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Portfolio Insurance Using Leveraged ETFs

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Abstract

This study examines the use of leveraged exchange traded funds (LETFs) within a portfolio insurance framework to reduce exposure to downside risk. Investors have learned the importance of mitigating this risk having experienced two "once in a century" events in the last 20 years with the tech crash in the early 2000s and the financial crisis in 2008. Current portfolio insurance strategies are either option based (Leland & Rubinstein, 1976) or constant proportional portfolio insurance (CPPI), (Black & Jones, 1987). The cost of option based strategies can be quite high while a CPPI strategy requires constant rebalancing.

This study combines the advantages of each by using LETFs to attain the leverage options provide, while at the same time allowing a greater percentage of the portfolio to be invested in bonds since a position in LETFs relative to a typical market index magnifies equity exposure. Thus, where a standard CPPI strategy may require 50% of the portfolio to be invested in equities, using a 3x LETF only requires approximately 16.7%. Results suggest the use of LETFs within a portfolio insurance framework result in better returns, higher Sharpe, Sortino, Omega, and cumulative prospect values while reducing Value at Risk (VaR) and Excess Shortfall below VaR. This twist on the use of LETFs will be of interest to any investor concerned with mitigating downside risk while allowing participation in increasing markets.

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II. Introduction

Background of the Issue

Leverage exchange-traded funds (LETFs) are a relatively new investment tool and their popularity has been rapidly increasing since their inception in 2006. LETFs increase an investor's exposure to the underlying index by 2x or 3x, or in regard to inverse LETFs, -1x, -2x or -3x, and they do this using financial derivatives, swaps and debt. This study examines the use of LETFs within a portfolio insurance (PI) framework to reduce exposure to downside risk. Investors have learned the importance of mitigating this risk having experienced two "once in a century" events in the last 20 years with the dot com crisis in the early 2000s and the financial crisis in 2008. Current portfolio insurance strategies are either option based portfolio insurance (OBPI), (Leland & Rubinstein, 1976) or constant proportional portfolio insurance (CPPI), (Black & Jones, 1987). The cost of option based strategies can be quite high while a CPPI strategy requires constant rebalancing.

This study combines the advantages of each by using LETFs to attain the leverage options provide, while at the same time allowing a greater percentage of the portfolio to be invested in bonds or treasury bills, since a position in LETFs relative to a typical market index magnifies equity exposure. Thus, where a standard CPPI strategy may require 50% of the portfolio to be invested in equities, using a 2x LETF only requires 25% and using a 3x LETF only requires 16.7%. In analyzing return and risk performance of the portfolios under study, this twist on the use of LETFs will be of interest to any investor concerned with mitigating downside risk while allowing participation in increasing markets.

Research Objective

The key question modern day investors are continuously asking is how can one mitigate huge losses without dramatically sacrificing returns. The research objectives of this study are to provide evidence into the benefits of using LETFs in a portfolio insurance framework. In applying LETFs to the context of a CPPI, less cash will be used in the risky asset which will manage dramatic losses in a more efficient manner. If evidence can be shown of risk mitigation and returns that are commensurate with the market and a standard CPPI strategy, the objectives will be satisfied. If evidence can be shown of risk mitigation and returns that exceed the market and a standard CPPI strategy, this will be even more beneficial to investors.

Contribution

There has been much research done on the formulation and use of PI strategies, and extensions of such. However, this specific extension of attaining leverage does not appear to have ever been researched. There have also been few studies done on the use of leveraged exchange-traded funds in a long-term portfolio. This can all be due to the relatively recent introduction of LETFs as an investment tool and a general recommendation to not include them in a long-term investment portfolio. This study will present a unique PI strategy using LETFs and attempt to demonstrate the benefits from them. The contribution will serve to represent a potential twist on investors interested in PI and a potential dismissal of previous accusations that LETFs should not be used over longer holding periods. For the greater purpose, the contribution will put to test a new portfolio insurance tactic in which all investors could profit from.

III. Literature Review

Prospect Theory

Prospect theory, first identified by Kahneman and Tversky (1979), describes behavior shared by investors when dealing with gambling and speculation. With a simple survey of questions it was noticed that risk aversion in the positive domain is associated with risk seeking in the negative domain, which violates what was thought to have been the standard of rational behavior in situations of risk known as the expected utility theory. For example, when asked to choose between (A) 80% chance of winning $4,000 or (B) 100% chance of winning $3,000, 80% of respondents chose (B). When asked to choose between (A) 80% chance of losing $4,000 or (B) 100% chance of losing $3,000, 92% or respondents chose (A). By merely changing the positive prospects to negative prospects, a different preference is witnessed.

Therefore, prospect theory investors are those who are risk averse in gains but risk seeking in losses, whereas expected utility theory assumes all investors are risk averse. As the basics of prospect theory indicate that investors fear losses more than they enjoy gains, this behavior modeling has gained popularity with the two major market crashes in the past 20 years. Tversky and Kahneman (1992) expanded on their prior formulation of prospect theory to include cumulative probabilities as opposed to individual probabilities, referred to as cumulative prospect theory. The calculation for a Cumulative Prospect Value (CPV) is given as,

$$\mathrm{CPV} = \sum_{i=1}^{N} \pi_i v(x_i)$$

where π_i is the decision weight of the individual positive and negative possible outcomes and x_i is an outcome. The decision weight π_i with a positive outcome is the difference between the capacities of events "the outcome is at least as good as x_i" and "the outcome is strictly better than

x_i". For a negative outcome, it is the difference between the capacities of events "the outcome is at least a bad as x_i" and the outcome is strictly better than x_i".

Portfolio Insurance

The concept of PI has been around for several decades stemming from Leland and Rubinstein (1976). In a bear market, which covered nearly all of 1973 and 1974, Leland noted that pension funds had withdrawn from the market causing them to miss the bull market which followed in 1975. Realizing that some sort of insurance would have mitigated losses, the idea of PI was created. PI is a framework where portfolios moderately capitalize on market gains while minimizing exposure to market losses. PI becomes even more important to investors when large market declines are experienced. Modern day investors have become all too familiar with dramatic losses, allowing PI and the research of new PI strategies to gain popularity. Current PI strategies are either option based portfolio insurance (OBPI), (Leland & Rubinstein, 1976) or constant proportionate portfolio insurance (CPPI), (Black & Jones, 1987). Extensions and modifications to the CPPI have been researched and tested.

