0$. Let $H_1 = rK_1$ and $H_2 = sK_2$.

If any vertex $u \in N(v)$ has three or more neighbors in $V - N[v]$, then $\partial(\{v, u\}) \geqslant \Delta(T) - 1 + 3 - 2 > \Delta(T) - 1 = \partial(T)$, a contradiction. Moreover, since T is a tree, $N(v)$ is an independent set. Hence, $\deg(u) \leqslant 3$ for all $u \in N(v)$.

Furthermore,

$$\begin{aligned} \partial(N(v)) &= \textstyle\sum_{u \in N(v)} (\deg(u) - 1) + 1 - |N(v)| \\ &= \textstyle\sum_{u \in N(v)} (\deg(u) - 1) + 1 - \Delta(T) \\ &\leqslant \partial(T) = \Delta(T) - 1. \end{aligned}$$

This implies that at least two vertices in $N(v)$ have degree at most two. Among all vertices of $N(v)$ with degree at most two, select two, say x and y, such that priority is given first to leaves and next to support vertices. If each of x and y is a leaf or a support vertex, then we have shown that $T \in \mathcal{T}$. Hence assume that this is not the case. Thus, with out loss of generality, y has a neighbor in $V(H_2)$. Label the vertices of $N(v) - \{x, y\}$, u_i for $1 \leqslant i \leqslant \Delta(T) - 2$. Thus by our choice of x and y, every vertex u_i in $N(v) - \{x, y\}$ has degree three or has exactly one neighbor, say w_i, in $V(H_2)$.

Let $U = \{u_i \mid \text{where } \deg(u_i) = 3\}$ and $W = \{w_i \mid \text{where } w_i \in V(H_2) \cap N(u_i) \text{ and } \deg(u_i) = 2\}$. Let y' be the neighbor of y in $V(H_2)$.

If $U \neq \emptyset$, then

$$\begin{array}{rcl}\partial(T) & \geqslant & \partial(U \cup W \cup \{y'\}) \\ & = & 1 + 2|U| + 2|W| + 2 - |U| - |W| - 1 \\ & = & |U| + |W| + 2 \\ & = & \Delta(T) \\ & > & \partial(T),\end{array}$$

a contradiction.

Hence, $U = \emptyset$, that is $\deg(u_i) \leqslant 2$ for all $u_i \in N(v)$.

If x is not a leaf, then x is either a support vertex or x has a neighbor in $V(H_2)$. It follows that

$$\begin{array}{rcl}\partial(T) & \geqslant & \partial(W \cup \{x, y'\}) \\ & \geqslant & 2|W| + 2 + 2 - |W| - 2 \\ & = & \Delta(T) \\ & > & \partial(T),\end{array}$$

a contradiction.

Hence, x is a leaf and $T \in \mathcal{T}$.

Assume that $T \in \mathcal{T}$. Then $\partial(T) \geqslant \partial(\{v\}) = \Delta(T) - 1$. To see that $\partial(T) \leqslant \Delta(T) - 1$, let D be a $\partial(T)$-set. If $N(v)$ contains at least two vertices where each one is a leaf or a support of degree two, then it follows that $v \in D$. Since $\deg(u) \leqslant 3$, all $u \in N(v)$ and $V - N[v]$ induces $rK_1 \cup sK_2$, adding another vertex to D does not increase the differential. Hence, $D = \{v\}$ and $\partial(T) = \Delta(T) - 1$. If $N(v)$ contains exactly one leaf and no other vertex of $N(v)$ has degree three, then at most $\Delta(T) - 1$ vertices of $V(T) - \{v\}$ can add one to the differential implying that $\partial(T) \leqslant \Delta(T) - 1$. Hence $\partial(T) = \Delta(T) - 1$. $\square$

