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An investigation of palindromes *and their place in mathematics*

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Introduction

What do the Honda Civic, the Mazda 626, and the Boeing 747 have in common? What about Weird Al's song *Bob*, the first name of Miley Cyrus' alter ego, and the 70s sensation Abba? What do all these things have in common? They all contain palindromes. While some people recognise a palindrome when they see one, fewer realise that a palindrome is a special case of a pattern and that these patterns are all around. Palindromes frequently occur in names, both of vehicles and people, and in music.

The traditional mathematical curriculum has often left palindromes out of the common vernacular. Consider question #4 from the 18th annual American Mathematics Competition-8 test in 2002:

> The year 2002 is a palindrome (a number that reads the same from left to right as it does from right to left). What is the product of the digits of the next year after 2002 that is a palindrome?

Notice that the question provides the meaning of the word *palindrome*. This test does not typically give hints, clues, or definitions for terms. However, in the case of palindrome, the test writers make an exception. The organisers of the test state, "A special purpose of the AMC 8 is to demonstrate the broad range of topics available for the junior high school mathematics curriculum" (Mathematical Association of America [MAA] 2009).

So where do palindromes fit in a school curriculum that wishes to incorporate the National Council of Teachers of Mathematics (NCTM) Standards? The NCTM Number and Operations Standard for grades 6–8 states that students should understand "numbers, ways of representing numbers, relationships between numbers, and number systems" (2000, p. 214). In the Australian Curriculum Assessment and Reporting Authority (ACARA) *Australian Curriculum: Mathematics* (ACARA, 2012) part of the description of the Number and Algebra content strand is "They build on their understanding of the number system to describe relationships and formulate generalisations." (ACARA, 2012). In this article, I present palindromes as a fun, interesting, and engaging way to work with numbers and make connections to events in life outside of school. In the following sections, I provide three areas that can be explored to engage students in the exciting world of palindromes.

Fun with numbers

Numbers provide people with endless interpretations. Adults concern themselves with credit scores, numbers representing body weight, figures in a check book, or retirement account balances. All of these experiences with numbers exist simply because we have a reason to pay attention to them. However, there is a lighter side to numbers that can be far more interesting and entertaining. And these interesting sequences of numbers have a name: palindromes!

Consider the following statement: "Most of the adults you know have lived through two palindromic years." When was the last generation of people alive where a majority of the adults lived during two palindrome years? This question can be further expanded to consider people who live by calendars other than our Gregorian calendar.

Another fun experience with dates is birthdays. What month/day combinations are possible with palindromes? Many other countries, for example the United Kingdom, reverse the order of the month and day in their standard representation of dates. How does this change a palindrome date? Also, digital clocks provide yet another opportunity to find palindromes. A small amount of investigation reveals that palindromes are plentiful in our numeric representations of dates and times.

Products

Eleven is the first two-digit palindromic positive integer. Eleven multiplied by the numbers 1–9 produces palindromes that many students encounter in the later years of elementary school as they are working with multiplication facts (see Dockweiler 1985 for similar ideas using sums). Schiller and Charles (2004) conclude their article *Moving Forward and Backward with Palindromes* by asking how many times you can multiply 11 and produce a row of Pascal's triangle. The first four powers of 11 produce palindromes that may interest middle school students whose teacher introduces them to this peculiarity. However, an extension to this idea reveals new patterns not found in Pascal's triangle. The first three powers of 111 also produce palindromes (see Table 1). This pattern with numbers containing only digits with 'ones' leads to a conjecture: "Do only the first two powers of 1111 produce palindromes?" Checking

Table 1. Powers of numbers with only ones as digits. Exponents extend as long as the resulting product is a palindrome.

Base	Exponent	Product
11	1	11
11	2	121
11	3	1331
11	4	14 641
111	1	111
111	2	12 321
111	3	1,367,631
1111	1	1111
1111	2	1 234 321
11 111	1	11 111
11 111*	2	123 454 321
111 111	1	111 111
111 111	2	12 345 654 321
1 111 111	1	1 111 111
1 111 111	2	1 234 567 654 321
11 111 111	1	11 111 111
11 111 111	2	123 456 787 654 321
111 111 111	1	111 111 111
111 111 111	2	12 345 678 987 654 321

* indicates the last row that can be computed on a TI-84 Plus calculator without encountering scientific notation

Wolfram Alpha (www.wolframalpha.com with queries such as "11 111 × 11 111"), the conjecture appears to be true. You and your students can continue the conjecture and use Wolfram Alpha to confirm the pattern. The pattern ends after one hundred eleven million one hundred eleven thousand one hundred eleven as shown in Table 1. Amazing! While teachers can use multiples of eleven to introduce palindromes, activities using Pascal's triangle offer another introductory context for this investigation (see Schiller & Charles, 2004; Lemon, 1997).