Option Based Portfolio Insurance

OBPI, as introduced in Leland and Rubinstein (1976) and summarized in Trainor, Chhachhi and Brown (2016), has two methods to which the portfolio insurance is established. One method is referred to as a protective put, where a put option is purchased on the risky asset which is held, and the other method is to hold the risky asset and write a call option on the risky asset, known as a covered call. In method one, the insurance is established through the put option and in method two the insurance comes from the premium attained on writing the call option as both of these define the minimum return of the portfolio. Each method can be constructed to have similar returns but most of the literature evaluating OBPI suggests the protective put method.

Constant Proportion Portfolio Insurance

CPPI is a strategy in which was introduced by Black and Jones (1987) to offer “an approach to portfolio insurance that is easy to understand, straightforward to implement and flexible under changing positions”. In applying a CPPI strategy an investor will choose a floor, the lowest acceptable value of the portfolio, which determines the cushion, the portfolio value minus the floor. The exposure, the amount invested in the risky asset, is calculated by multiplying some predetermined multiplier, based off the investors risk tolerance, and the cushion value. A higher multiplier results in a relatively higher percentage invested in the risky asset. As the cushion value approaches zero, the exposure will also approach zero regardless of the multiplier value, as this keeps the value of the portfolio from dropping below the floor.

For clarification, assume an investor has an initial portfolio value of $100,000. The floor is set at $90,000, which makes a cushion of $10,000, and the predetermined multiplier is 5. This means that the initial exposure to the risky asset will be $50,000 (multiplier times the cushion) and the remaining $50,000 will be placed in a riskless asset. As the portfolio moves up and down over time, the ratio of the exposure to the cushion will always rebalance to 5. As for when rebalancing occurs, that is also predetermined as a certain percentage move in the market, usually between 2-5%, or a period of time, such as monthly or yearly.

OBPI vs. CPPI

For comparing the two PI strategies there is substantial documented research. Cesari and Cremonini (2003) put to test different dynamic strategies which include CPPI and OBPI amongst buy-and-hold and constant mixture strategies for comparison in historical simulation and Monte Carlo simulation. CPPI strategies were dominant in bear and no-trend markets and considered more beneficial overall. Bertrand and Prigent (2005) provide research into the difference in sensitivity to the risky asset fluctuations and the volatility in the market itself but are left to consider OBPI as a generalized CPPI. Pezier and Scheller (2013) compare the two strategies in realistic market settings and note the CPPI preferable in all of them. Zieling, Mahayni and Balder (2014), while also stating the CPPI a more optimal strategy, extends the strategy to include a time-varying multiplier based upon the expected future volatility. The empirical results are robust in favor of the time-varying multiplier. Bertrand and Prigent (2011) expand their search for an advantage between the two strategies as they feel previous literature fails to determine a concrete upper hand. By analyzing the OBPI and CPPI strategies using downside risk measures and performance measures which take into account the non-normality of returns, otherwise known as Kappa performance measures, the CPPI method outperforms the OBPI method using the Omega measure. The Omega measure and all the other performance measures will be elaborated on in the next section.

Leverage Exchange-Traded Funds and Their Decay

It is also important to understand LETFs as it is the differentiation in comparing the PI in this research to others. LETFs are always set to manage daily returns of the underlying index multiplied by their specified leverage ratio. However, the key here is that they mimic daily returns, not returns over an extended period of time. As holding periods of a LETF increase, the leverage ratios associated with them tend to break down due to compounding. Trainor and Baryla (2008) refer to this as the constant leverage trap, which describes the magnified compounding problem.

For example, an investor invests $100 into a market index. On the first day, the market falls 10% and the next day the market gains 20%. Therefore, after the first day the account value would be $90 and after the second day it would be $108, giving the investor a total return of 8%. If the investor would've invested that same $100 in a 2x LETF for the same holding period, the investor would've lost 20% on the first day and gained 40% on the second day. The account value would've fallen to $80 after the first day and risen to $112 after the second day. This is only a 12% return compared to the underlying index receiving an 8% return, making for a leverage ratio of only 1.5 over the two days (12% divided by 8%). Granted this represents an extremely volatile couple of days but it serves to represent the leverage ratio break down over a long-term holding period.

Lu, Wang and Zhang (2012) generalized this break down occurrence. Their results showed that over holding periods no greater than one month, an investor can assume that a 2x/-2x LETF would maintain its leverage ratio and provide the expected return applied to the underlying index. As the holding period gets longer than this, investors should be wary of the relationship. Over the period of a quarter (3 months), the -2x LETF ratio begins to break down and stray away from the expected leverage ratio. For 2x LETFs, the break down seems to be more likely around the one-year mark.

In Trainor and Baryla (2008), Monte Carlo simulation is applied to analyze what the daily return sequences could look like for a 2x LETF in the long run. A daily return of 0.04% and daily standard deviation of 1.1% (≈ 10% yearly return, 20% annual standard deviation) were used. The results show that for a one year holding period the leveraged ratio drops to 1.4 times the underlying index. Looking at the performance of the ProShares 2x S&P 500 LETF compared to the S&P 500 in June 2007, the leverage ratio break down is visualized quicker and more significantly. After one week, the ratio drops to 1.93 and after just four weeks it drops to 1.29. For this being a relatively short holding period, this deviation from the expected leverage ratio of 2 is quite substantial but it is noted that the market volatility during this time was high, which strongly affects the decay of the leverage ratio.

Decay exists when the LETFs return over a particular time frame minus the leverage ratio of the underlying index multiplied by the underlying index return is negative (Trainor and Carroll, 2013). Decay is known to be influenced by the daily leverage ratio, time, the underlying index return and the volatility, volatility being the primary cause (Cheng and Madhavan, 2009). As most studies discourage using LETFs as a long-term investment tool, LETFs can actually be held for longer periods without experiencing extreme decay given the right market conditions. In fact, during periods of low volatility, LETFs can actually magnify underlying index returns to a leverage ratio greater than the expected ratio. As shown by Trainor (2011), with theoretical returns from periods of low volatility (VIX < 18.6%) the leverage ratio exceeds that of the expected ratio. From 1990-1999 the index averages 15.2% annual return, a theoretical 2x LETF averages 33.5% and a theoretical 3x LETF averages 55.5%. Even though this only includes low volatility months from the period it merely represents how there can be better than expected returns in certain market trends.