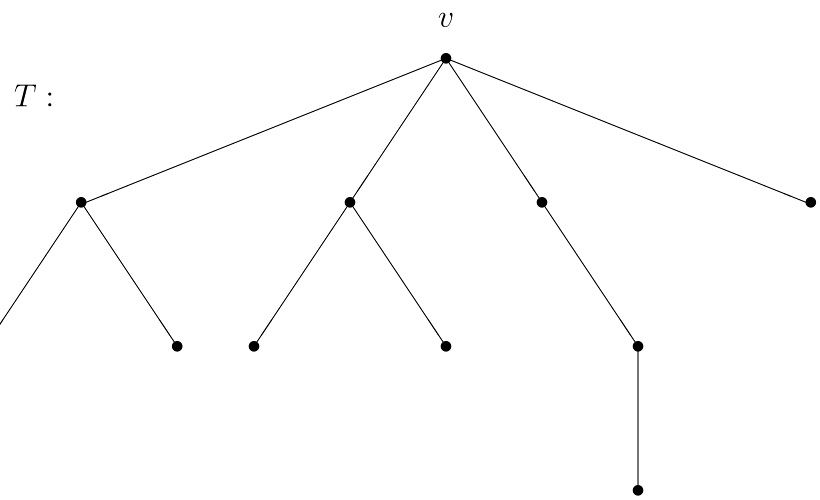

Figure 4: A tree $T \in \mathcal{T}$.

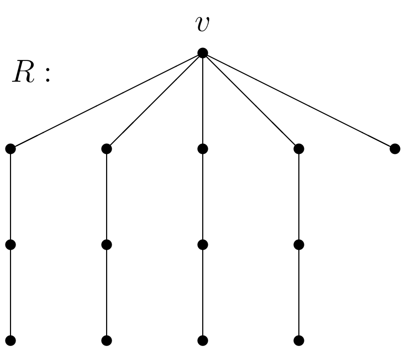

Figure 5: A tree $R \in \mathcal{T}$.

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Jason R. Lewis: Domination Critical Graphs with Arbitrary Independent Domination Numbers. *Eighth Annual North Carolina Mini-Conference on Graph Theory, Combinatorics, and Computing*, Appalachian State University (April, 2003)

Jason R. Lewis, Roby Ellis: Investigations into the Kaprekar Process. *Rose-Hulman Undergraduate Mathematics Journal*, volume 3 issue 2, 2002

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Dayla Pike, **Jason R. Lewis**: Six Proofs of an Elementary Combinatorial Identity. *Sixth Annual North Carolina Mini-Conference on Graph Theory, Combinatorics, and Computing*, Appalachian State University (April, 2001)

Jason R. Lewis, Roby Ellis, Katherine Richards: 6174 - Kaprekar's Constant. *Sixth Annual North Carolina Mini-Conference on Graph Theory, Combinatorics, and Computing*, Appalachian State University (April, 2001)

Honors and Awards

Graduated Cum Laude, GPA 3.524, ETSU, May, 2002:

Academic Performance Scholarship, ETSU,
August, 1998–May, 2002

Charles Wilkey Scholarship, ETSU Department of Mathematics,
August, 2001–May, 2002

Faber-Neal Scholarship, ETSU Department of Mathematics,
August, 1999–May, 2001

Wilson-Hartsell Scholarship, ETSU Department of Mathematics,
August, 1998–May, 1999

University Service

Mathematics Laboratory Tutor, ETSU Mathematics Department, August 1999–May 2004
Tutored ETSU undergraduate students in freshman level mathematics classes. These classes include basic Probability and Statistics, Calculus I, II and III, College Algebra, and Linear Algebra. The tutoring environment was both individual tutoring and small group tutoring. I have also tutored less frequently in almost all of the other freshmen through junior level mathematics classes.

Professional Memberships

Tennessee Beta Chapter of Kappa Mu Epsilon - Mathematics Honor Society, President, March 2000–Present

East Tennessee Student Chapter of the Mathematical Association of America, President, August, 1999–August, 2003