With tools such as Wolfram Alpha able to compute incredibly large numbers, any class with access to a computer and an Internet connection can check conjectures that arise naturally. In the past, conjectures with numbers of such magnitude would require a computer lab, a programming language, and a carefully written set of syntax (Donahue, 1984; Lulli, 1983). Palindromes of this sort and any very large numbers in general, are now easily accessible to all students.

2,	3,	5,	7,
11,	13,	17,	19,
23,	29,	31,	37,
41,	43,	47,	53,
59,	61,	67,	71,
73,	79,	83,	89,
97,	101,	103,	107,
109,	113,	127,	131,
137,	139,	149,	151,
157,	163,	167,	173,
179,	181,	191,	193,
197,	199,	211,	223,
227,	229,	233,	239,
241,	251,	257,	263,
269,	271,	277,	281,
283,	293,	307,	311,
313,	317,	331,	337,
347,	349,	353,	359,
367,	373,	379,	383,
389,	397,	401,	409,
419,	421,	431,	433,
439,	443,	449,	457,
461,	463,	467,	479,
487,	491,	499,	503,
509,	521,	523,	541,
547,	557,	563,	569,
571,	577,	587,	593,
599,	601,	607,	613,
617,	619,	631,	641,
643,	647,	653,	659,
661,	673,	677,	683,
691,	701,	709,	719,
727,	733,	739,	743,
751,	757,	761,	769,
773,	787,	797,	809,
811,	821,	823,	827,
829,	839,	853,	857,
859,	863,	877,	881,
883,	887,	907,	911,
919,	929,	937,	941,
947,	953,	967,	971,
977,	983,	991,	997.

Figure 1. Primes less than 1000.

Primes

As fun with numbers continues, prime numbers are an important and intriguing set of numbers to investigate. Consider the prime numbers up to 1000. How many, or what percent of these, would you suspect to be palindromes? Again, Wolfram Alpha allows the query "primes <=1000" to generate a list of primes less than or equal to 1000. Observing the list, we see 168 prime numbers. Examining the palindromes in this list: 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, and 929, we see there are 16 palindrome primes less than 1000. This is close to 10% of the total number of primes in that range. An interesting follow-up question may be, "Would any list of consecutive primes contain 10% palindromes?" I leave it to the reader to determine the values for which this conjecture holds, as well to explore further conjectures.

Fun with words and language

Words provide another context from which to investigate palindromes and are actually the original context for palindromes. An Internet search of "palindromes" reveals approximately 704 000 hits, most dealing with words and sentences. Consider the mathematically-related palindrome phrase 'never odd or even'. Interestingly enough, many palindrome sentences make sense. Are there numbers in mathematics in which the phrase 'never odd or even' makes sense? By the middle school years, students should be able to provide entire sets of numbers that are never odd or even (e.g., π, $\sqrt{2}$, any non-integer) by calling on their knowledge of number systems. By definition, odd or even refers only to integers, which offer a chance to discuss the value of paying attention to definitions.

Another twist to palindrome words is to combine them with numbers. For instance, I know a mailbox with a combination lock whose sequence is 31-24-31. Looking at this, you do not see an obvious palindrome. However, say the words "thirty-one, twenty-four, thirty-one" and you will hear the three-part sequence that has the same beginning and ending, with one part in the middle. Would you call this a palindrome? Of course, a combination such as 31-8-13 appears visually as a palindrome, but saying "thirty-one, eight, thirteen" does not produce the palindromic speech sounds from the first example. The mathematical connection here is dependent on the definition being utilised.

What do we define to be a palindrome? Originally, the Greeks described certain words as palindromes around 300 BC, and the people of the Indian sub-continent used the term for numbers in AD 850 (Schiller & Charles, 2004). This serves to illustrate the role definition plays, not only in mathematics, but also within language in general. Teachers can expand the unique topic of palindromes to allow students to determine ways of defining types of palindromes. In the final section, I offer my own definition of palindrome as related to geometry.

Fun with shapes

Geometry may be an unexpected place to look for a palindrome, considering that a palindrome is a word, phrase, or number that is the same forward as backward. In antiquity, the term palindrome was originally used for words and later used for numbers. More recently, entire books and websites were created to document phrases that were palindromes. But if we extend the idea of a palindrome, something that is the same forward as it is backward, we can extend our mathematical connection to one further strand: geometry.

The NCTM Standard for geometry in grades 6–8 states that students should "apply transformations and use symmetry to analyse mathematical situations." The very concept of symmetry itself is a visual palindrome that uses neither words nor numbers. From a given line of symmetry, an object's halves are the same forward as backward. Figure 2 presents a square with a line of symmetry drawn through it. A line of symmetry, by definition, creates the position where a mirror could be set to reflect half of the image and still complete the image. Figure 3 presents a pentagon with one line of symmetry drawn in. Using geometry to provide "visual palindromes" with lines of symmetry allows another connection to be created between number and geometry.