Portfolio Performances Holding LETFs in Some Capacity

It is noted that that due to the recent introduction of LETFs as an investment asset, there is a lack of study about the impact of holding them in a long-term portfolio. DiLellio, Hesse and Stanley (2014) use LETFs as a diversification tool in portfolios with three different allocations of stocks, bonds and a small amount (about 10%) in a -1x or 2x LETF. When applied to two different market trends, “rising” (1991-2000) and “flat” (2001-2010), the portfolios show signs of reducing the standard deviation. Terminal wealth of the portfolios did not show signs of risk-reward benefits but the Sharpe ratio did show positive signs of risk-adjusted return. DiLellio, et al. (2014) believe this study showed sufficient evidence to call into question the previous recommendation to not hold LETFs in a long-term passive strategy.

Trainor and Gregory (2016) attempt to show the results of using two basic option strategies with LETFs. The first is a covered call, when an investor owns the underlying asset and writes a call option on the asset, and the second is a protective put, when an investor owns the underlying asset and buys a put option on the asset. Using one-month call options with delta of .5, the strategies are applied to calculate annual returns. The study shows that both strategies reduce risk with standard deviation and provide a better risk-return tradeoff measured by higher Sharpe ratios. In some cases, significant returns are sacrificed though.

IV. Methodology

Data

This study explores the benefits of LETFs in a CPPI format. In the literature review the basics of a CPPI is demonstrated. Three different CPPI's are put to test using different risky assets and a different multiplier (m) based on the known risk of the risky asset. All of the CPPI's use a floor (F) of 90% of the total portfolio value, and this will be rebalanced annually. The risk-free asset being used in all the CPPI's will be the Treasury Ladder calculated as the average of the 1 year, 2 year, 5 year, 7 year, and 10 year treasury return.

What will be labeled as CPPI S&P, uses the SPDR S&P 500 ETF (SPY), which tracks the daily performance of the S&P 500, as the risky asset at a multiplier of 5. CPPI S&P, also referred to as the standard CPPI, is used for comparison to the CPPI portfolios containing a

LETF as the risky asset. CPPI 2x uses the ProShares Ultra S&P 500 ETF (SSO), which seeks 2x the daily performance of the S&P 500, as the risky asset at a multiplier of 2.5. CPPI 3x uses the ProShares UltraPro S&P 500 ETF (UPRO), which seeks 3x the daily performance of the S&P 500, as the risky asset at a multiplier of 1.6667. For reference using a portfolio value (V_P) of $100,000 see the table below:

CPPI	*Floor (F)*	*Cushion*	*Multiplier (m)*	*Allocation to Risky Asset*	*Allocation to Risk-free Asset*
CPPI S&P	$90,000	$10,000	5	$50,000	$50,000
CPPI 2x	$90,000	$10,000	2.5	$25,000	$75,000
CPPI 3x	$90,000	$10,000	1.6667	$16,667	$83,333

The discrepancy in the amount allocated to the risky asset is dictated by the leverage associated with SSO and UPRO. The exposure to market changes will remain the same but it allows less cash to be invested. The proportion in the risky asset at time t is calculated as the $\max\{\min[(m(V_P – F),V_P],0\}/V_P$ allowing the exposure to fluctuate between 0% to 100% depending on the movement of the assets and the overall portfolio performance. The exposure is rebalanced monthly to the value of the cushion multiplied by m, so that even if the market were to shed 20% over the month the floor would not be breached. As justified by Trainor, et al. (2016), since a 20% monthly drop has only occurred once post World War II (21% in October 1987), this is a reasonable position for even the most risk-averse investors. Table 1 demonstrates how the CPPIs are adjusted from month to month. The exposure is directly affected by the previous months' portfolio value, which is based off the performance of the risky asset and riskless asset. The floor is rebalanced annually to 90% of the V_P.

Table 1: Actual CPPI calculations based on the performance of the assets. Taken from the months of December 2009 to November 2010. It shows the performance of the underlying risky asset, the allocation to the risky asset, the portfolio value (V_P) and the floor (F) for all three CPPI strategies.

S&P Returns (SPY)	% in SPY	V_P	F		2x Returns (SSO)	% in SSO	V_P	F		3x Returns (UPRO)	% in UPRO	V_P	F
		1.000	.9				1.000	.9				1.000	.9
1.90%	0.500	0.998	.9		3.65%	0.250	0.992	.9		5.20%	0.167	0.990	.9
-3.63%	0.492	0.988	.9		-7.50%	0.233	0.986	.9		-11.19%	0.151	0.986	.9
3.12%	0.446	1.000	.9		6.11%	0.219	1.001	.9		8.94%	0.145	1.001	.9
6.09%	0.514	1.031	.9		12.37%	0.253	1.028	.9		18.90%	0.168	1.027	.9
1.55%	0.636	1.044	.9		2.81%	0.311	1.043	.9		4.08%	0.206	1.042	.9
-7.95%	0.691	0.992	.9		-16.09%	0.342	0.995	.9		-24.15%	0.227	0.997	.9
-5.17%	0.462	0.977	.9		-10.93%	0.239	0.982	.9		-16.58%	0.161	0.984	.9
6.83%	0.393	1.007	.9		14.07%	0.208	1.015	.9		21.17%	0.142	1.018	.9
-4.50%	0.530	0.990	.9		-9.26%	0.284	0.999	.9		-13.46%	0.194	1.004	.9
8.96%	0.453	1.031	.9		18.44%	0.249	1.047	.9		27.94%	0.173	1.054	.9
3.82%	0.635	1.056	.9		7.57%	0.350	1.075	.9		11.44%	0.244	1.084	.9

The data analyzed will be empirical results and theoretical results. The SPY has been available since 1993, the SSO since 2006 and the UPRO since 2009 and their return data was obtained through the Center for Research in Security Prices (CRSP). Return data on the S&P 500 including dividends and the Treasuries is also taken from the CRSP. When applying the CPPI to each of the risky assets it is important that they all begin on the same date for ease in comparison. For this, the CPPI S&P, CPPI 2x and CPPI 3x all begin on November 30, 2009 using empirical data. Monthly returns on the CPPIs under study are then calculated through December 31, 2015. To capture more empirical results, the CPPI S&P and the CPPI 2x are applied from June 30, 2006 for a separate comparison. This is to observe how the actual leveraged CPPI strategy would perform when experiencing a drop in the market (financial crisis in 2008).

With only being able to use roughly five years of empirical data (8 years on CPPI 2x), in which the market has generally only been increasing, it is important to establish theoretical data to be analyzed. The theoretical data used is from 1942 to 2015 and is used in the Monte Carlo simulation. Empirical daily returns from the S&P 500, which now represents SPY, are used to calculate the daily returns for a LETF seeking 2x the daily performance, like the SSO, and the daily returns for a LETF seeking 3x the daily performance, like the UPRO. This is done by a simple multiplication of 2 and 3 on the daily return of the S&P 500. Monthly returns are then calculated, which are used in the CPPI formulation. For robustness, Monte Carlo simulation is then used to produce 10,000 simulations where monthly returns sampled from the 1942 to 2015 data are used to create unique annual returns. Three succeeding months are sampled at a time to capture any possible momentum in the returns.

Empirical results, theoretical results and the Monte Carlo results are analyzed using the same return and risk measures. These measures include the average return, minimum return, maximum return, standard deviation, Sharpe ratio, Sortino ratio, Value at Risk (VaR), Excess Shortfall (ES) and Omega ratio. Cumulative prospect values are calculated for the theoretical data and Monte Carlo simulation data.

The average return is calculated as the sum of all the returns in the time frame divided by the count of all the returns in the time frame. Standard deviation measures the dispersion of the data from the mean return and reflects the historical volatility. The minimum will be the lowest return in the time frame and the maximum will be the highest return in the time frame. The Sharpe ratio addresses risk-adjusted return by analyzing the return in excess of the risk-free rate with the portfolio volatility represented by standard deviation. The higher the ratio, the better risk-adjusted return. Due to the existence of returns that are not normally distributed it is important to use further performance measures.

The Sortino ratio is a modification of the Sharpe ratio that only considers the downside deviation removing the aspect of "good volatility". It is more appropriate due to the fact that the portfolios at study are designed to mitigate large losses and that returns are not normally distributed. The Sortino ratio is written as,

$$\mathrm{S} = \frac{R - T}{\mathrm{TDD}} \text{ where } \mathrm{TDD} = \sqrt{\frac{1}{\mathrm{N}}\sum_{\mathrm{i}=1}^{\mathrm{N}}(\mathrm{Min}(0, \mathrm{X_i} - \mathrm{T}))^2}$$

where R is the return, T is the target return, N is the total number of returns and X_i is the ith return. The higher the ratio, the greater the return per unit of downside risk and the lower the probability of a large loss. Omega, reported by Keating and Shadwick (2002), shows similarities to the Sortino ratio measuring downside risk. Omega is the sum of the returns above a certain threshold divided by the sum of the returns below that threshold, which is represented by zero in this study. The Omega ratio is written as,

$$\Omega_{\mathrm{x}}[\Gamma] = \frac{\int_{\Gamma}^{\infty}(1 - F_x(x)dx}{\int_{-\infty}^{\Gamma} F_x(x)dx}$$

where F is the cumulative distribution function and Γ is the threshold return. Value at Risk measures the expected maximum loss with a given confidence level over a specific time frame. A 5% level is used in this study which leaves the VaR to calculate the loss that can be expected to be seen less than 5% of the time. As pointed out in Acerbi, Nordio and Sirtori (2001), Excess Shortfall represents the average loss beyond the VaR threshold to truly represent the severity of a dramatic loss. It addresses the "what if" factor and makes up for the discrepancies with the VaR calculation. Acerbi and Tasche (2002) confirm the appropriateness of this definition compared to other shortfall calculations.

Since there is an expectation that PIs appeal more to prospect theory investors, Cumulative Prospect Values (CPV) are calculated using the function and parameters set forth in Tversky and Kahnemen (1992). CPVs are calculated for the three CPPI strategies and the market for comparison.

Statistical Tests

Two-tail t tests are used to determine statistical significance between average returns. Opdyke (2007) puts forth a method providing Sharpe ratios a p-value when in comparison to determine statistical significance. The Sharpe ratio may not be the best risk measure in this context but ruling statistical significance is an important feature that can be performed on the Sharpe ratios. To test for differences in VaR, an unconditional coverage test is applied as put forth in Annert, Osselaer and Verstraete (2009) and it is written as,

$$(\frac{1}{N}\sum_{n=1}^{N} Hit_n - \alpha)/\sqrt{\alpha(1-\alpha)/N}$$

where Hit_n equals one if the leveraged CPPI return is lower than the S&P 500 VaR and the standard CPPI VaR, zero otherwise, N is the number of returns and α is the VaR percentile level, 5%. All statistical tests test for significant differences at the 90%, 95% and 99% confidence intervals between the three CPPI strategies and the S&P 500 and between the two leveraged CPPI strategies and the standard CPPI strategy.

V. Results

Theoretical Data

Table 2 shows the monthly performance and risk measures from 1942 to 2015 for the CPPI S&P, CPPI 2x and CPPI 3x using the theoretical data. The multipliers of 5, 2.5 and 1.667 respectively, determine the exposure to the risky asset which is rebalanced monthly. The remaining allocation is invested in the Treasury Ladder, which is on display. The floor is set to 90% of the portfolio value and is rebalanced annually. Theoretical data is provided on the respective risky assets as well, excluding the S&P, which is empirical data.

Table 2: Monthly performance and risk measures using theoretical data for the three CPPI strategies, their corresponding underlying risky assets and the risk-free asset, the Treasury Ladder. Data is observed from 1942 to 2015.