Perhaps it is this relationship with symmetry that makes palindromes appealing. Symmetry is a natural form of beauty.

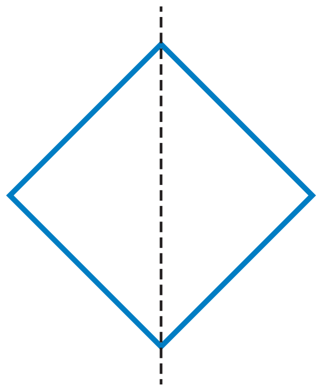

Figure 2. A square with line of symmetry drawn.

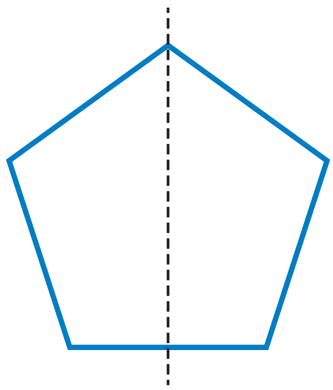

Figure 3. A pentagon with line of symmetry drawn.

Tying it all together

So how do words, numbers, and shapes converge in respect to palindromes? I begin with a simple word, such as MOM. Mom (in all capitals) is a nice palindrome to start with since a line of symmetry can be drawn through the

O, and no changes must be made to the Ms (Note: This is a font-dependent example).

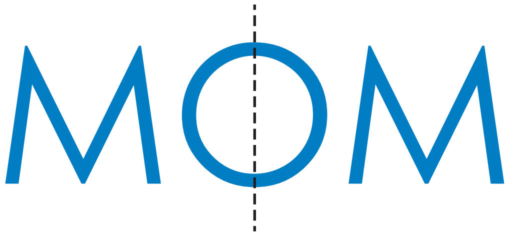

Figure 4. The word "MOM" written so that a vertical line of symmetry is possible.

A further conversion of MOM would be to change the letters to numbers. M is the 13th letter of the alphabet, and O is the 15th letter. As numbers then, MOM would be 13-15-13. This palindrome is similar to the combination of my mailbox example earlier.

Another word, such as DAD, provides a basis for both numbers and geometry. As numbers, DAD is simply 4-1-4. This numeric palindrome works in the traditional definition as well as in the spoken definition. However, in the geometric definition DAD encounters a new problem. As you can see in Figure 5, the visual palindrome is not quite right. One of the D's needs to be rotated or reflected. Figure 6 presents the necessary modifications to make DAD a visual palindrome. I reflected the second D, but I could have used a rotation as well. Furthermore, the first D could have been modified to meet the requirements for DAD to become a geometric palindrome.

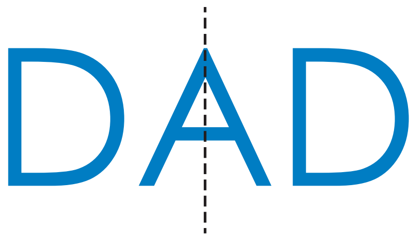

Figure 5. The word "DAD" written in an attempt to draw a line of symmetry.

Figure 6. The word "DAD" written with one transposed letter so that a line of symmetry can be drawn.

As an extension activity, teachers and students can explore the concepts of symmetry and other transformations on palindromic words. For instance, some words have another line of symmetry when written vertically. Consider the palindrome WOW as shown in Figure 7. Lines of symmetry can be drawn whether WOW is written vertically or horizontally. Notice that if DAD were written vertically, this line of symmetry would not exist.

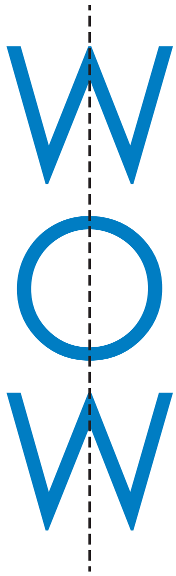

Figure 7. The word "WOW" written vertically with line of symmetry drawn.

Finally, palindromes offer an element to teaching mathematics that is always appreciated: motivation. Whether it is the idiosyncrasies of powers of 11, investigations into sets of numbers such as primes, combinations on locks or basic shapes in geometry, palindromes offer a wonderful context from which to start. In my teaching experience, I have had students approach me and say things like, "My birthday is December 21. I never realised what a cool birthday that is!" This date can be written as 12/21 in the United States, and as 21/12 in many other countries making this date especially unique. The experience of having students find new interest in a subject you love is a particularly rewarding aspect of being a teacher.

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