	T Ladder	S&P	2x	3x	CPPI S&P	CPPI 2x	CPPI 3x
Average	0.44%	1.01%	2.01%	3.01%	0.89%	1.07%	1.13% ^
St Dev	1.19%	4.16%	8.41%	12.72%	2.75%	3.31%	3.42%
Max	8.76%	16.72%	34.91%	54.49%	11.13%	17.91%	18.63%
Min	-4.66%	-21.61%	-43.72%	-64.73%	-21.61%	-31.64%	-29.81%
Sharpe	0.10	0.17	0.20	0.21	*0.21	**0.23	***0.24
Sortino	0.16	0.25	0.32	0.34	0.32	0.36	0.39
VAR 5%	-1.28%	-6.10%	-12.16%	-18.09%	***-3.19%	***-3.50%^^	***-3.61%^^
ES	-2.02%	-8.64%	-17.19%	-25.53%	-5.17%	-6.08%	-6.13%
Omega	3.04	1.86	1.85	1.84	2.38	2.52	2.59

*Significantly different from the S&P 500 at the 10% level or better, ** 5% level, *** 1% level.
^Significantly different from the CPPI S&P at the 10% level or better, ^^ 5% level, ^^^ 1% level.

In comparing the returns of the three CPPI strategies, CPPI 2x and CPPI 3x show better monthly averages over the standard CPPI S&P with CPPI 3x being significantly better than the standard CPPI S&P. CPPI 2x and CPPI 3x also outperform the S&P 500 which is somewhat surprising when considering the fact that the PI strategies are designed to mitigate downside risk by giving up some of the upside.

The Sharpe, Sortino and Omega ratios show superior risk-return performance for the CPPI 2x and CPPI 3x over the CPPI S&P. All three CPPI strategies dominate the S&P 500 in these categories showing significance with the Sharpe ratio. The minimums in the time frame are from October 1987 in which the market lost 21% and by excluding this month the minimums for the S&P 500, CPPI S&P, CPPI 2x and CPPI 3x are -16.71%, -13.81%, -12.88% and -12.04% respectively. These minimums show further downside protection over the S&P 500 and the standard CPPI S&P. The VaR and ES for the three CPPI strategies dominate that of the S&P 500. The VaRs of the leveraged CPPIs are statistically better than both the S&P 500 and the standard CPPI.

Overall, the CPPI strategies show positive signs of downside risk mitigation when compared to the S&P 500. Where returns are thought to have been slightly lower than the S&P 500 with the PI strategies, there is a slight outperformance in average return with the LETF CPPIs. Maximum monthly losses are reduced with the CPPIs. Figure 1 shows the cumulative ln growth of $1 for the three CPPI strategies and the S&P 500 from 1942 to 2015. The outperformance of the LETF CPPI strategies are visualized where the CPPI S&P trails slightly behind the S&P 500.

Figure 1: Cumulative ln growth of $1 for the three CPPI strategies using theoretical data and the S&P 500 from 1942 to 2015.

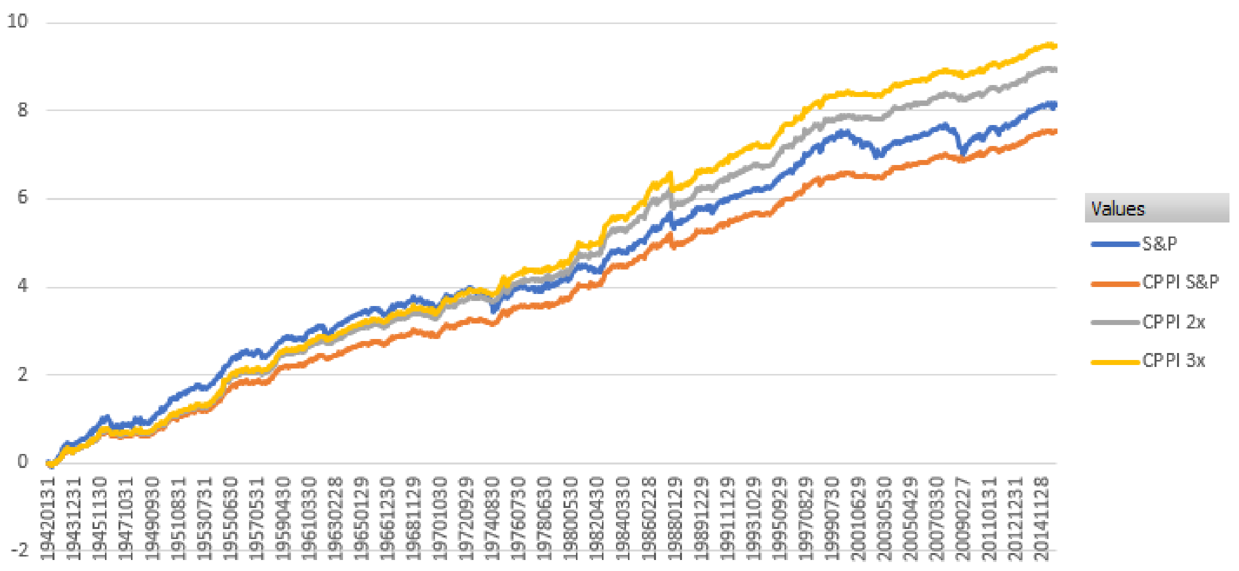


Table 3 displays the annual performance and risk measures using the theoretical data from 1942 to 2015. As annual trends can be more dramatic than monthly trends it may be more appropriate to analyze annual data on the market and the CPPI's.

Table 3: Annual performance and risk measures using theoretical data for the three CPPI strategies, their corresponding underlying risky assets and the risk-free asset, the Treasury Ladder. Data is observed from 1942 to 2015.

	T Ladder	S&P	2x	3x	CPPI S&P	CPPI 2x	CPPI 3x
Average	5.44%	13.00%	27.92%	45.05%	11.26%	13.75%	14.69%
St Dev	5.65%	17.11%	37.59%	62.56%	11.58%	15.16%	16.25%
Max	28.16%	52.85%	131.68%	248.21%	46.38%	66.67%	71.36%
Min	-2.64%	-36.65%	-66.09%	-84.77%	-5.58%	-11.06%	-9.33%
Sharpe	0.27	0.53	0.64	0.66	0.63	0.65	0.66
Sortino	0.70	1.03	1.58	1.96	1.98	2.53	2.89
VAR 5%	-1.27%	-12.96%	-27.12%	-41.32%	*-3.55%	*-3.62%	*-3.62%
ES	-2.26%	-25.08%	-46.89%	-63.76%	-4.68%	-6.45%	-5.96%
Omega	34.95	6.17	6.47	6.67	21.92	24.57	28.43

*Significantly different from the S&P 500 at the 10% level or better, ** 5% level, *** 1% level.
^Significantly different from the CPPI S&P at the 10% level or better, ^^ 5% level, ^^^ 1% level.

The theoretical annual performance and risk measures have results similar to that of the theoretical monthly results. The leveraged CPPIs outperform S&P 500 and the standard CPPI in terms of average return. Maximums for the leveraged CPPIs are better than the market and minimums are more than three times better.

Downside protection for the leveraged CPPIs is confirmed with stronger Sharpe, Sortino and Omega ratios. CPPI 2x more than doubles the S&P 500 Sharpe ratio and almost quadruples the Omega ratio. CPPI 3x almost triples the S&P 500 Sharpe ratio and more than quadruples the Omega ratio. The CPPI strategies provide significantly better VaRs and far superior ESs over the market, dramatically limiting the worst-case scenario returns for the PIs.

When assuming the existence of LETFs back to 1942, it is obvious that the CPPI 2x and CPPI 3x dominate the performance over the S&P 500 and provide extensive downside protection. In comparison to the CPPI S&P, performance is still stronger as is risk-adjusted return when observing the Sharpe, Sortino and Omega ratios. Higher CPVs, as seen in Table 4, elaborate on the satisfaction a prospect theory investor would experience using the leveraged CPPI strategies.

Table 4: Cumulative Prospect Values (CPVs) for the S&P 500 and three CPPI strategies using the Theoretical results from 1942 to 2015.

	S&P 500	CPPI S&P	CPPI 2x	CPPI 3x
CPV	0.092	0.143	0.163	0.175

Monte Carlo Simulation

To check on the robustness of the data it is important to run a Monte Carlo simulation on the theoretical data formulated from 1942 to 2015. The simulation calculates 10,000 annual data measures based on a random set of 12 months, taking 3 months at a time to capture any market momentum. The annual performance and risk measures in Table 5 are based on the 10,000 simulations. This removes particular trends and sequences from the historically based data.

Table 5: Annual performance and risk measures for the 10,000 Monte Carlo simulations. The simulations are calculated from the theoretical data from 1942 to 2015.

	T Ladder	S&P	2x	3x	CPPI S&P	CPPI 2x	CPPI 3x
Average	5.37%	12.70%	26.90%	42.92%	***10.41%	12.44%^^^	12.97%^^^
St Dev	4.80%	16.25%	36.73%	62.57%	11.81%	14.46%	15.06%
Max	32.30%	87.69%	246.35%	528.55%	78.81%	150.81%	203.75%
Min	-10.31%	-42.83%	-72.91%	-90.58%	-13.00%	-12.59%	-12.69%
Sharpe	0.30	0.54	0.63	0.62	0.55	**0.59^^	**0.60^^
Sortino	0.63	1.20	1.76	2.08	1.78	2.55	2.75
VAR 5%	-1.45%	-14.11%	-29.82%	-45.28%	***-4.91%	***-4.46%^^^	***-4.43%^^^
ES	-3.04%	-21.16%	-41.48%	-59.67%	-6.82%	-6.41%	-6.31%
Omega	29.17	7.33	7.18	7.19	16.81	23.67	25.31

*Significantly different from the S&P 500 at the 10% level or better, ** 5% level, *** 1% level.
^Significantly different from the CPPI S&P at the 10% level or better, ^^ 5% level, ^^^ 1% level.

The average annual returns confirm the results in the theoretical analysis with the CPPI 2x and CPPI 3x being statistically significant over the standard CPPI. The latter surprisingly outperforming the S&P 500. The CPPI minimums are better than the S&P 500 and the LETF CPPI minimums are greater than the standard CPPI. The Sharpe ratios of the leveraged CPPI's demonstrate a stronger risk-return performance with statistical significance over the standard CPPI and the S&P 500. The downside protection of the leveraged CPPI's are consistently better than the standard CPPI and substantially better than the S&P 500, as demonstrated by Sortino, VaR, ES and Omega. Table 6 contains the CPVs for the S&P 500 and the three CPPI strategies.

Table 6: CPVs for the S&P 500 and three CPPI strategies using the Monte Carlo results.

	S&P 500	CPPI S&P	CPPI 2x	CPPI 3x
CPV	0.091	0.127	0.161	0.167

The higher the CPV the better it is to be perceived by a prospect theory investor. Assuming the average investor is described by prospect theory, the CPPIs are superior to the S&P 500 and the leveraged CPPIs are preferred to the standard CPPI.

Empirical Results

With actual data being available on LETFs within the past 10 years, it is beneficial to apply the framework under study to analyze real results. In using the ProShares Ultra S&P 500 ETF (SSO) as the 2x LETF, the CPPI's can begin at the beginning of 2007. The SPDR S&P 500 ETF (SPY) is also substituted as the risky asset in the standard CPPI S&P. The framework and execution is consistent with the other studies and the results can be observed in Table 7.

Table 7: The monthly performance and risk measures for empirical data from 2007 to 2015. The CPPI S&P represents the strategy for using the SPY, which mimics the daily return of the S&P 500, as the risky asset and the CPPI 2x represents the strategy using the SSO, which mimics 2x the daily return of the S&P 500, as the risky asset. The S&P 500 and Treasury Ladder are also given.

	T Ladder	S&P	SPY	SSO	CPPI S&P	CPPI 2x
Average	0.32%	0.63%	0.62%	0.92%	0.54%	0.54%
St Dev	1.08%	4.51%	4.52%	9.24%	2.21%	2.20%
Max	4.64%	10.89%	10.91%	21.81%	4.64%	4.72%
Min	-2.24%	-16.71%	-16.52%	-34.93%	-5.56%	-6.10%
Sharpe	0.24	0.13	0.13	0.09	0.22	0.22
Sortino	0.45	0.18	0.18	0.13	0.33	0.34
VAR 5%	-1.43%	-7.71%	-7.55%	-16.21%	**-3.55%	**-3.52%
ES	-1.78%	-10.11%	-10.20%	-20.90%	-4.41%	-4.31%
Omega	2.21	1.44	1.43	1.29	1.82	1.84

*Significantly different from the S&P 500 at the 10% level or better, ** 5% level, *** 1% level.
^Significantly different from the CPPI S&P at the 10% level or better, ^^ 5% level, ^^^ 1% level.

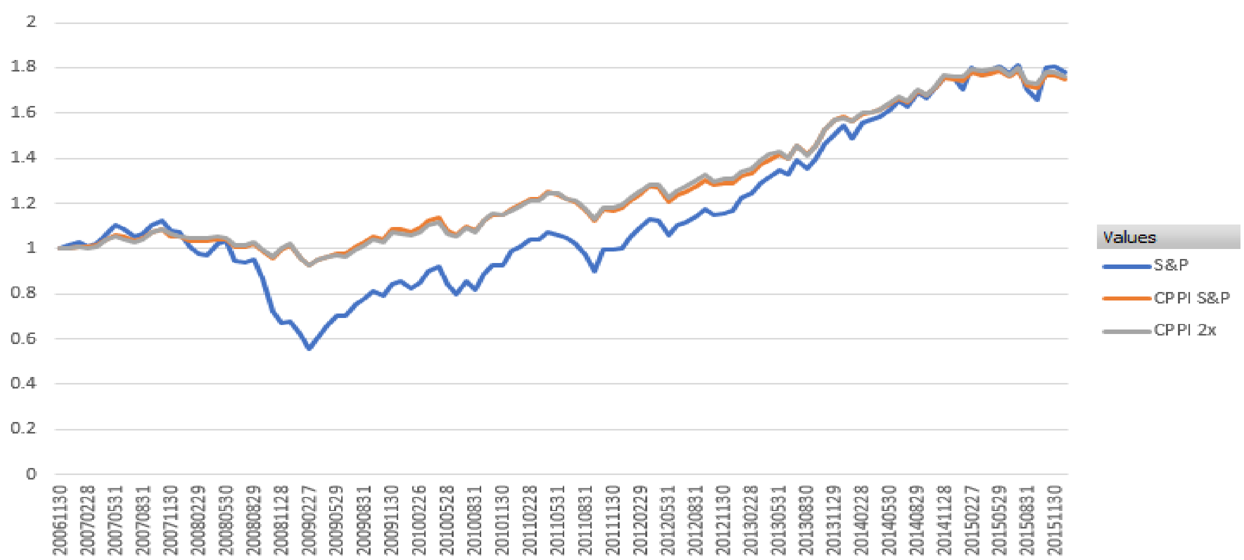
Empirical data is vital for analysis as it shows the consistent performance of the leveraged CPPI strategies. When dealing with such a small-time frame, little difference is expected between the market and the CPPI strategies but they do show positive loss mitigation addressing the housing market crisis in 2008 with lower volatility measured by standard deviation. This can be seen in the annual returns in Table 8 and the cumulative growth graph in

Figure 2. The CPPIs perform great in 2008 but do give up gains in the other years. But, in looking at the geometric annual returns it can be suggested that mitigating the large losses can make up for the smaller gains when the market is doing well. The CPPI return averages are in line with the S&P 500 and have better minimums. Sharpe, Sortino and Omega ratios are all better than the S&P 500. VaRs for the CPPIs are significantly better and Expected Shortfalls confirm.

Table 8: Annual returns for the S&P 500, CPPI S&P and the CPPI 2x using empirical data from 2007 to 2015. The annual geometric return is also given.

	S&P 500	CPPI S&P	CPPI 2x
2007	5.74%	4.70%	5.48%
2008	-36.65%	-3.36%	-3.36%
2009	26.21%	6.99%	4.33%
2010	15.12%	8.16%	9.97%
2011	1.84%	0.57%	1.69%
2012	16.04%	9.30%	10.11%
2013	32.32%	22.37%	20.25%
2014	13.60%	10.62%	11.61%
2015	1.45%	0.07%	-0.04%
Geo Annual Return	6.44%	6.37%	6.46%

Figure 2: Cumulative growth of $1 for the S&P 500, CPPI S&P and the CPPI 2x using empirical data from 2007 to 2015.



With the induction of the ProShares UltraPro S&P 500 ETF (UPRO) in mid-2009, empirical data is now provided on a 3x LETF. All three CPPI strategies are executed from 2010 to 2015, which is a small-time frame but still serves importance in analysis. Empirical monthly results are given in Table 9.

Table 9: The monthly performance and risk measures for the empirical data from 2010 to 2015. CPPI S&P, CPPI 2x and CPPI 3x are all given along with their respective risky asset, the Treasury Ladder and S&P 500. The CPPI 3x uses the UPRO as its risky asset, which seeks 3x the daily return of the S&P 500.

	T Ladder	S&P	SPY	SSO	UPRO	CPPI S&P	CPPI 2x	CPPI 3x
Average	0.21%	1.10%	1.09%	2.03%	2.97%	0.68%	0.70%	0.71%
St Dev	0.90%	3.76%	3.76%	7.66%	11.65%	2.14%	2.16%	2.19%
Max	2.22%	10.89%	10.91%	21.81%	32.61%	4.63%	4.72%	4.98%
Min	-2.24%	-8.02%	-7.95%	-16.09%	-24.15%	-5.05%	-4.55%	-4.36%
Sharpe	0.22	0.29	0.29	0.27	0.26	0.31	0.32	0.32
Sortino	0.38	0.51	0.51	0.46	0.44	0.52	0.56	0.56
VAR 5%	-1.34%	-5.65%	-5.70%	-11.81%	-17.96%	*-3.46%	*-3.48%	*-3.55%
ES	-1.75%	-6.75%	-6.75%	-13.81%	-21.20%	-4.31%	-4.01%	-3.97%
Omega	1.79	2.08	2.06	1.93	1.89	2.15	2.20	2.20

*Significantly different from the S&P 500 at the 10% level or better, ** 5% level, *** 1% level.
^Significantly different from the CPPI S&P at the 10% level or better, ^^ 5% level, ^^^ 1% level.

Following the housing market crisis in 2008, the market was sent into a relatively low volatile, bullish trend, which lasted a majority of this time-frame. As PIs are designed to mitigate large losses while giving up some upside, it makes sense that the S&P 500 outperformed all three CPPI strategies in average monthly return. The CPPIs still provided better downside protection with lower minimums than the S&P 500 while the leveraged CPPIs beat the standard CPPI by at least 0.5%. Strong portfolio insurance was also demonstrated by higher Sortino ratios, significantly better VaRs, better ESs and higher Omegas. Overall volatility was also lower with the CPPIs.

Additional Results

In examining the data, it was noticed that in some years, given great portfolio performance, the exposure in the S&P 500 for the CPPI S&P would max out at 100%. As this would happen, our exposures in our leveraged CPPIs would continue to increase as they had much smaller initial exposure to their risky asset. This would end up giving the leveraged CPPIs more than 100% exposure to the underlying market due to the leveraged they are providing.

For example, in 1997 the CPPI S&P reached an exposure of 100% in July. As the market continued to perform well throughout the year the exposure in the 2x and 3x LETF went to 69% and 47% respectively. This basically means that the CPPI 2x ended with 138% exposure to the S&P 500 and the CPPI 3x ended with 141% exposure to the S&P 500. This is a very risky position considering how susceptible the PIs would be to a dramatic decline before rebalancing could occur.

A perfect example of this occurred in 1987. The start of this year began with impressive gains leaving the CPPI S&P maxed out in 100% of the market by April. The market rallies continued and the CPPI 2x reached 75% exposure to the 2x LETF (≈ 150% exposure to the S&P 500) by September and the CPPI 3x reached 49% exposure to the 3x LETF (≈ 147% exposure to the S&P 500). The following month the market experienced its worst monthly decline post WWII (21% loss). With so much exposure to the market, all the CPPIs took a large hit but the leveraged CPPIs had it the worst due to their leveraged positions. The returns for this period were -21.60% for the CPPI S&P, -31.64% for the CPPI 2x and -29.81% for the CPPI 3x.

For this reason, a maximum exposure was tested for the leveraged CPPIs. For the CPPI 2x, the maximum exposure to the risky asset would be 50% and for the CPPI 3x, the maximum exposure to the risky asset would be 33.33%. This limits the exposure to the underlying market to 100% for all the CPPIs. The proportion in the risky asset for CPPI 2x at time t is now calculated as the $\min\{\max(\min[(m(V_P - F), V_P], 0)/V_P, 0.5\}$. The proportion in the risky asset for CPPI 3x at time t is now calculated as the $\min\{\max(\min[(m(V_P - F), V_P], 0)/V_P, 0.33\}$.

This new exposure limit was tested with all of the data. For the most part, the performance and risk measures were very similar compared to the results not using the exposure limit. The severe downsides were the only thing significantly affected, which makes the exposure limit very beneficial for this study.

VI. Conclusion, Limitations, and Recommendations

Conclusion

With the introduction of LETFs within the last decade, empirical data was not going to be sufficient enough for this study. By integrating theoretical data from 1942 to 2015. Substantial data was available for application in this CPPI framework. Monte Carlo results of 10,000 simulations added credibility to the theoretical results. With this being said, the minimal empirical data did serve a strong purpose in validating the CPPIs with real LETF returns and proving that the framework can be executed.

This study served to try and mitigate large losses while participating in upside gains. As standard CPPI adequately behaves like so, could a CPPI be even more efficient when using a LETF. Therefore, not only were the leveraged CPPIs compared to the market but also to the standard CPPI.

Using a 2x and 3x LETF in a CPPI context appears to give better returns with lower risk relative to the S&P 500. This is based on smaller standard deviations, higher Sharpe ratios, higher Sortino ratios, better VaRs, better ESs and higher Omega values. The leveraged CPPIs do lag in returns for the empirical results as a result of the strong performance of the market. As seen in the empirical results from 2007 to 2015, strong loss mitigation can make up for the smaller gains.

Relative to a standard CPPI strategy, return from using CPPI 2x and CPPI 3x is higher with possible increased risk considering standard deviation. However, Sharpe ratios, Sortino ratios and Omega ratios suggest using LETFs are superior to the composition of a standard CPPI. CPVs also argue that the average investor would prefer leveraged CPPIs over the standard CPPI and the S&P 500.

Limitations

As LETFs are relatively new investment tools, there is a lack of empirical data. This prompted reliance on theoretical data for much of the study. As the formulation of the theoretical data was practical, in the sense that LETFs simply attempt to mimic the daily performance of the underlying index by some ratio, minor discrepancies can be made in the actual management of these funds. The use of the limited empirical data was present to validate the results from the theoretical data.

In this study trading costs associated with the strategies were not acknowledged as neither were the expenses associated with holding the exchange-traded funds. Bid-ask spreads were also not considered. DiLellio et al. (2014) justify applying a 1% fee to the LETFs to address all of the issues mentioned above. For reference, the SPY has a 0.10% expense ratio, the SSO has a 0.89% expense ratio and the UPRO has a 0.94% expense ratio. It is know that the expense ratios for these funds are quite trivial now but it is hard to make the same conclusion on the expense ratios from the past.

Recommendations

There is a substantial amount of literature dismissing the use of LETFs in long-term portfolios. As this study has shown that there serves to be benefits in using LETFs in longer holding periods, it would be interesting to test them in different portfolio contexts, such as a constant ratio portfolio. There are several variables used in this study that could be adjusted to further analyze the results such as different risk-free assets, different Floors and different multipliers. A time-varying multiplier based on expected volatility, as put forth by Zieling, et al. (2014), could be interesting as Trainor (2011) recommends the use of predicting volatility to increase the performance of LETFs. Rebalancing based on percentage moves in the asset instead of time increments would be interesting to look at too. Any further research into the use of LETFs in long-term portfolios would be beneficial to many investors.

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